

Synchrotron Radiation: Production & Properties

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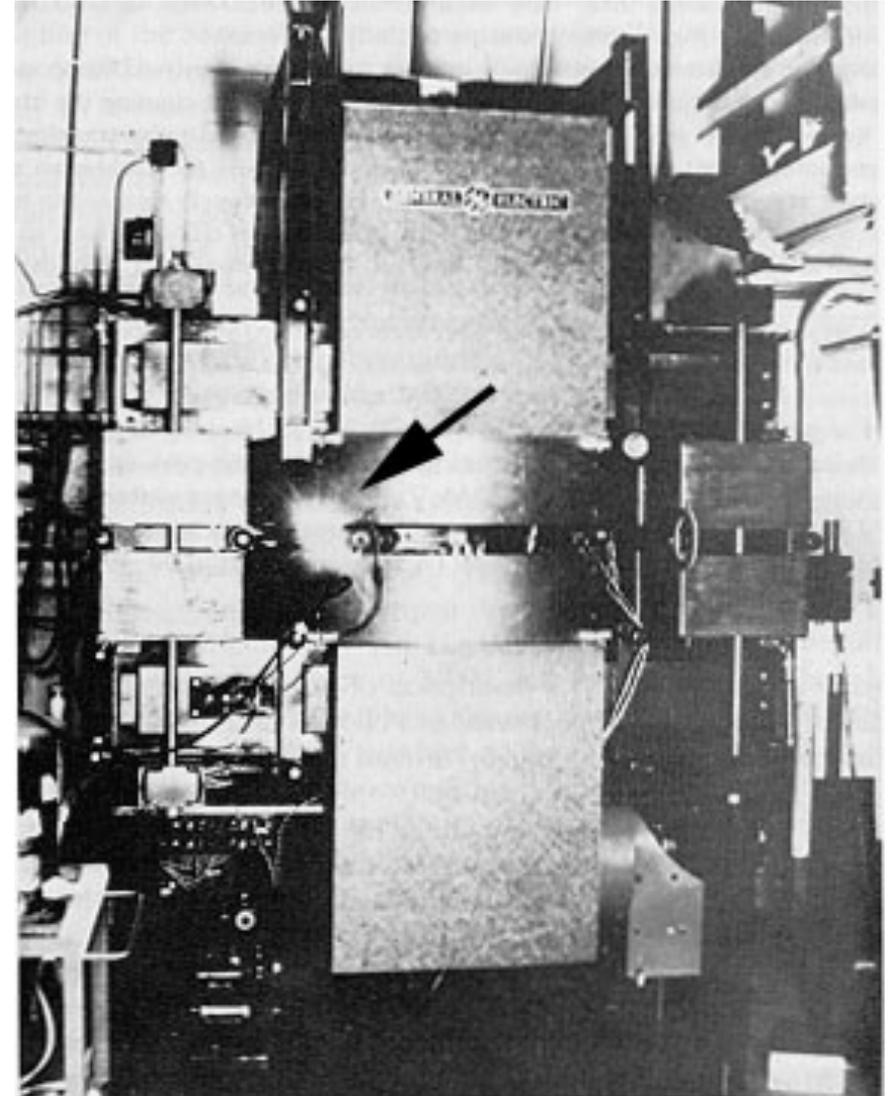


Synchrotron Radiation - Some Background

Synchrotron Radiation (SR) - radiation from charged particles traveling in circular orbits - was first observed from a 70 MeV synchrotron at GE in Schenectady in 1947.

On April 24, [1947] Langmuir and I [Herbert Pollack] were running the machine... Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.

Excerpted from Handbook on Synchrotron Radiation, Volume 1a, Ernst-Eckhard Koch, Ed., North Holland, 1983.



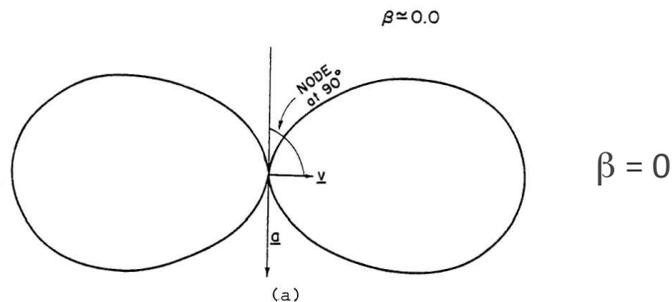
Radiation Patterns From Accelerating Charges

Definitions:

$$\beta = v/c$$

$$\gamma = 1/\sqrt{1-\beta^2} = E/m_0c^2$$

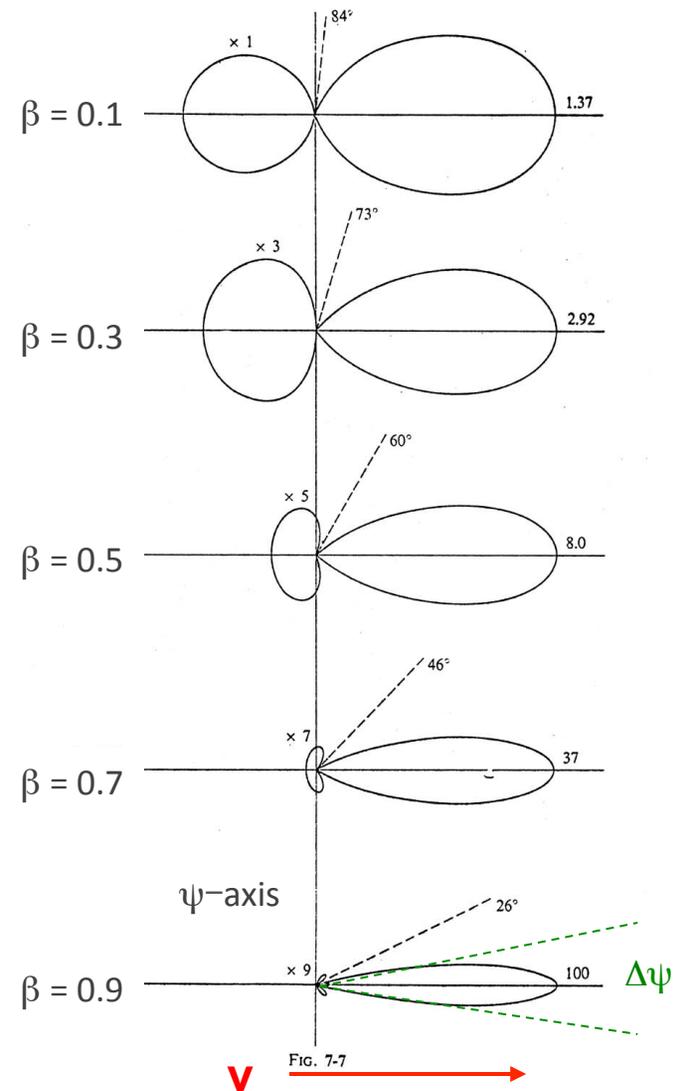
When $v \ll c$, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.



But as β approaches 1:

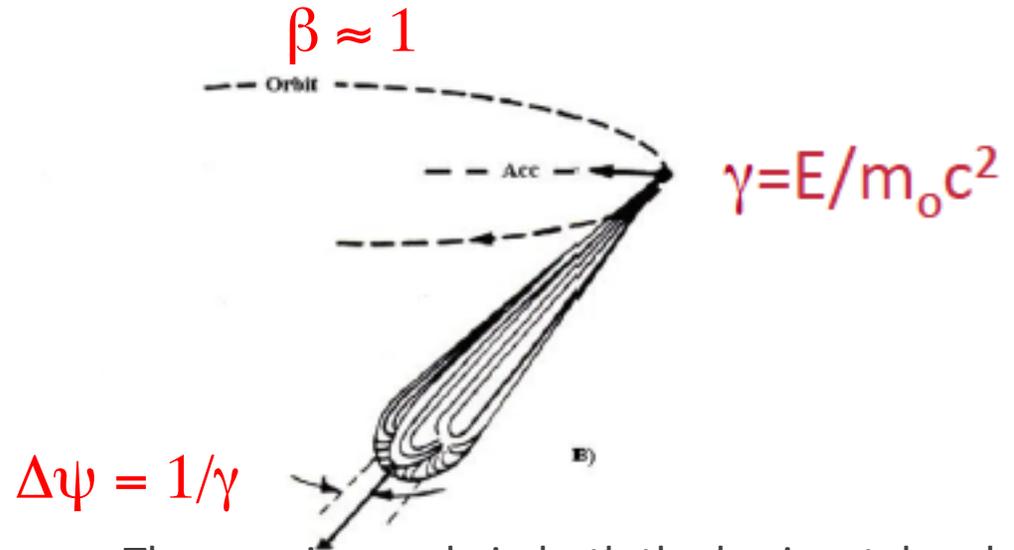
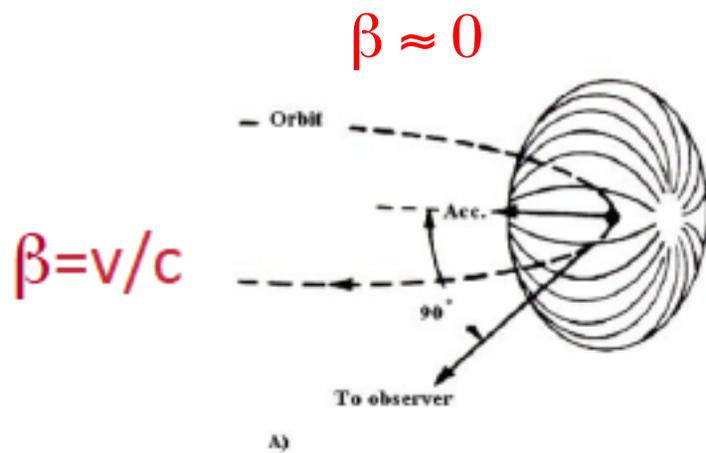
- The shape of the radiation pattern changes; it is more forward directed

$$\tan(\psi)_{lab} = \frac{\sin \psi'}{\gamma(\cos \psi' + \beta)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$



The angular divergence of the radiation (sometimes called the opening angle), $\Delta\psi$, is approximately $1/\gamma$.

Radiation from Highly Relativistic Particles



At the APS with $E = 7 \text{ GeV}$,

$$\gamma = E/m_0 c^2 = 7 \text{ GeV}/0.511 \text{ MeV}$$

$$\gamma = 1.4 \times 10^4$$

$$1/\gamma = 73 \times 10^{-6}$$

- The opening angle in both the horizontal and vertical directions is given approximately by:

$$\Delta\psi_{\text{vert}} = \Delta\psi_{\text{hor}} \approx 1/\gamma,$$

when $\beta \approx 1$.

- Relativistic velocities are good!!
 - radiation forward directed
 - radiated power $\propto E^4$

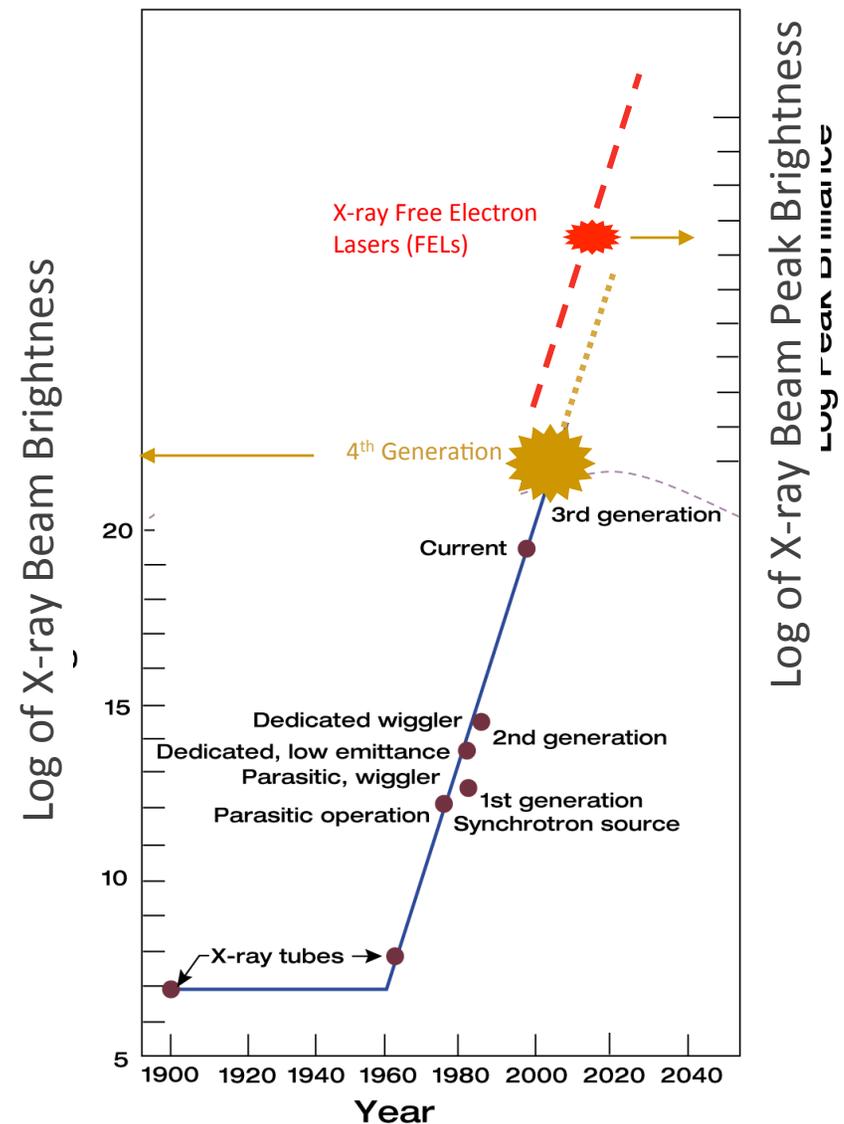
See Appendix 1 for more details

The Evolution of Brightness from SR Sources

Synchrotron radiation (from IR to hard X-rays) has been used as a research tool for nearly 50 years.

- 1st Generation Sources
 - Ran parasitically on accelerations for high-energy physics (CHESS and SSRL originally)
- 2nd Generation Sources
 - Built to optimize synchrotron radiation from the bending magnets (NSLS)
- 3rd Generation Sources
 - Built to optimize synchrotron radiation from insertion devices (ALS, APS, NSLS II [const.])
- 4th Generation Sources
 - fully coherent sources
 - X-ray Free Electron Lasers (X-FELs)
 - Diffraction Limited Storage Rings (DLSRs)
 - FEL-Oscillator (FEL-O)
 - Energy Recovery Linacs (ERLs)

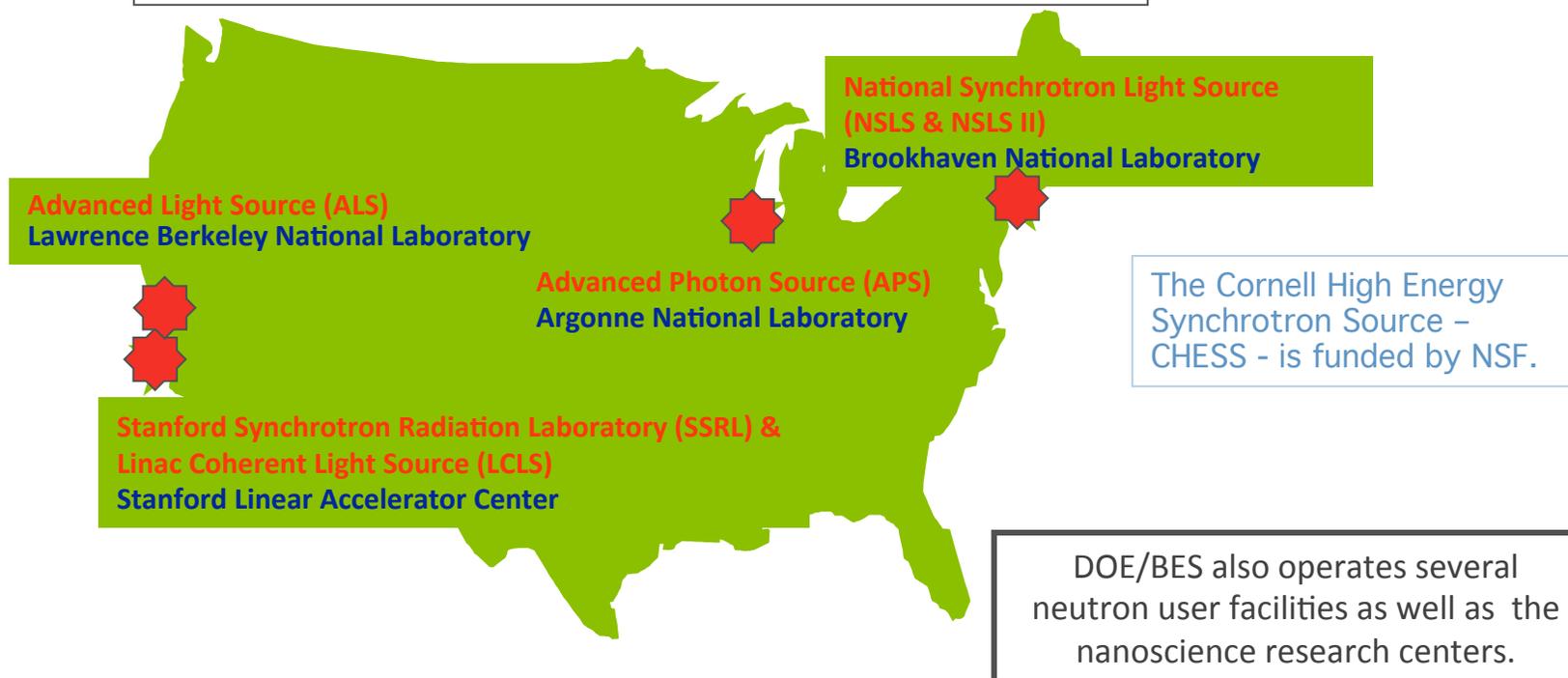
History of (8-keV) X-Ray Sources



A Vital National Resource for Science and Technology

The **Advanced Photon Source (APS)** is a fully optimized, insertion-device-based, third-generation x-ray source for the production of high intensity (brightness) hard x-ray beams.

ALS, APS, NSLS & SSRL are funded by the Department of Energy, Office of Science, Basic Energy Science (DOE/BES).



Over **5500 unique researchers** from around the world came to Argonne last year to perform experiments at the APS.

The Anatomy of Synchrotron Radiation Facility

ADVANCED PHOTON SOURCE BEAM ACCELERATION & STORAGE SYSTEM

(A) ELECTRON GUN

(B) ELECTRON LINEAR ACCELERATOR
Output energy: **375 MeV**

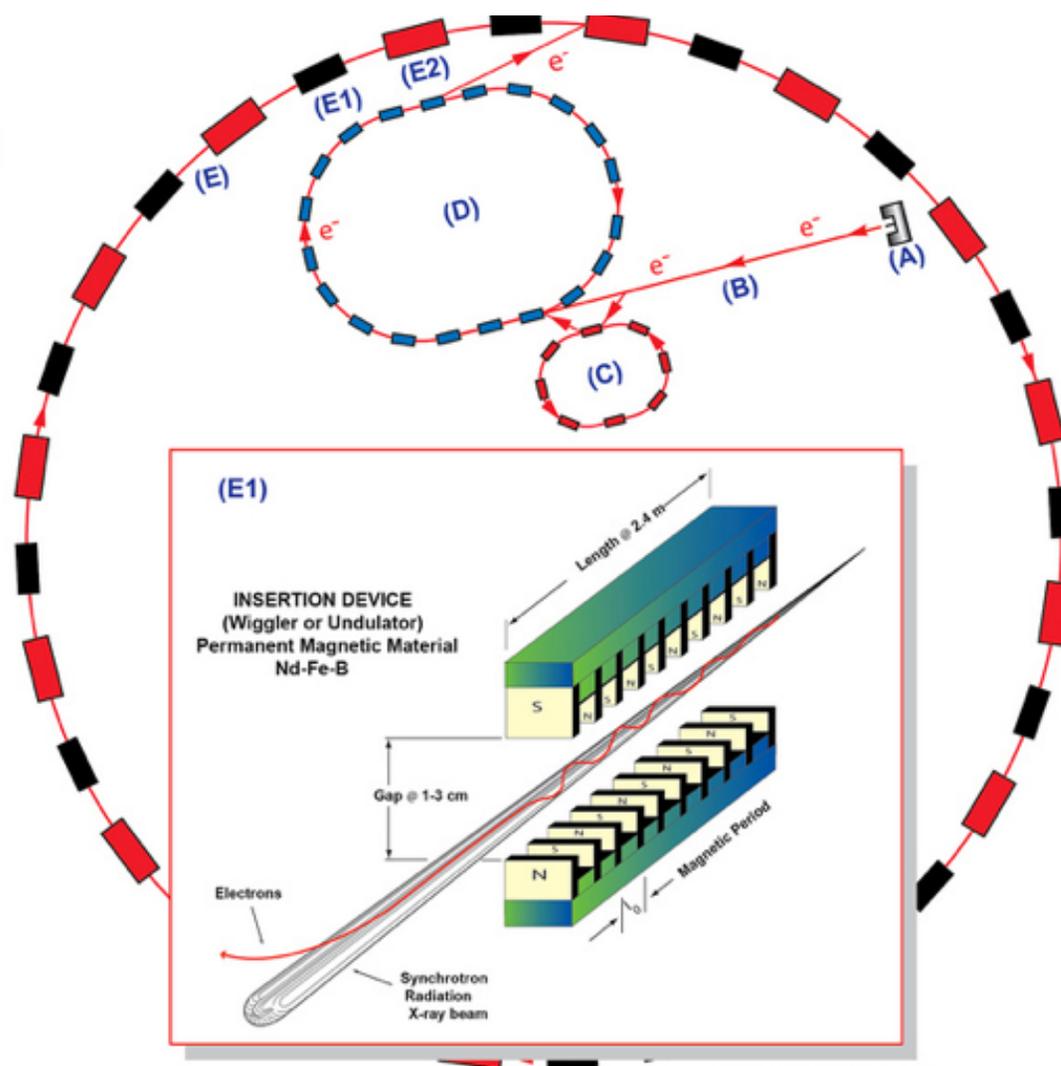
(C) PARTICLE ACCUMULATOR RING

(D) BOOSTER SYNCHROTRON
Nominal extraction energy: **7.0 GeV**

(E) STORAGE RING
Nominal energy: **7.0 GeV**

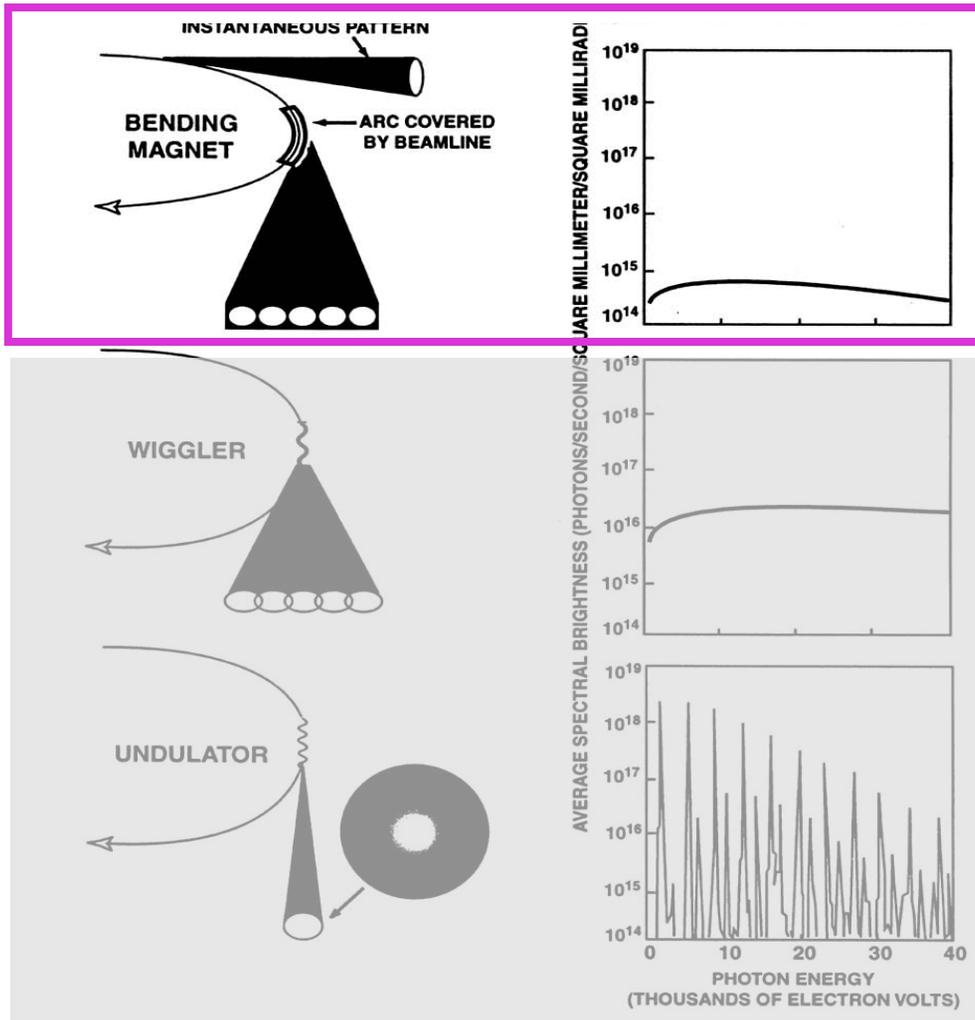
(E1) INSERTION DEVICE

(E2) BENDING MAGNET



NOTE: Diagrams not to scale

BM Spectral Properties



Bend Magnet Radiation

- Spectrum characterized by the critical energy:

$$E_c = 3hc\gamma^3/4\pi r \quad (E_c \approx 20 \text{ keV @ APS})$$

$$= 0.66 \text{ B[Tesla]} E^2 [\text{GeV}]$$
- Vertical opening angle ($\Delta\psi_v$) is $1/\gamma$.
 At the APS:

$$\gamma = E/m_0c^2 = 7 \text{ GeV}/0.511 \text{ MeV}$$

$$\gamma = 1.4 \times 10^4$$
 so $1/\gamma = 73 \times 10^{-6} \text{ radians}$
- Horizontal opening angle determined by apertures
- In the plane of the orbit, the polarization is linear and parallel to the orbital plane.

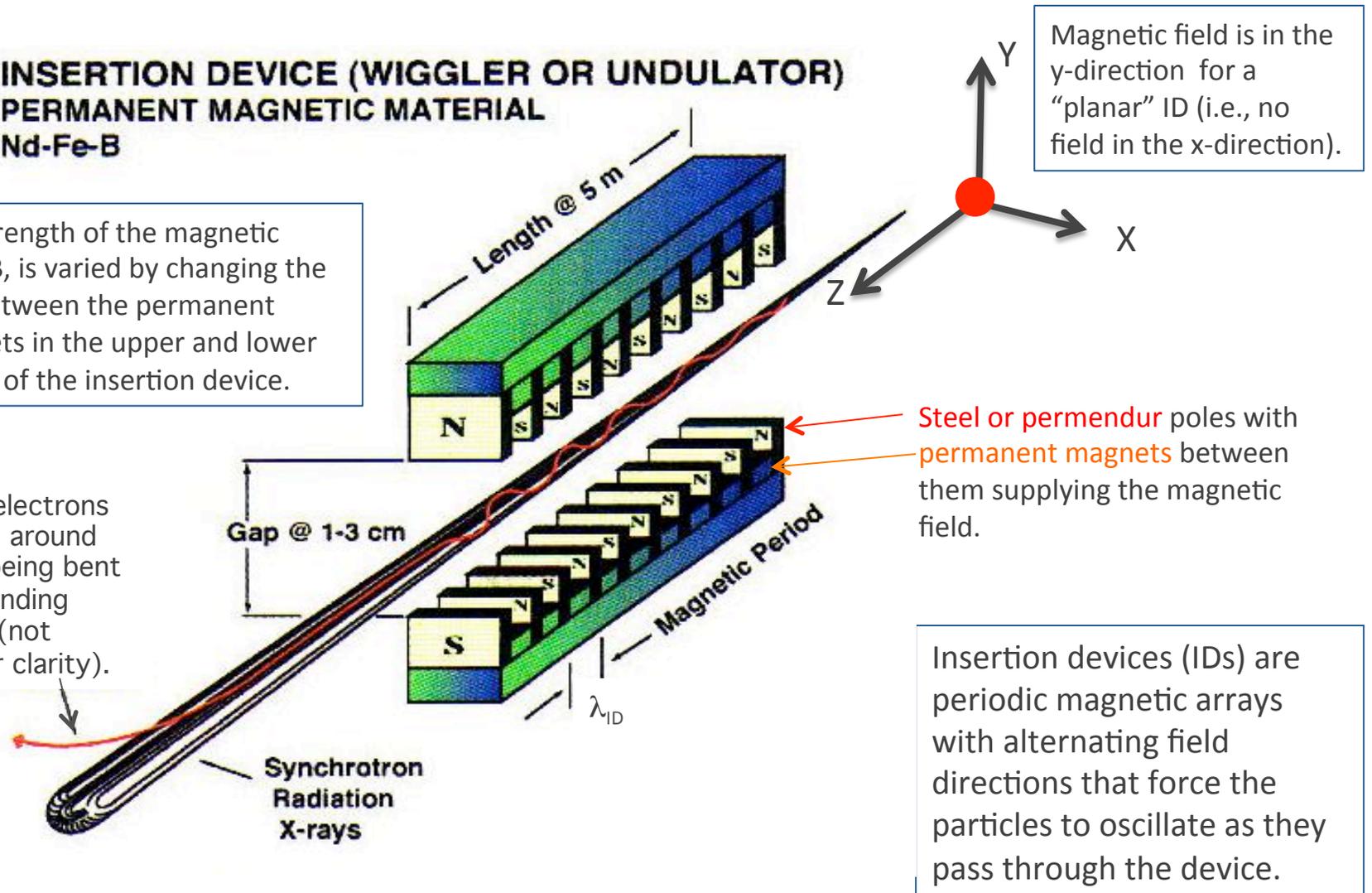
See Appendix 2 for more details

Planar Insertion Devices

INSERTION DEVICE (WIGGLER OR UNDULATOR) PERMANENT MAGNETIC MATERIAL Nd-Fe-B

The strength of the magnetic field, B , is varied by changing the gap between the permanent magnets in the upper and lower halves of the insertion device.

Beam of electrons continues around the ring being bent by the bending magnets (not shown for clarity).



Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

Characterizing Insertion Devices

- IDs are characterized by the so-called field index or deflection parameter, K (See Appendix 3):

$$K = eB_0\lambda_{ID}/2\pi m_0c = 0.0934 \lambda_{ID}[\text{cm}] B_0[\text{kG}]$$

where λ_{ID} is the period of the insertion device and B_0 the peak magnetic field. (The length of the ID, L , is equal to the number of periods, N , times the length of the period, i.e., $L = N\lambda_{ID}$.)

- The maximum deflection angle of the particle beam, θ_{\max} , is given by:

$$\theta_{\max} = \pm(K/\gamma)$$

and the amplitude of the oscillation of the particles, x_{\max} , by:

$$x_{\max} = (K/\gamma)(\lambda_{ID}/2\pi)$$

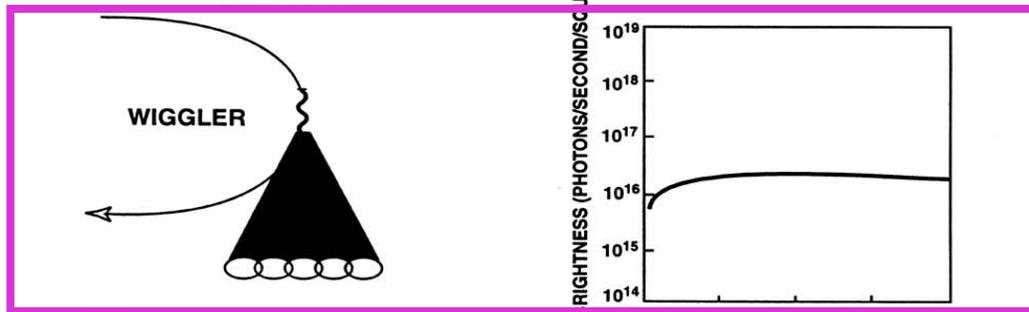
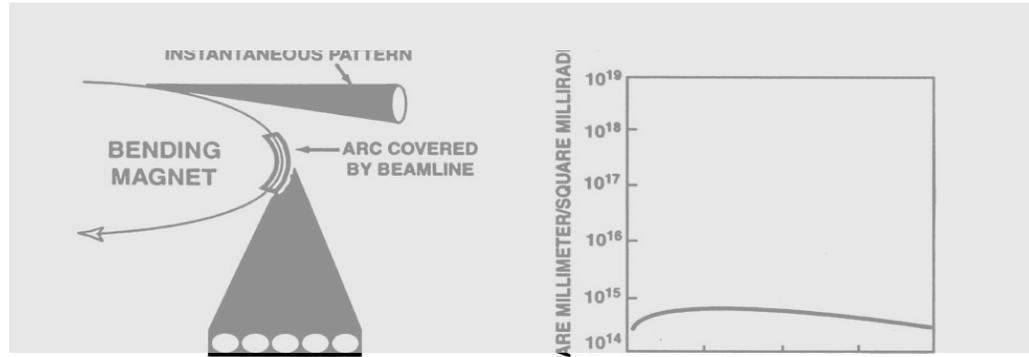
APS Undulator A has a period of 3.3 cm and operates with $K \approx 1$, therefore:

$$\theta_{\max} \approx 1/\gamma \quad \text{and} \quad x_{\max} \approx 0.38 \text{ microns.}$$



Wiggler Radiation Pattern and Spectrum

Wiggler Radiation ($K \gg 1$)



- Radiation spectrum looks like $2N$ dipole sources (N = number of periods)
- $\theta_{\max} = (K/\gamma) \gg 1/\gamma$, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation ($1/\gamma$).
- Spectrum characterized by the critical energy (which may be different than BM critical energy)
- Presently, there are NO planar wigglers installed at the APS.
- Wigglers with fields in both the horizontal and vertical directions produce elliptically polarized radiation. These are sometimes called elliptical multipole wigglers (EMWs).

(a) Wiggler



(b) Undulator



Undulators

Undulators:

- $K \approx 1$
- θ_{\max} is comparable ($1/\gamma$) and so the radiation from each pole overlaps causing interference effects in the spectral distribution.
- On-axis ($\psi_v = \psi_H = 0$), the constructive interference occurs at a particular x-ray energy (wavelength) when:

$$\lambda_n^{\text{x-ray}} = (\lambda_{\text{ID}}/2\gamma^2 n)(1 + K^2/2)$$

- The wavelength, $\lambda_n^{\text{x-ray}}$, where constructive interference occurs, can be adjusted by varying the magnet field, B , in the undulator.

$$K = 0.0934 \lambda_{\text{ID}}[\text{cm}] B_o[\text{kG}]$$

- Since most undulators are made with permanent magnets, the B-field is changed by changing the gap between the magnets.

APS 2.4 m long Undulator A ($\lambda_{\text{ID}} = 3.3 \text{ cm}$)



(a) Wiggler



(b) Undulator



Tuning the Peaks of Undulator Radiation

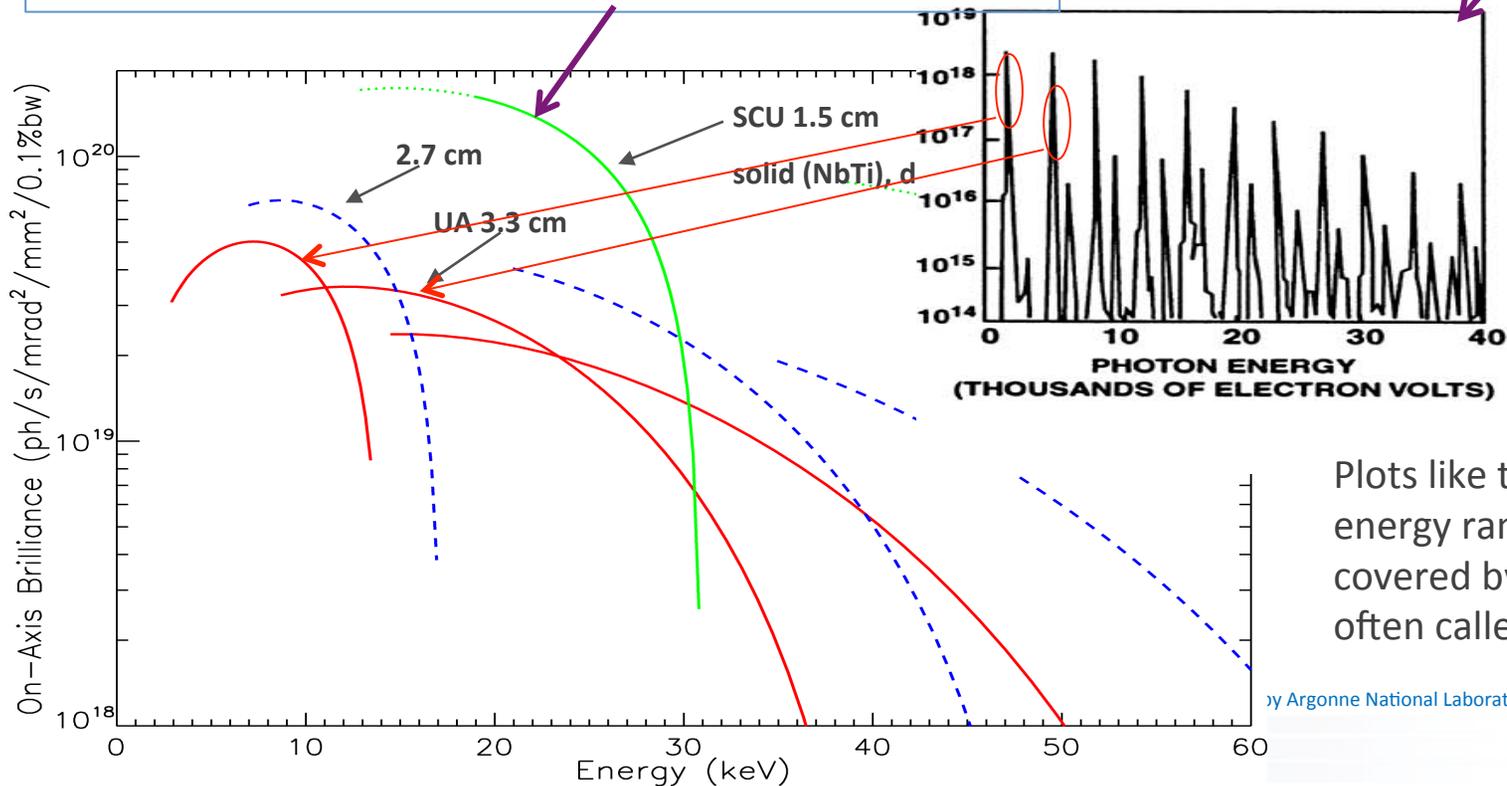
On-axis ($\psi_v = \psi_H = 0$), the constructive interference occurs when:

$$\lambda_n^{\text{x-ray}} = (\lambda_{ID}/2\gamma^2n)(1 + K^2/2)$$

where n is the harmonic number ($n=1$ is the fundamental). $\lambda_n^{\text{x-ray}}$, can be adjusted by changing the gap, which changes K since $K = 0.94 \lambda_{ID}[\text{cm}] B_o[\text{Tesla}]$.

Here is the spectrum over the tuning range of the undulator. The low energy is fixed by the minimum magnet gap (determined by the vacuum chamber). As the gap is opened, the x-ray energy increases, but as you open it further, the magnetic field decreases and so does the intensity of the x-rays.

Here is the spectrum for a particular gap (i.e., a particular K value).



Plots like this that show the energy range that can be covered by an undulator are often called "tuning curves".

Undulator Radiation

- Undulators defined as IDs with deflection angle $\approx 1/\gamma$, i.e., $K \approx 1$
- The spectrum from an undulator is spiky, but peaks are tunable by varying K ($K = 0.94 B[T] \lambda_{ID}[cm]$)
- At the peaks (harmonics) the horizontal and vertical opening angles of the radiation is given by:

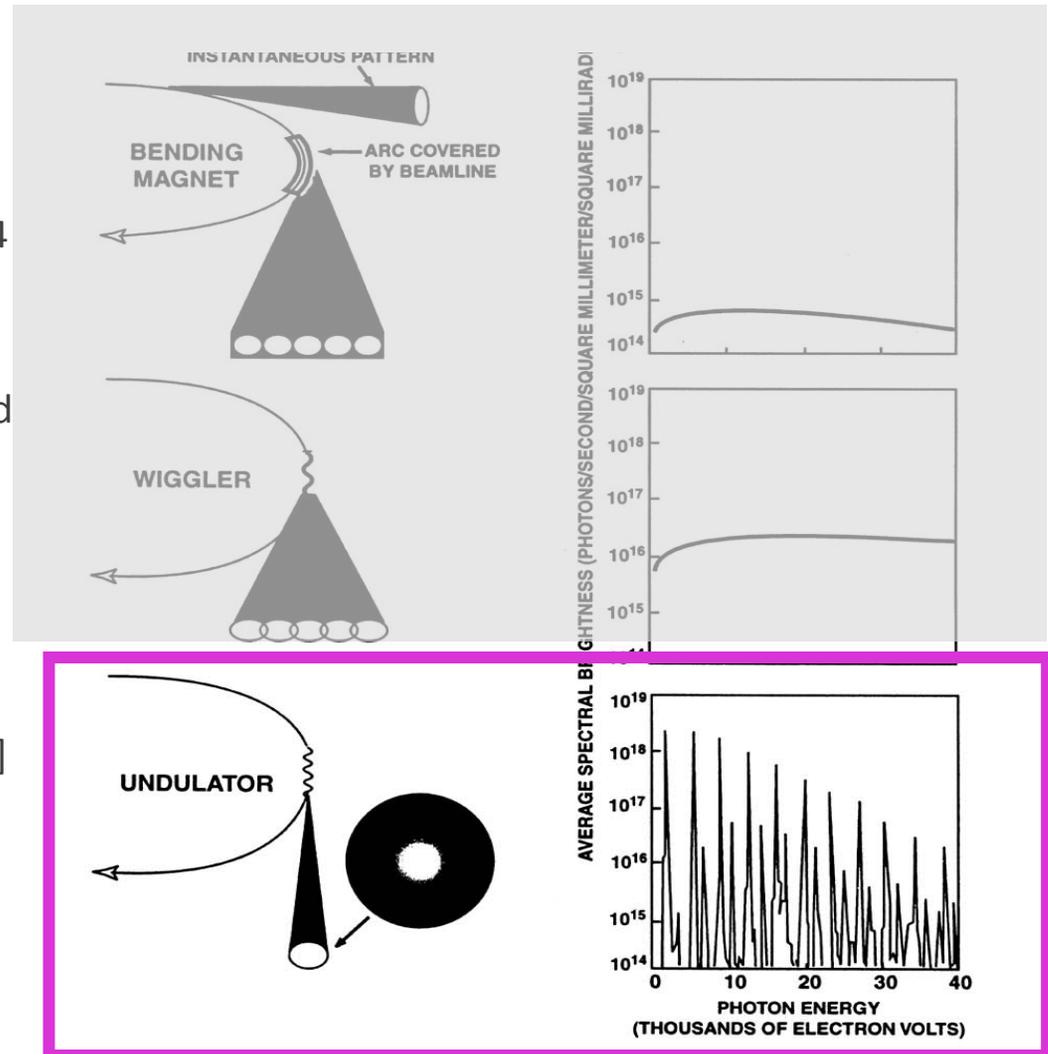
$$\Delta\psi_V = \Delta\psi_H = (1/2\gamma) [1/N]^{1/2} \approx (\lambda_{x\text{-ray}} / 2L)^{1/2}$$

where N = number of periods [typically 100]

- To get the true opening angle, need to consider the opening angle of the emitting particles (- more later on!)

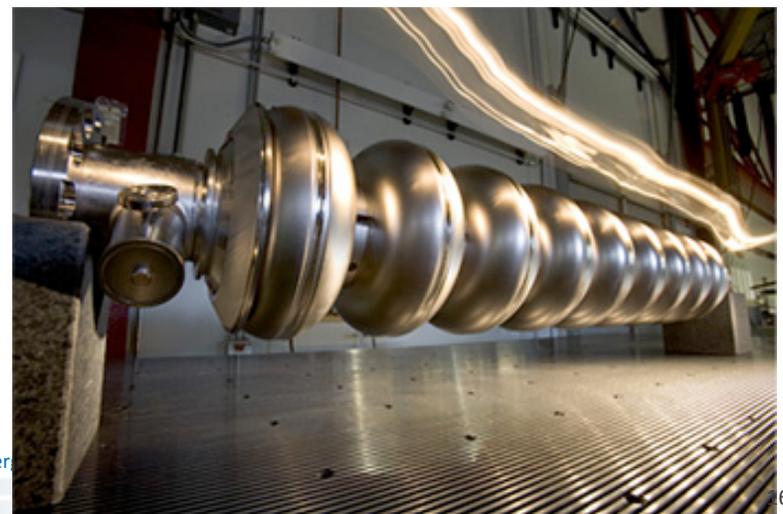
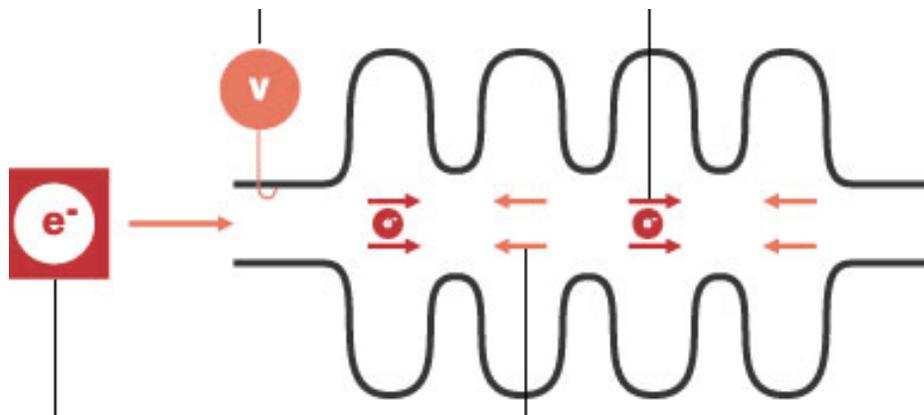
See Appendix 4 for more details

Undulator Radiation Spectra



Time Structure of the Radiation

- Because the electrons are radiating x-rays, they are constantly losing energy.
- To restore the energy loss on each revolution, radio-frequency (RF) cavities are installed in the storage ring to replenish the radiative energy losses.
- Particles are grouped together by the action of the radio-frequency (RF) cavities into bunches. At the APS:
 - typically about 100 psec FWHM (about 3 cm in length)
 - 1104 m circumference (3.68 microsecond period)
 - there are 1296 evenly spaced “RF buckets” (stable orbit positions” around the ring)
 - minimum spacing is 2.8 nsec between bunches (determined by the RF frequency- 352MHz)
- Details of the time structure depends on the fill pattern, i.e. which RF buckets have electrons in them.

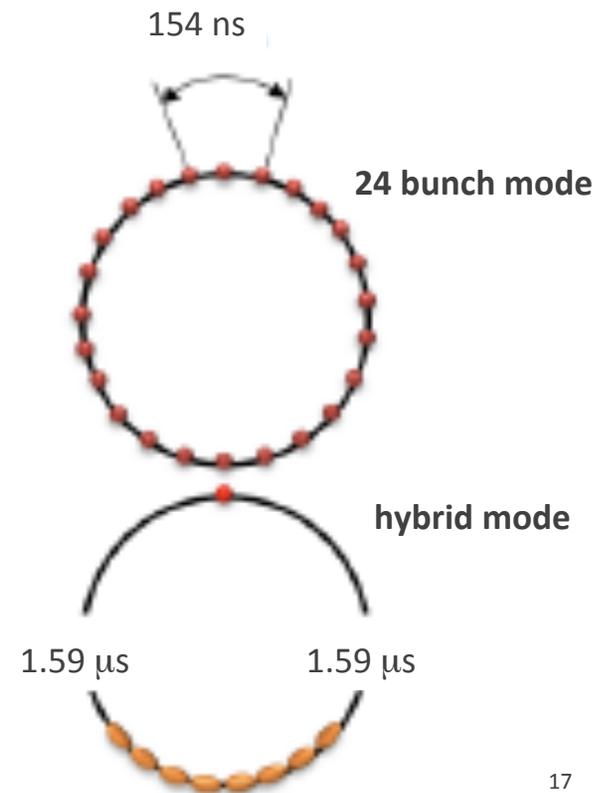


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Typical APS Filling Patterns

- The APS has a 1104 m circumference (3.68 microsecond period).
- There are 1296 evenly spaced “RF buckets”(stable orbit positions) around the ring that electrons could be stored in.
- This would correspond to a spacing of 2.8 nsec between bunches
- 324 equally spaced bunches
 - approximately 11 nsec between bunches
 - approximates a continuous source
- 24 equally spaced bunches
 - approximately 154 nsec between bunches
 - compromise between quasi-continuous source and pulsed source
- 1 + 7x8 (hybrid mode)
 - a single bunch followed by 8 groups of 7 bunches
 - timing experiments



Transverse Properties of Particle Beams

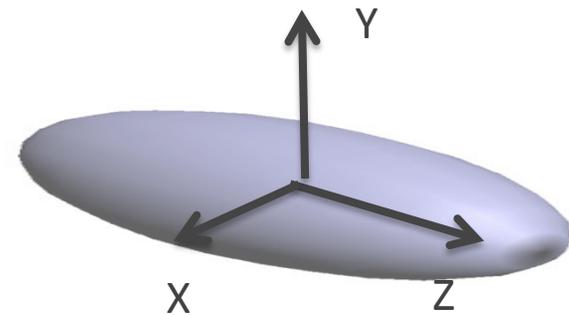
- Up until now, we have calculated the radiation properties from a single electron, however in a storage ring, the radiation is emitted from an ensemble of electrons with some finite size and divergence distribution.
- Both the transverse and longitudinal properties of the particle beam in a storage ring are the equilibrium properties of the particle beam, but here we are interested in the **transverse** properties.
- The product of the particle beam size and divergence is proportional to a parameter of the beam called the **emittance** (units are **length x angle**).

- There is a separate horizontal emittance (ϵ_x) and vertical emittance (ϵ_y), typically

$$\epsilon_y \text{ is 1\% to 10 \% of } \epsilon_x$$

- The **emittance is a constant of the storage ring**, although one can trade off beam size for divergence as long as the area of the phase-space remains constant.

X, Y are transverse directions
Z is the longitudinal direction



$$\epsilon_H = 3 \times 10^{-9} \text{ m-rad @ APS and}$$

$$\epsilon_V = 0.03 \times 10^{-9} \text{ m-rad.}$$

Why Do We Need to Know about the Transverse Particle Beam Properties?

- Although the flux from undulators can be determined without detailed knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely **brightness**, requires a more detailed knowledge of the particle beam's size and divergence.

Brightness has units of: photons/sec/0.1% BW/source area/source solid angle

$$\text{Flux}/4\pi^2 \Sigma_h \Sigma_v \Sigma_h' \Sigma_v'$$

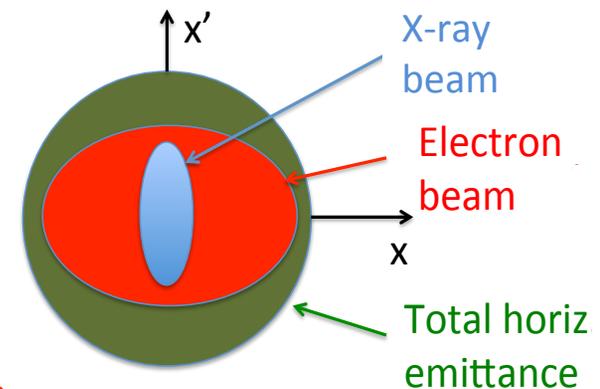
where Σ_i (Σ_i') is the **effective** one sigma value of the source size (divergence) in the i^{th} direction. If Gaussian distributions are assumed for both the particle beam and the radiation itself, the resultant source size and divergence is the quadrature sum of the two components, namely:

$$\Sigma_i = [\sigma_r^2 + \sigma_i^2]^{1/2} \quad \text{and}$$

$$\Sigma_i' = [\sigma_r'^2 + \sigma_i'^2]^{1/2}$$

↑ ↑
radiation electron beam
beam size size (i= x or y)

↑ ↑
radiation beam electron beam
divergence divergence (i= x or y)



- When the electron emittance ↓ then brightness ↑

See Appendix 5 for an example

Diffraction Limited Sources and Coherence

- The Heisenberg Uncertainty Principle sets a lower limit for the emittance of radiation. Recall:

$$\Delta x \Delta p_x \geq \hbar / 2$$

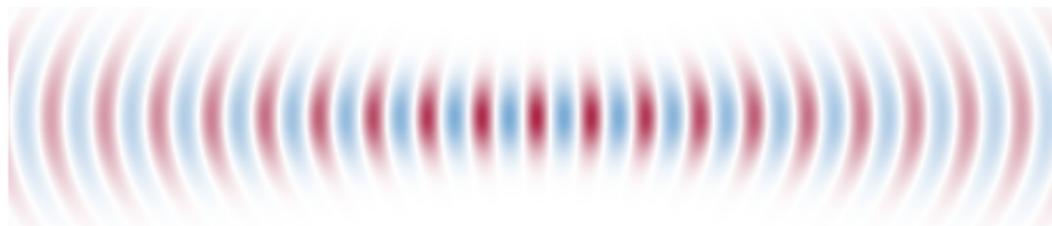
$$\frac{p_x}{p_z} = \phi_x \text{ or } \frac{\Delta p_x}{p_z} = \Delta \phi_x \text{ and } p_z = \hbar k = \hbar(2\pi / \lambda)$$

$$\text{so : } \Delta x \Delta p_x = \Delta x \Delta \phi_x p_z = \Delta x \Delta \phi_x [\hbar(2\pi / \lambda)] \geq \hbar / 2$$

$$\Delta x \Delta \phi_x \geq \lambda / 4\pi$$

- So once the particle beam emittance for a given x-ray wavelength, λ , is less than $\lambda/4\pi$, the source is said to be at the **diffraction limit** in that direction. If both the horizontal and vertical emittance of the beam is less than $\lambda/4\pi$, then the radiation emitted is **fully coherent and the source is diffraction limited**. For 1Å (12 keV) x-rays, the particle beam emittance would have to be less than:

$$1\text{\AA} / 4\pi = 10^{-10} \text{ meters} / 4\pi \approx 8 \times 10^{-12} \text{ m or } 8 \text{ picometers} - \text{radian} \text{ (radians are dimensionless)}$$



- Coherence can be an important parameter in some experiments such as photon correlation spectroscopy, x-ray holography, imaging, etc.

See lecture Friday morning on Coherence Based Imaging



Partial Coherent Sources

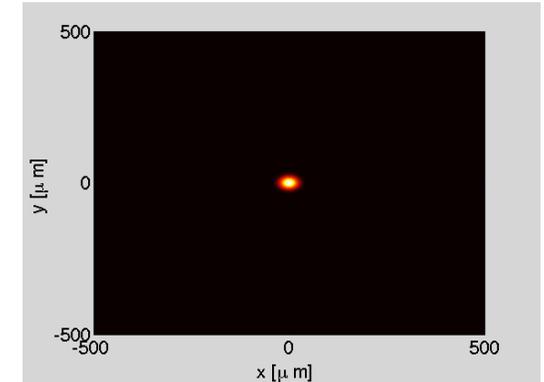
For 1Å (12 keV) x-rays → 8 picometers – radian for fully coherent beam.

- APS operates with:

$$\varepsilon_H = 3 \times 10^{-9} \text{ m-rad or } 3000 \text{ picometer-radian}$$

$$\varepsilon_V = 0.025 \times 10^{-9} \text{ m-rad or } 25 \text{ picometer-radian}$$

- Hence the APS is a partially coherent source at 1 Å.
- Partially coherent sources are sometimes characterized by the coherent fraction.
 - Coherent fraction = ratio of diffraction-limited emittance to actual beam emittance, or the fraction of the x-ray flux that is coherent.
- For the APS at 1Å, the coherent fraction is $\approx 10^{-3}$. Marginally good enough for coherence-based experiments so the trend is to try to reduce the particle beam emittance to increase coherence.
- Why not just squeeze the electron beam tighter in the existing storage ring?
 - Intra-bunch scattering of the particles results in beam loss and shorter lifetimes (*increase the number of bunches and lengthen the bunch longitudinally to reduce particle density*)
 - Need stronger focusing magnetic fields than can be reached with iron magnets with present electron beam energy and size of the storage ring vacuum chamber (*reduce the size of the vacuum chamber*)
 - Present bend magnet configuration limits emittance (*change from 2 bend magnets per sector to multiple bend geometry, i.e., a larger number of shorter bending magnets*)



An MBA-Lattice at APS: A New Generation of Storage Ring Sources

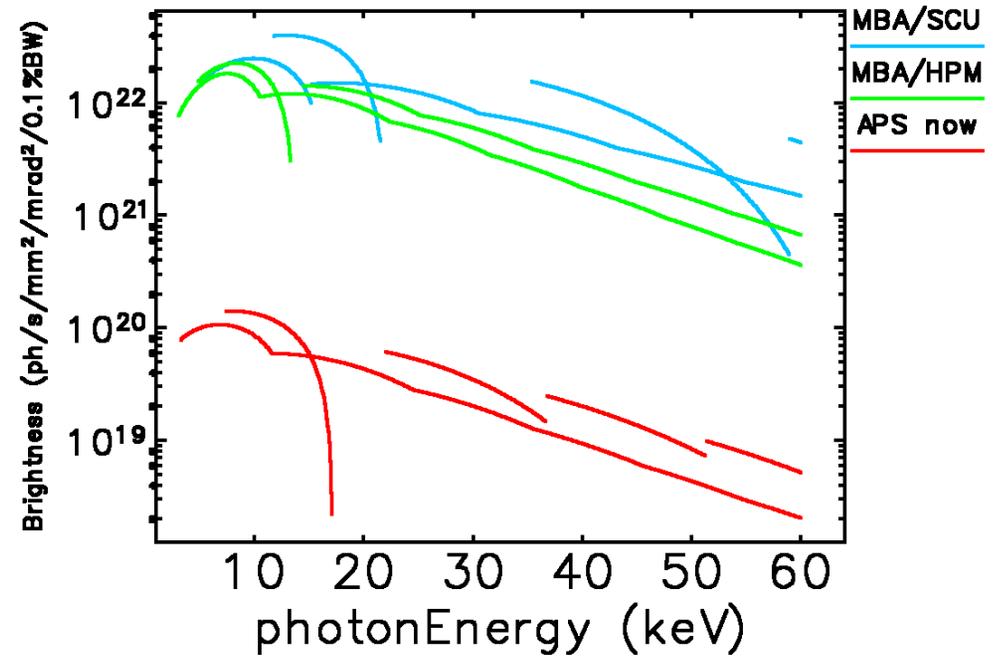
$$\varepsilon_x = C_L \frac{E^2}{N_D^3}$$

$C_L = \text{constant}$
 $E = \text{beam energy}$
 $N_d = \text{dipoles per sector}$

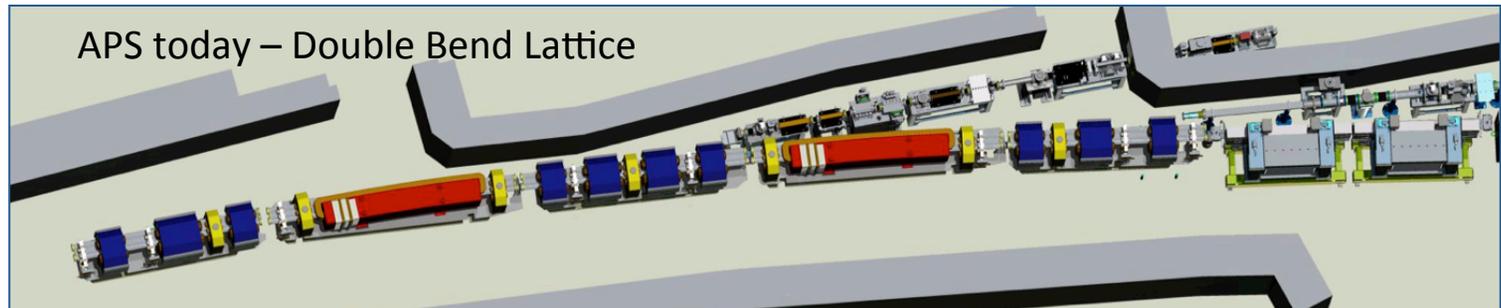
APS MBA Upgrade:

Beam energy: 7 GeV \Rightarrow 6 GeV

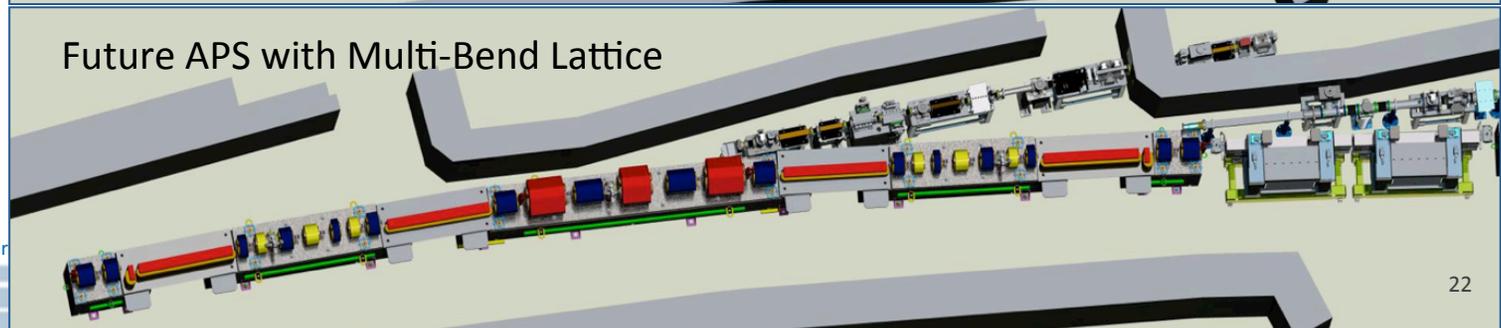
Dipoles per sector: 2 \Rightarrow 7



APS today – Double Bend Lattice



Future APS with Multi-Bend Lattice



~50-fold
reduction in
horizontal
emittance

The Advanced Photon Source



Parameters comparisons: APS today vs APS MBA Upgrade

Quantity	Symbol	Units	APS Now	APS MBA Timing Mode	APS MBA Bright. Mode
Beam Energy	E	GeV	7	6	6
Beam current	I	mA	100	200	200
Number of bunches	N_b		24	48	324
Bunch duration	σ_t	ps	34	70	68
Bunch spacing	T_b	ns	153	77	11
Bunch rep. rate	f_b	MHz	6.5	13	88
Emittance ratio	$K = \epsilon_x / \epsilon_y$		0.016	1.0	0.1
Horizontal emittance	ϵ_x	pm	3100	42	60
Horizontal beam size	σ_x	μm	275	7.4	8.8
Horizontal beam divergence	σ_x'	μrad	11	5.7	6.8
Vertical emittance	ϵ_y	pm	40	42	6
Vertical beam size	σ_y	μm	10	10.9	4.1
Vertical beam divergence	σ_y'	μrad	3.5	3.8	1.4

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MBA Lattice Upgrades - a Competitive Landscape

ESRF (France)

- Council approved second phase of upgrade, which incorporates an MBA lattice
- ESRF plans to resume operation in 2020, complete four state-of-the-art beamlines by 2022

SPRING-8 (Japan)

- Capable of upgrading in 2020 timeframe

MAX-IV (Sweden)

- Construction underway; inauguration June 2016

SIRIUS (Brazil)

- Completing final design; operational in 2018



The uncertain U.S. position within the competitive landscape was a central theme in the July 2013 BESAC Report:

*'...It is essential that the facilities this science community [of storage ring users] relies on remain **internationally competitive** in the face of the innovative developments... The Office of Basic Energy Sciences should ensure that U.S. storage ring x-ray sources **reclaim their world leadership position**...'*

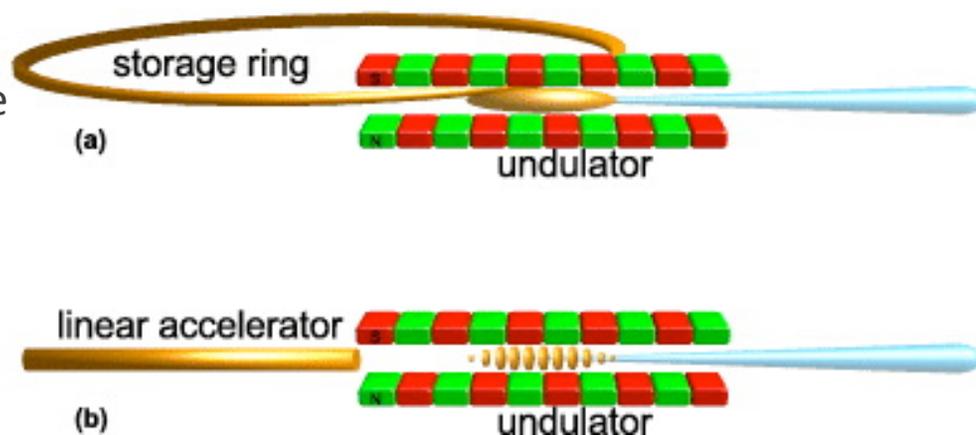
Our plans for the APS Upgrade enable the U.S. to establish a world leadership position in storage-ring-based x-ray sources

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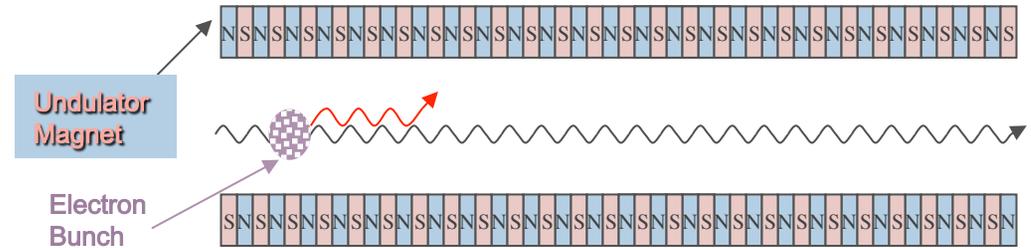
Non-Storage Ring Coherent X-ray Sources - XFELs

- Another way to reduce the particle beam emittance is through linac-based x-ray free electron lasers (XFELs)
 - Full transverse (spatial) coherence and femtosecond pulses
- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an **emittance-preserving linac**.
- The problem with a linac is you only use the electrons once, and so have to accelerate new electrons for each pulse (in the storage ring you “recycle” the electrons after they have been accelerated).
- The gain in the laser is obtained through a process called **Self-Amplified Spontaneous Emission** or **SASE**.



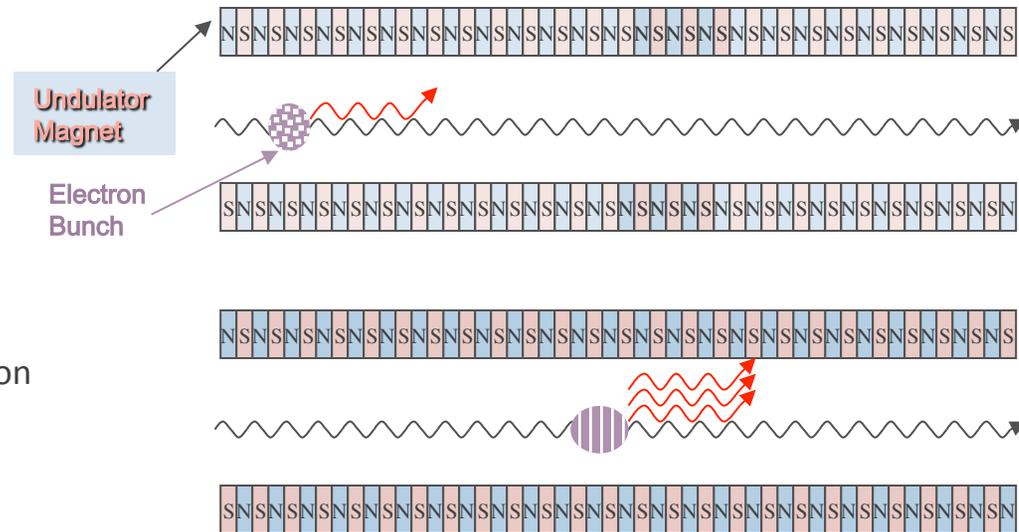
Self-Amplified Spontaneous Emission - Undulator Radiation

The LCLS produces extraordinarily bright pulses of synchrotron radiation in a process called “self-amplified spontaneous emission” (SASE). In this process, an intense and highly collimated **electron beam** travels through an undulator magnet. The alternating north and south poles of the magnet force the electron beam to travel on an approximately sinusoidal trajectory, emitting **synchrotron radiation** as it goes.



Linac Coherent Light Source (LCLS) at SLAC is an x-ray FEL using 1 km of the 3 km linac there to produce 15Å to 1.5Å beams.

Self-Amplified Spontaneous Emission - Energy Modulation → Density Modulation (micro-bunching)



The electron beam and its synchrotron radiation are so intense that the electron motion is modified by the electric and magnetic fields of its own emitted synchrotron light. Under the influence of both the undulator magnet and its own synchrotron radiation, the electron beam is forced to form micro-bunches,



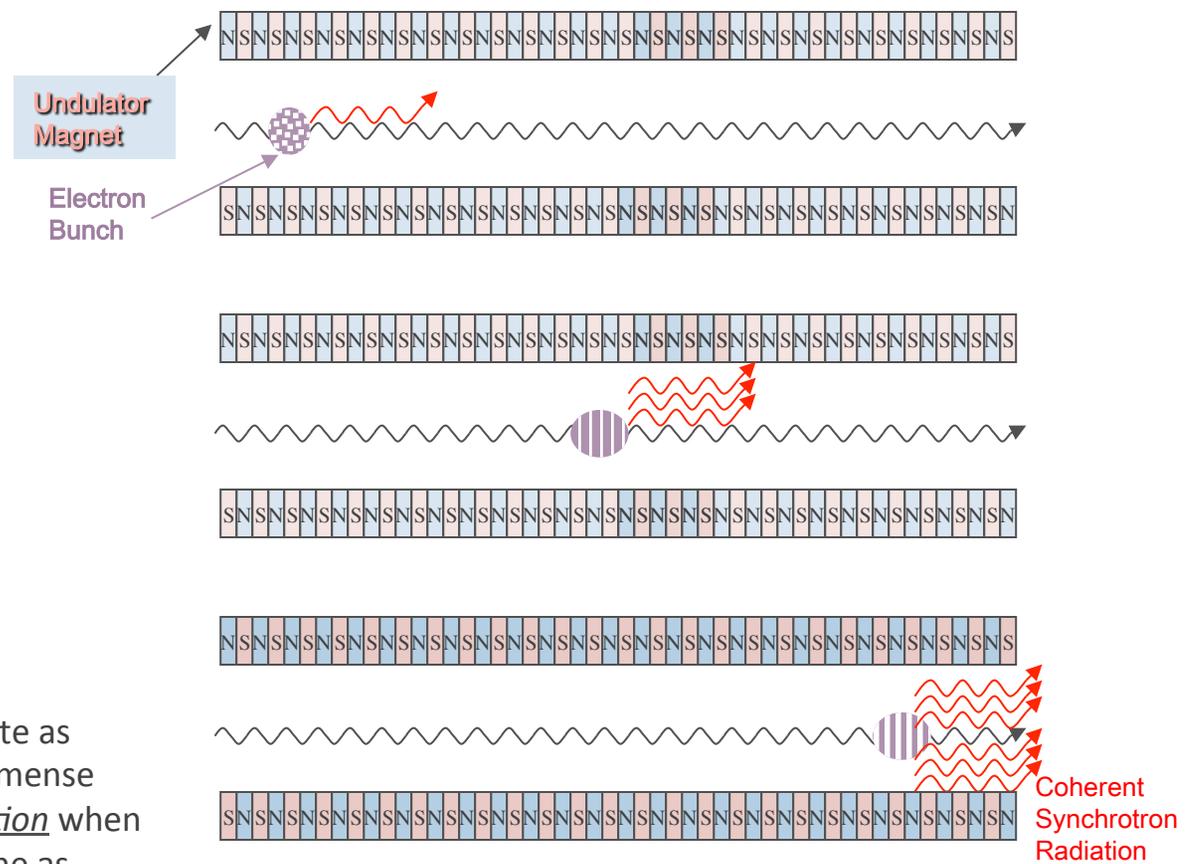
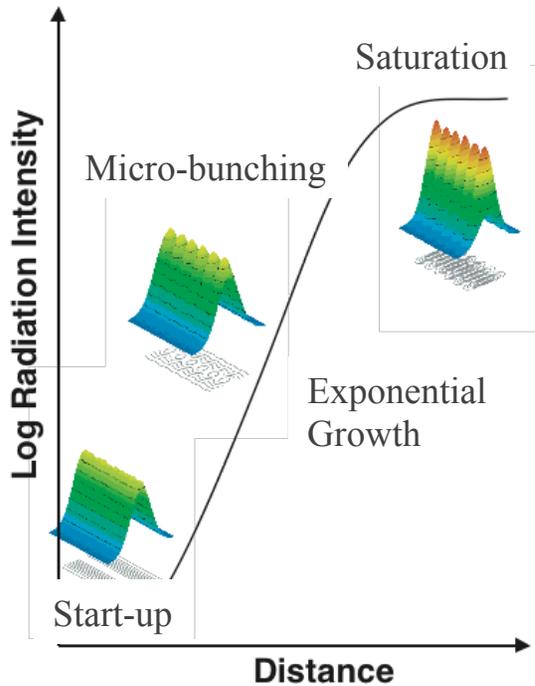
separated by a distance equal to the wavelength of the emitted radiation.

John Galayda, LCLS

Energy modulation → density modulation

$$Power\ Radiated = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

Self-Amplified Spontaneous Emission - Exponential Growth of Intensity



These micro-bunches begin to radiate as if they were single particles with immense charge. The process reaches *saturation* when the micro-bunching process has gone as far as it can go.



John Galayda, LCLS



Linac Coherent Light Source at SLAC

X-FEL based on last 1-km of existing 3-km linac

1.5-15 Å (14-4.3 GeV)

Existing 1/3 Linac (1 km)
(with modifications)

Injector (35°
at 2-km)

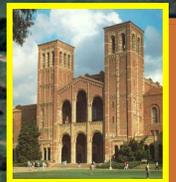
New e^- Transfer Line (340 m)

X-ray
Transport
Line (200 m)

Undulator (130 m)

Near Experiment Hall

Far Experiment
Hall



UCLA



Hard X-Ray FELs in Operation & Under Construction

LCLS-I, II 2009, 2018
14.5 GeV, 120 Hz NC



SACLA 2011
8.5 GeV, 60 Hz NC



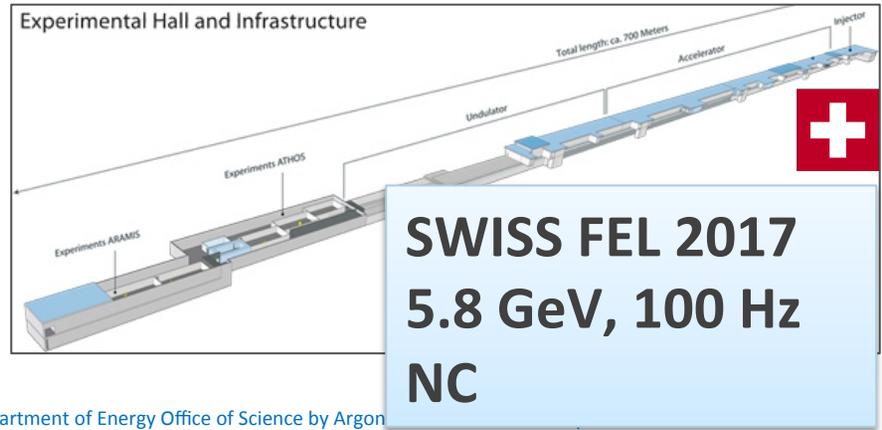
XFEL 2015
17.5 GeV, 3000 x 10 Hz SC



PAL XFEL 2015
10 GeV, 100 Hz NC



Experimental Hall and Infrastructure

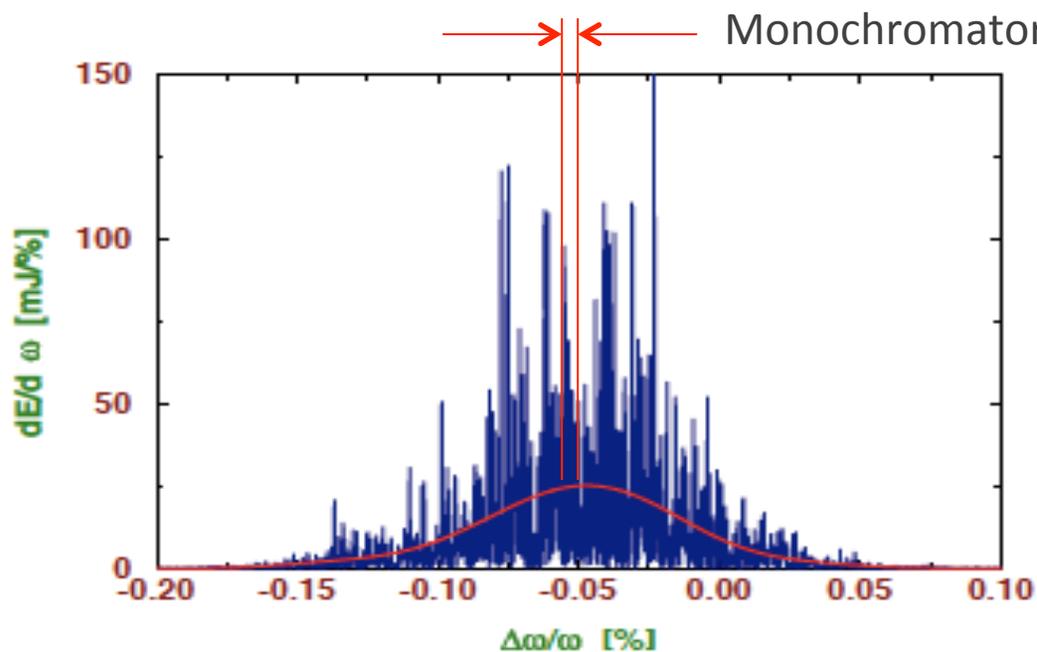
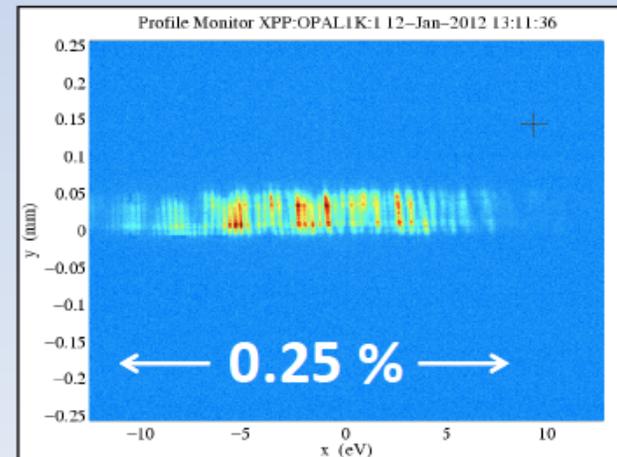


SWISS FEL 2017
5.8 GeV, 100 Hz NC

The Self-Amplified Spontaneous Emission (SASE) Process

- The lasing starts up from the random microbunching (i.e., shot noise) on the electron beam instead of being coherently produced by an input “seed” source.

Measured SASE spectrum (LCLS)



High flux on sample

Summary

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.
- There is a strong science case for a new generation of sources such as:
 - X-ray free electron lasers
 - Low-emittance storage rings
- FELs can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate is currently limited to hundreds of hertz.
 - The Linac Coherent Light Source II (LCLS II), under construction at Stanford, will increase that to the 10's of kilohertz.
- Storage rings can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak power.
 - The APS is working to incorporate a low emittance lattice into the proposed upgrade of the facility (APS-U) that will produce beams of high coherence at megahertz rates.
- Both LCLS II and APS-U will keep US SR facilities at the cutting edge to produce world class science in the years to come.



References

- **Sources**

“Introduction to Synchrotron Radiation” by Giorgio Margaritondo

“Synchrotron Radiation Sources – A Primer” by Herman Winick

“Undulators, Wigglers, and their Applications” Edited by H. Onuki and P. Elleaume

“Elements of Modern X-ray Physics” by Jens Als-Nielsen and Des McMorrow

“Third-Generation Hard X-Ray Synchrotron Radiation Sources: Source Properties, Optics, and Experimental Techniques”, Edited by Dennis M. Mills



Appendix 1a: Radiated Power from Charges at Relativistic Velocities

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r , in the non-relativistic case, α is just the centripetal acceleration, v^2/r . In the relativistic case:

$$a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r}$$

Where $\tau = t/\gamma =$ proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$



Appendix 1b: Dependence on Mass and Energy of Radiated Power

$$P = \frac{2e^2 \gamma^4 v^4}{3c^3 r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e⁻ with the same total energy, E.)
2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)



Appendix 2: BM Spectral Distribution

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c .

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and ρ is the radius of curvature of the trajectory. Note that the **critical energy scales as γ^3** . In practical units, the critical energy can be written as:

$$E_c[\text{keV}] = 2.218 E^3[\text{GeV}] / \rho[\text{m}] = 0.06651 B[\text{kG}] E^2[\text{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at $E = 7 \text{ GeV}$. The critical energy of the radiation emitted from the BM is:

$$E_c[\text{keV}] = 0.06651 B[\text{kG}] E^2[\text{GeV}]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5 \text{ keV}} \text{ or } \underline{0.64 \text{ \AA}}.$$

Appendix 3: Where did “K” come from?

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

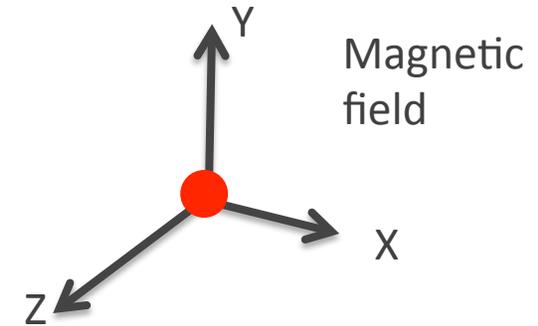
$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \quad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0} \frac{\lambda_{ID}}{2\pi c} \cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0} \frac{\lambda_{ID}}{2\pi} \cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c} \left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0} \frac{\lambda_{ID}}{2\pi c}\right] \frac{1}{\gamma} \left[\frac{\lambda_{ID}}{2\pi}\right] \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K \frac{1}{\gamma} \left[\frac{\lambda_{ID}}{2\pi}\right] \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$x_{\max} = K \frac{1}{\gamma} \left[\frac{\lambda_{ID}}{2\pi}\right] \quad \text{and} \quad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \quad \text{where} \quad K = \left[\frac{eB_0}{2\pi} \frac{\lambda_{ID}}{m_0 c}\right]$$

Equation of motion for a relativistic charged particle in a magnetic field



Appendix 4: How Do You Get 1 Å X-rays from a 3 cm Period Magnetic Field?

Where does the $1/\gamma^2$ come from in the equation: $\lambda_n^{\text{x-ray}} = (\lambda_{\text{ID}}/2\gamma^2n)(1 + K^2/2)$?

1) Consider the electron in its rest frame:

- It does not see a static magnetic field from the undulator, but rather a time-varying B-field and associated E-field (due to the relativistic transformation of the magnetic field of the device).
- The period of the E and B field are **Lorentz contracted** so that: $\lambda_{\text{e-frame}} = \lambda_{\text{ID}} / \gamma$ and so the electron oscillates (and hence radiates) with that same period driven by the EM fields.

2) Back in the lab frame:

- Due to the fact that the electron is traveling towards us, the radiation emitted by the electron is Doppler shifted to higher frequencies (shorter wavelengths). The **relativistic Doppler shift** goes as $\sqrt{1-\beta} / \sqrt{1+\beta} \approx 1/2\gamma$, and so the wavelength observed in the lab is:

$$\lambda_{\text{lab}} \approx (\lambda_{\text{ID}}/\gamma)(1/2\gamma) = (\lambda_{\text{ID}}/2\gamma^2)$$



Appendix 5: Calculating X-ray Beam Brightness

APS Electron Beam Parameters

APS operates with $\epsilon_H = 3 \times 10^{-9}$ m-rad and a coupling (ratio of vertical emittance to horizontal emittance) of 0.9%, therefore

$$\epsilon_V = 0.025 \times 10^{-9} \text{ m-rad.}$$

The particle beam source size and divergence at the locations of the IDs are:

$$\begin{aligned}\sigma_H &= 270 \text{ microns} \\ \sigma'_H &= 11 \text{ microradians} \\ \sigma_V &= 9 \text{ microns} \\ \sigma'_V &= 3 \text{ microradians}\end{aligned}$$

APS Undulator X-ray Beam Parameters

APS Undulator A has a length of 2.4 meters. For 1Å radiation the natural opening angle is:

$$\begin{aligned}\sigma'_{x\text{-ray}} &= 1/\gamma (1/N)^{1/2} \\ &= [\lambda/2L]^{1/2} = 4.5 \times 10^{-6} \text{ rad}\end{aligned}$$

The corresponding source size of the radiation is:

$$\sigma_{x\text{-ray}} = [\lambda L/8\pi^2]^{1/2} = 1.7 \text{ microns.}$$

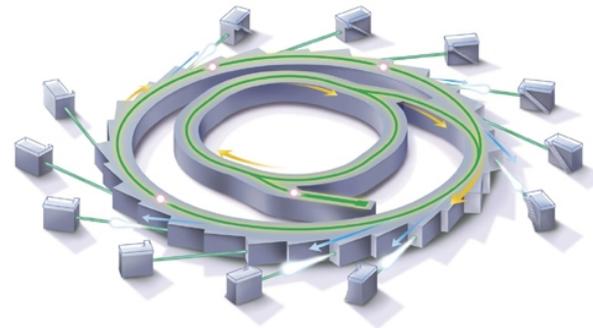
For source size $\Sigma = [\sigma_{x\text{-ray}}^2 + \sigma_{\text{electron}}^2]^{1/2}$ and source divergence $\Sigma' = [\sigma'_{x\text{-ray}}^2 + \sigma'_{\text{electron}}^2]^{1/2}$

$$\mathcal{B} = \text{Flux}/4\pi^2 \Sigma_H \Sigma_V \Sigma_H' \Sigma_V'$$

Appendix 6: The Case for Low Emittance Storage Rings

- In July 2013 the DOE's Basic Energy Sciences (BES) convened a Subcommittee of the BES Advisory Committee (BESAC) to provide them with "advice on the future of photon sources and science."
- *Diffraction Limited Storage Rings and U.S. Storage Ring Upgrades:*
 - *At best the present plans for upgrades of U.S. storage rings will leave the U.S. **behind the international community** in this area of x-ray science. The Office of Basic Energy Sciences should ensure that U.S. storage ring x-ray sources reclaim their world leadership position. **This will require a careful evaluation of present upgrade plans to determine paths forward that will guarantee that U.S. facilities remain at the cutting edge of x-ray storage ring science.***

Grand Challenge Science on Diffraction-Limited Storage Rings



A consensus report on future opportunities from scientists at

ALS, LBNL

APS, ANL

NSLS-II, BNL

SSRL, SLAC

together with a broad community of scientists at laboratories and universities.

BESAC Subcommittee on Future Light Sources

July 10-12, 2013

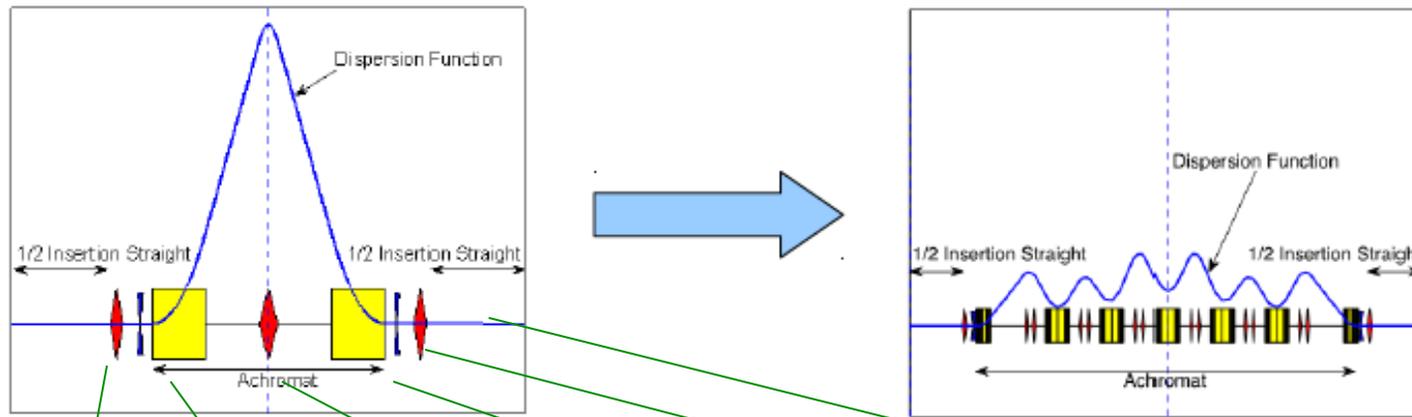


Appendix 7: Reducing the Beam Emittance in Storage Rings with Multi-Bend Achromat (MBA) Magnetic Lattices

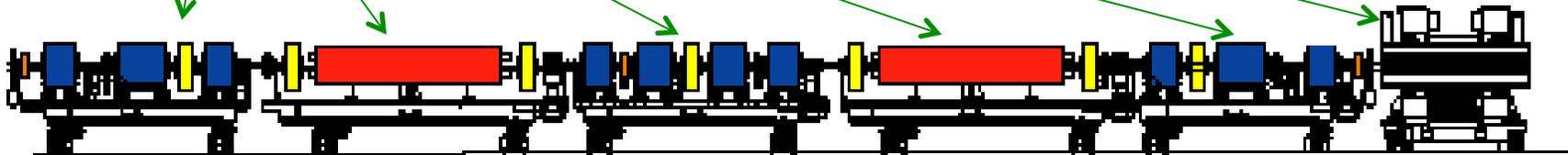
The particle beam emittance scales as:

$$\epsilon_0 \propto \frac{E^2}{N_D}$$

Where E is the particle energy and N_D is the number of dipole magnets in the ring



All figures courtesy C. Steier, LBNL.



- 7 bending magnets in one sector (a seven bend achromat or 7BA) at the APS would have ~40x lower emittance than today's 2B(A) lattice.

