Synchrotron Radiation: Production & Properties

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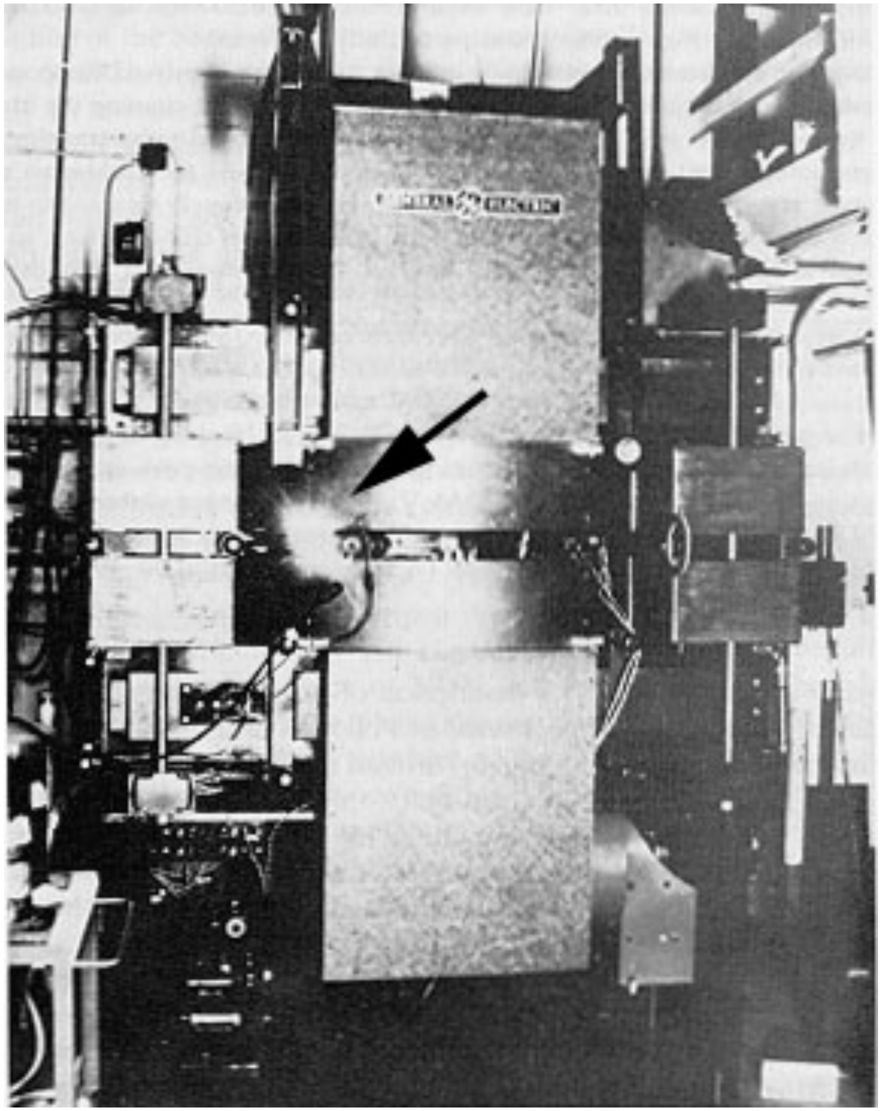
Synchrotron Radiation - Some Background

Synchrotron Radiation (SR) - radiation from charged particles traveling in circular orbits - was first observed from a 70 MeV synchrotron at GE in Schenectady in 1947.

On April 24,[1947] Langmuir and I [Herbert Pollack] were running the machine… Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.

Excerpted from Handbook on Synchrotron Radiation, Volume 1a, Ernst-Eckhard Koch, Ed., North Holland, 1983.



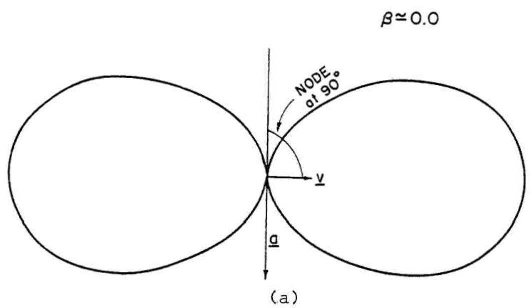
Radiation Patterns From Accelerating Charges

Definitions:

$$\beta = v/c$$

$$\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$$

When v << c, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.



$\beta = 0$

But as β approaches 1 :

- The shape of the radiation pattern changes; it is more forward directed
- The intensity of the radiation pattern increases

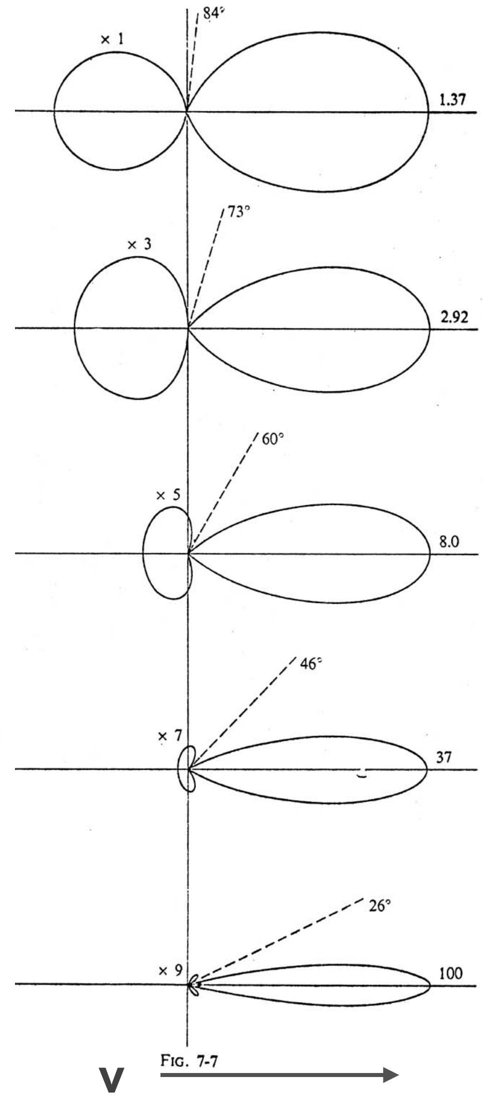
$\beta = 0.1$

$\beta = 0.3$

$\beta = 0.5$

$\beta = 0.7$

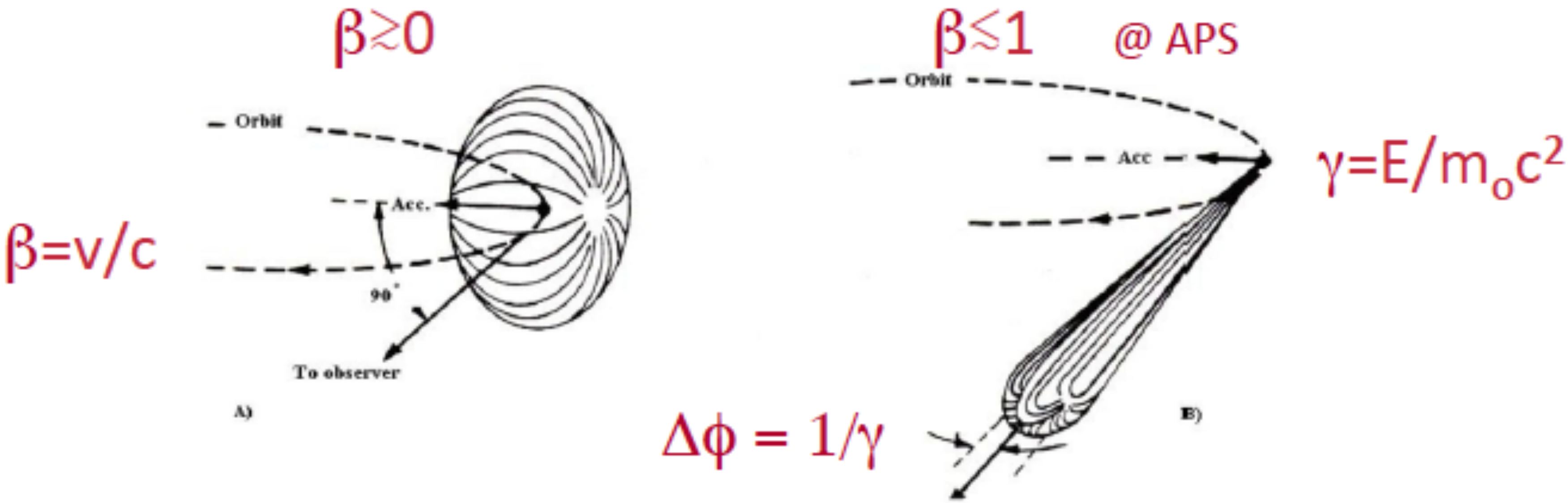
$\beta = 0.9$



v

$$\tan\theta_{lab} = \frac{\sin\theta'}{\gamma\left(\cos\theta' + \beta\right)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$

Radiation from Highly-Relativistic Particles



At the APS with E = 7 GeV,

$\gamma = E/m_o c^2 = 7\ GeV/0.511\ MeV$

$\gamma = 1.4 \times 10^4$

$1/\gamma = 73 \times 10^{-6}$

- The opening angle in both the horizontal and vertical directions, is given approximately by:

$$\Delta\phi_{vert} = \Delta\phi_{hor} \approx 1/\gamma,$$

when $\beta \approx 1$.

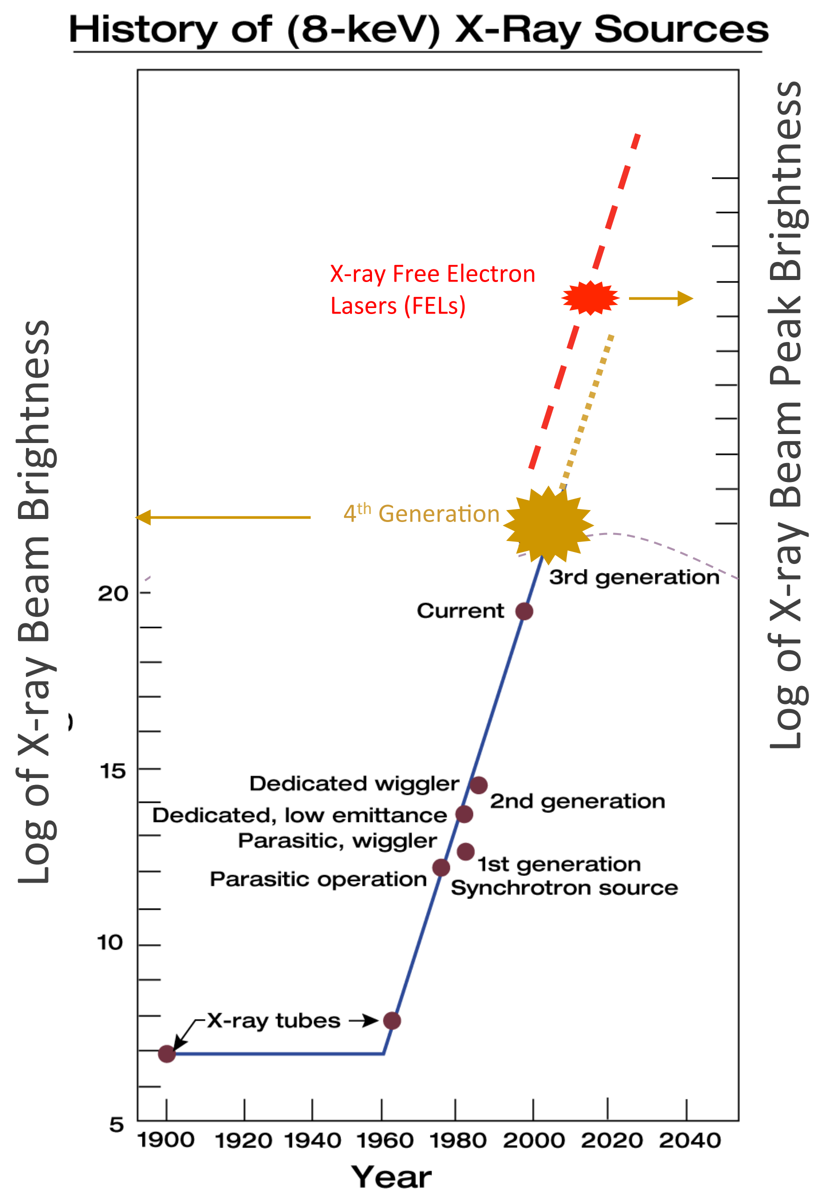
- Relativistic velocities are good!!
  - radiation forward directed
  - radiated power $\propto E^4$

See Appendix 1 for more details

The Evolution of Brightness from SR Sources

Synchrotron radiation (from IR to hard X-rays) has been used as a research tool for nearly 50 years.

- 1st Generation Sources
  - *Ran parasitically on accelerations for high energy physics (CHESS and SSRL originally)*
- 2nd Generation Sources
  - *Built to optimize synchrotron radiation from the bending magnets (NSLS)*
- 3rd Generation Sources
  - *Built to optimize synchrotron radiation from insertion devices (ALS, APS, NSLS II [const.])*
- 4th Generation Sources
  - *fully coherent sources*
    - *X-ray Free Electron Lasers (X-FELs)*
    - *Diffraction Limited Storage Rings (DLSRs)*
    - *FEL-Oscillator (FEL-O)*
    - *Energy Recovery Linacs (ERLs)*



A Vital National Resource for Science and Technology

The **Advanced Photon Source (APS)** is a fully optimized, insertion-device-based, third generation x-ray source for the production of high intensity (brightness) hard x-ray beams.

ALS, APS, NSLS & SSRL are funded by the Department of Energy, Office of Science, Basic Energy Science (DOE/BES).

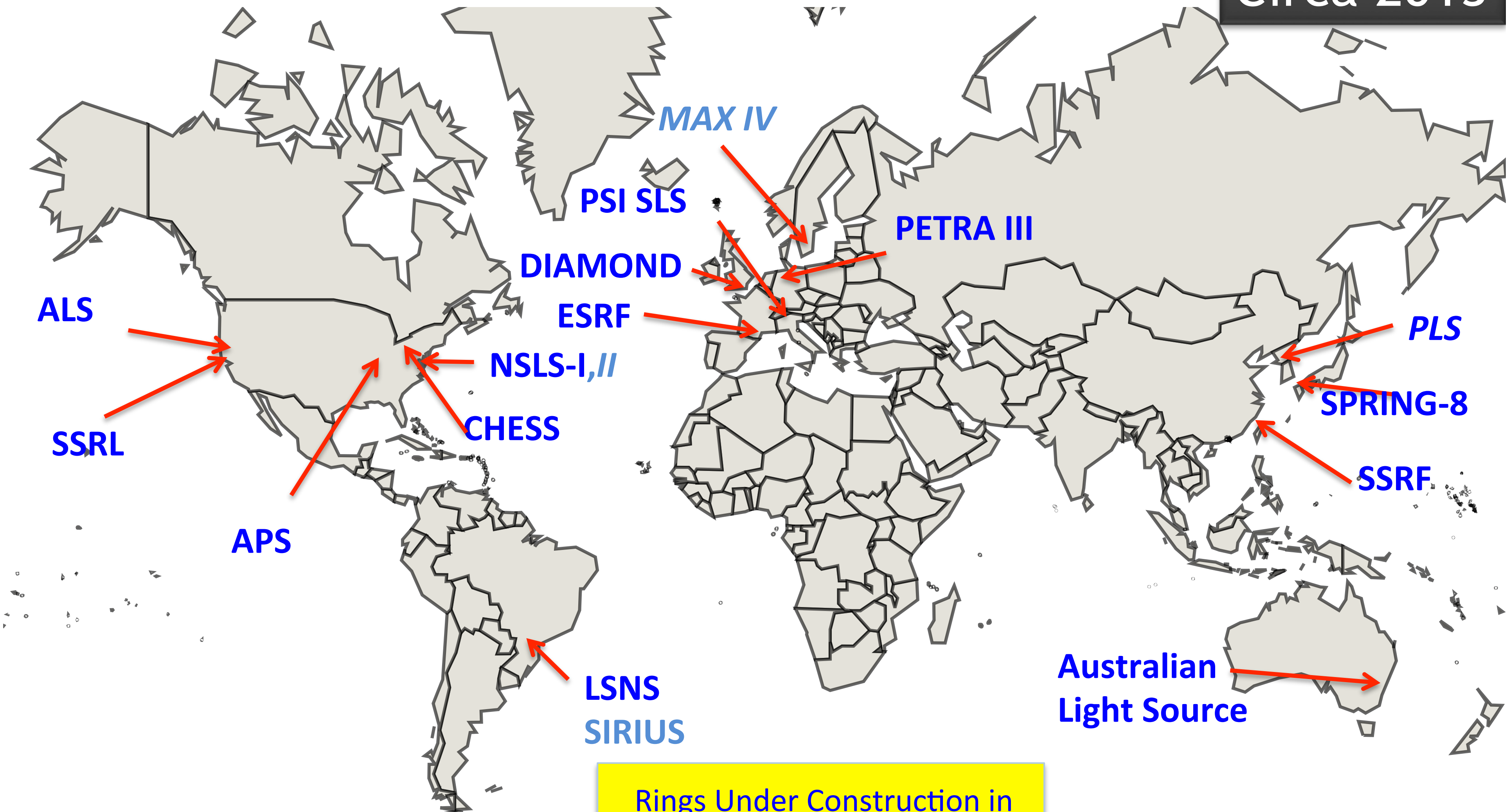
The Cornell High Energy Synchrotron Source – CHESS- is funded by NSF.

DOE/BES also operates several neutron user facilities as well as the nanoscience research centers.

Over **5000 unique researchers** from around the world came to Argonne last year to perform experiments at the APS.

Storage Ring Light Sources Worldwide

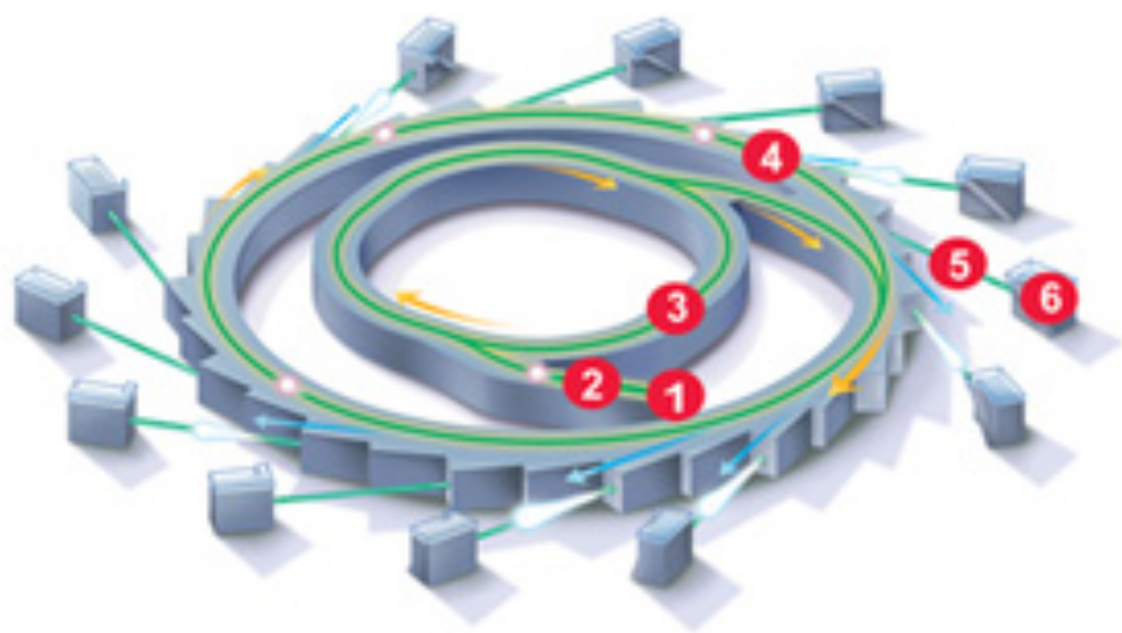
Circa 2013



Rings Under Construction in Light Blue

Adapted from presentation given to Basic Energy Sciences (BES) Advisory Committee in July 2013

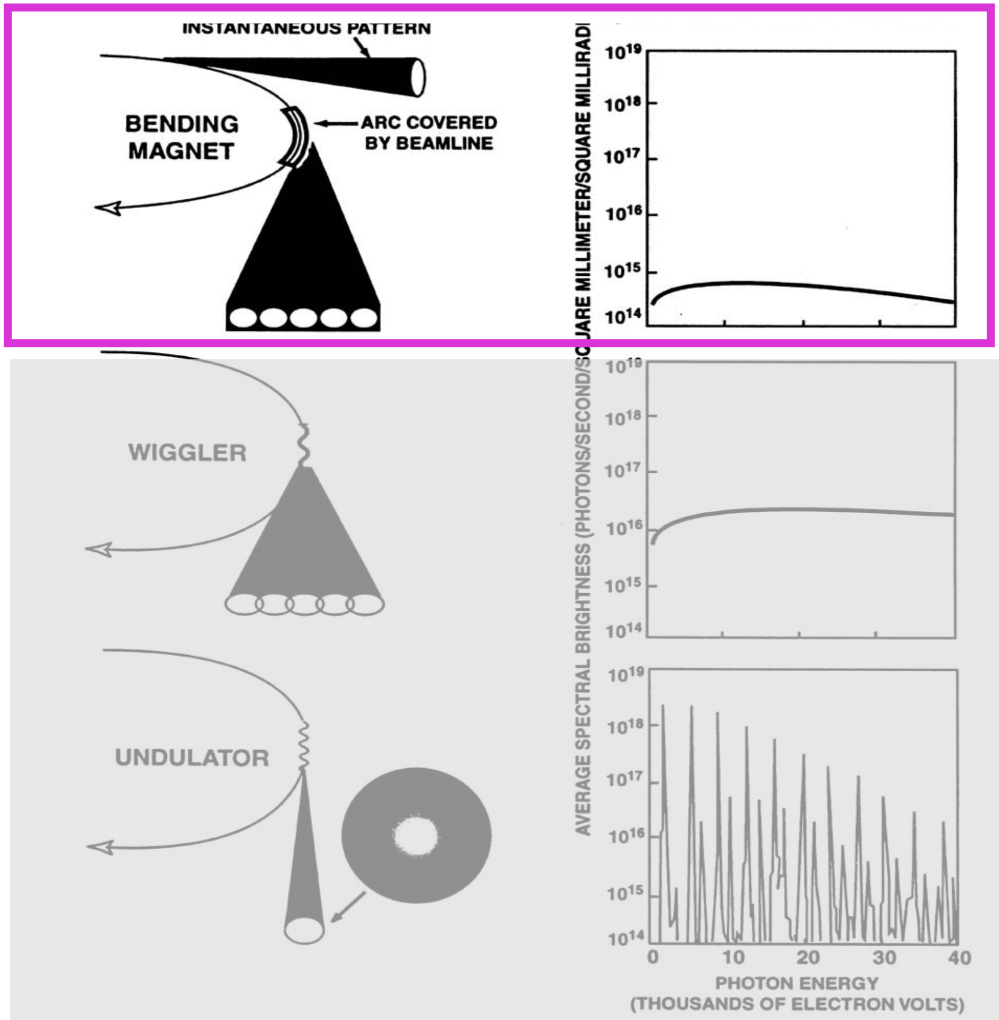
Typical SR Facility Complex



1. Electron gun
2. Linear Accelerator LINAC
3. Booster Synchrotron
4. Storage Ring (SR)
5. Beamline
6. Experiment station

(Courtesy: Australian Synchrotron,
Illustrator: Michael Payne)

BM Spectral Properties



Bend Magnet Radiation

- Spectrum characterized by the critical energy:

$$E_c = 3hc\gamma^3/4\pi r.$$

- Vertical opening angle ($\Delta\phi$) is $1/\gamma$. For the APS:

$$\Delta\phi_{vert} = 1/\gamma = 73 \times 10^{-6} \text{ radians}$$

- Horizontal opening angle determined by apertures

- In the plane of the orbit, the polarization is linear and parallel to the orbital plane.

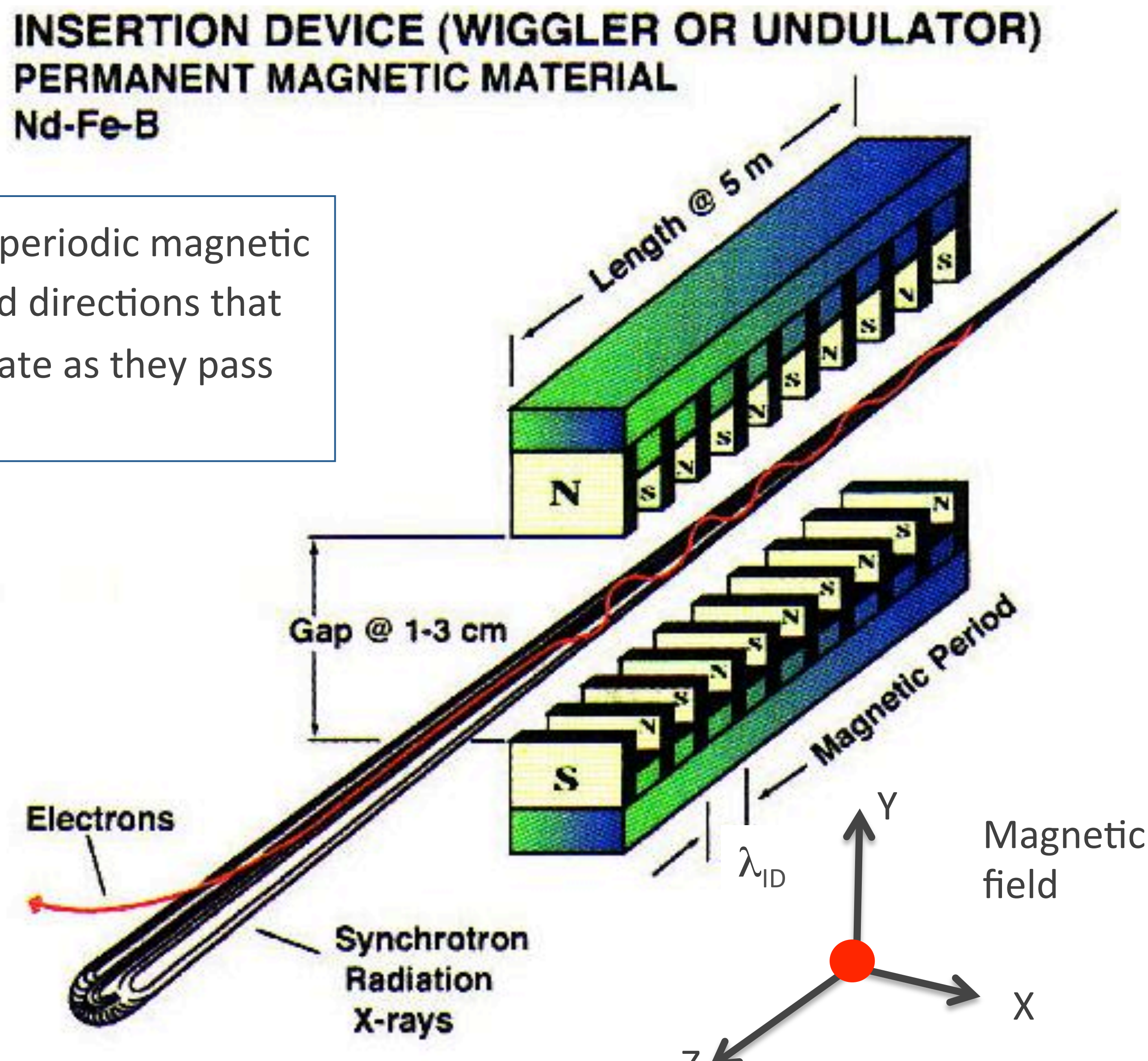
See Appendix 2 for more details

Planar Insertion Devices

Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

$L_{ID} = N\lambda_{ID}$ where:

L_{ID} = length
N = number of periods
λ_{ID} = magnetic period



Characterizing Insertion Devices

- IDs are characterized by the so-called field index or deflection parameter, K **(See Appendix 2)**:

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[cm]\ B_o[kG]$$

where λ_{ID} is the period of the insertion device and B_o the peak magnetic field. (The length of the ID, L, is equal to the number of periods, N, times the length of the period, i.e., $L = N\lambda_{ID}$.)

- The maximum deflection angle of the particle beam, θ_{max}, is given by:

$$\theta_{max} = \pm(K/\gamma)$$

See Appendix 3 for more details

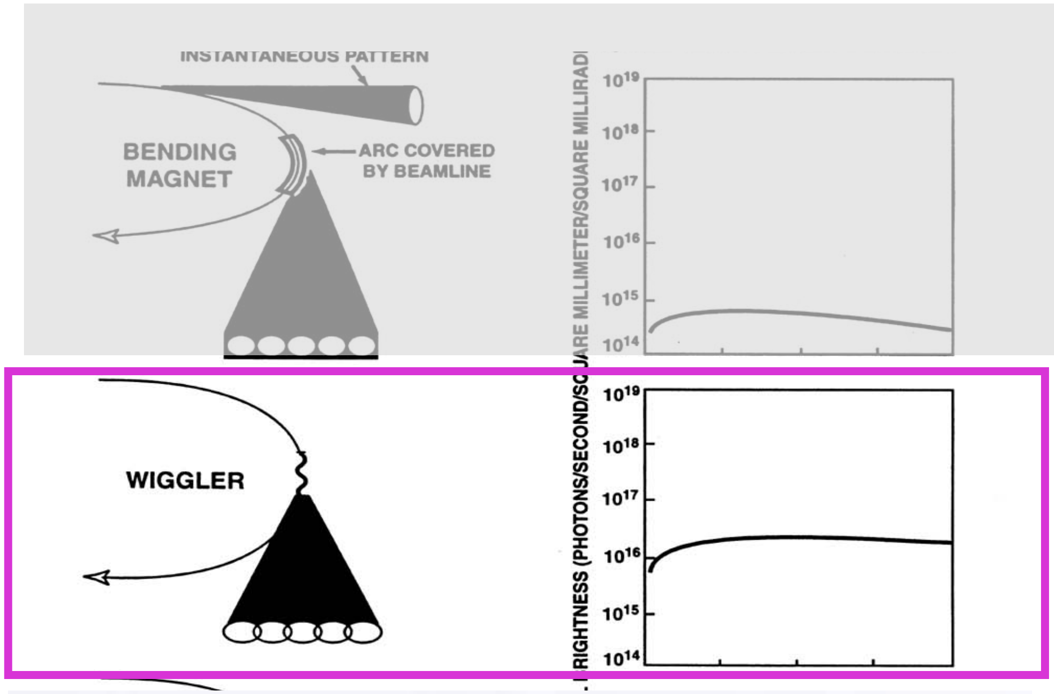
and the amplitude of the oscillation of the particles, x_{max}, by:

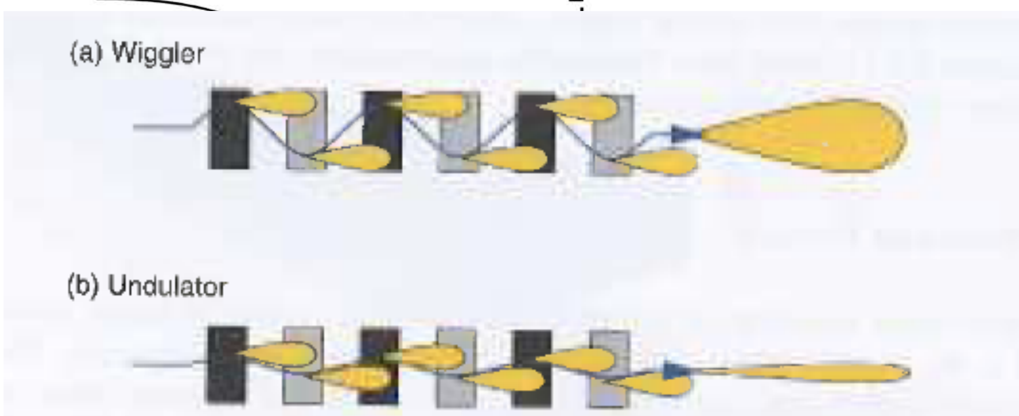
$$x_{max} = (K/\gamma)(\lambda_{ID}/2\pi)$$

APS Undulator A has a period of 3.3 cm and operates with $K \approx 1$, therefore:

$\theta_{max} \approx 1/\gamma$ **and** *$x_{max} \approx 0.38$ microns.*

Wiggler Radiation Pattern and Spectrum





Wiggler Radiation (K>>1)

- θ_{max} = (K/γ) >> 1/ γ, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation (1/ γ).
- radiation spectrum looks like 2N dipole sources (N = number of periods)
- spectrum characterized by the critical energy (which may be different than BM critical energy)
- Presently, there are NO planar wigglers installed at the APS. Wigglers with fields in both the horizontal and vertical directions produce elliptically polarized radiation. These are sometimes called elliptical multipole wigglers (EMWs).

Undulators

Undulators:

- $K \approx 1$
- θ_{max} is comparable $(1/\gamma)$ and so the radiation from each pole overlaps causing interference effects in the spectral distribution.
- Constructive interference occurs when:

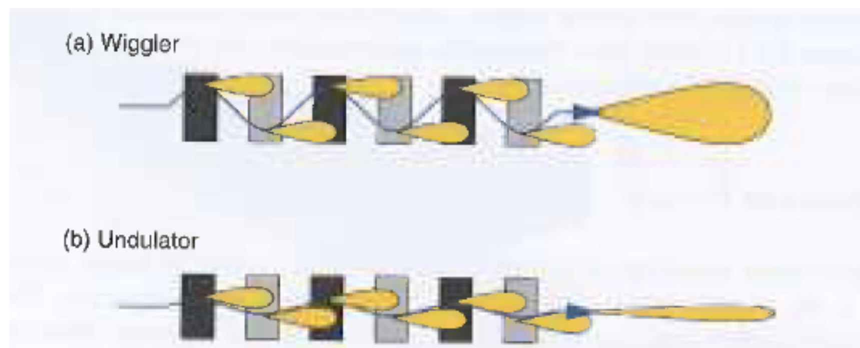
$$\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

- The wavelength where constructive interference occurs, $\lambda_n^{x\text{-}ray}$, can be adjusted by varying the magnet field, B, produced by the undulator.

$$K = 0.0934\,\lambda_{ID}[cm]\,B_o[kG]$$

- Most undulators are made of permanent magnets, so to vary the B-field that the electrons see, the gap between the upper and lower magnet arrays is varied.

APS 2.4 m long Undulator A (λ_{ID} = 3.3 cm)



Tuning Curves for Undulators

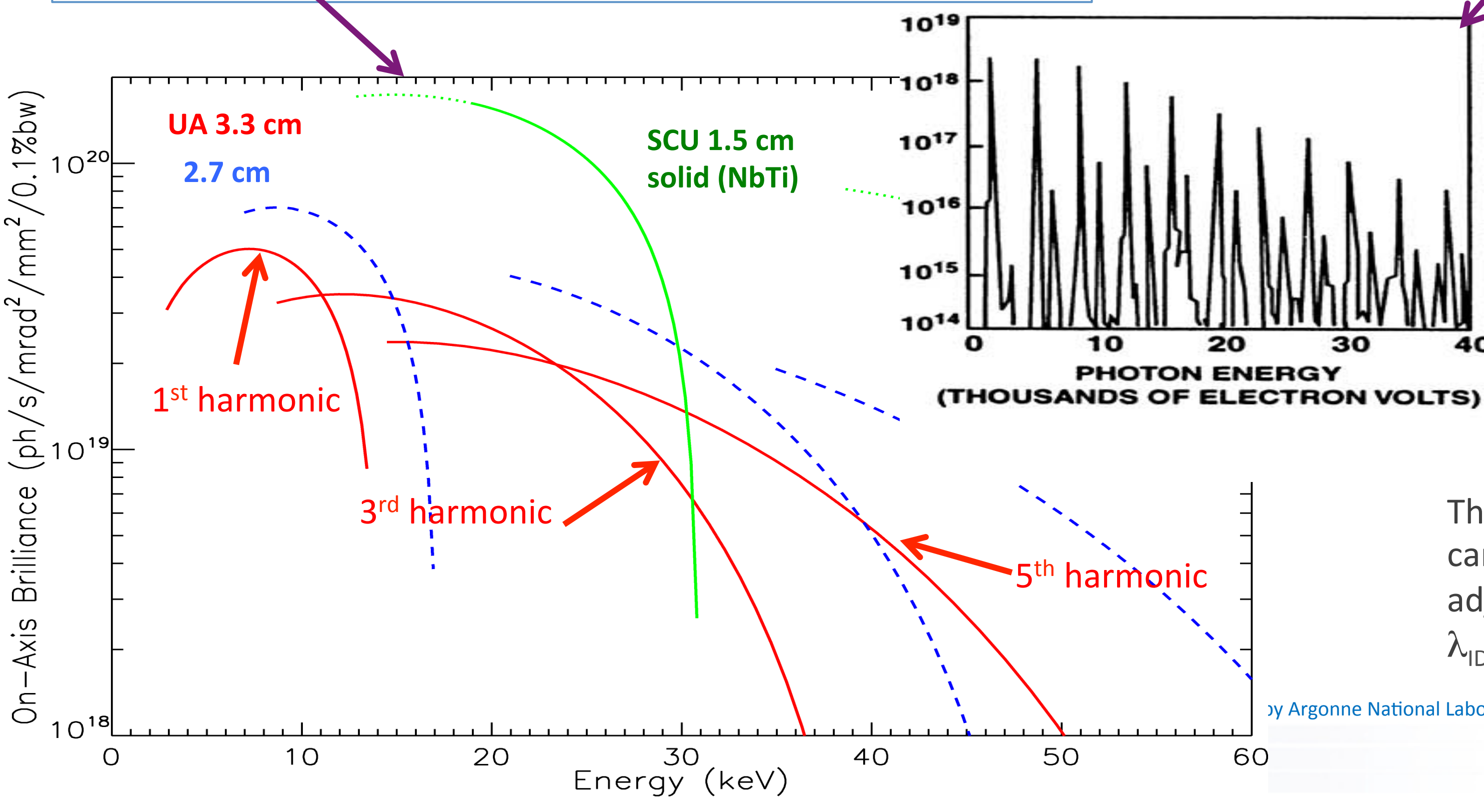
- Recall:

$$\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

Note that you do not get just the fundamental (n=1), but you get all the odd harmonics (n=3,5,7…)

Here is the spectrum over the tuning range of the undulator. The low energy is fixed by the minimum magnet gap (determined by the vacuum chamber). As the gap is opened, the x-ray energy increases, but as you open it further, the magnetic field decreases and so does the intensity of the x-rays.

Here is the spectrum for a particular gap (i.e., a particular K value).



The range of energy that can be covered can be adjusted varying the period, λ_{ID}, of the undulator.

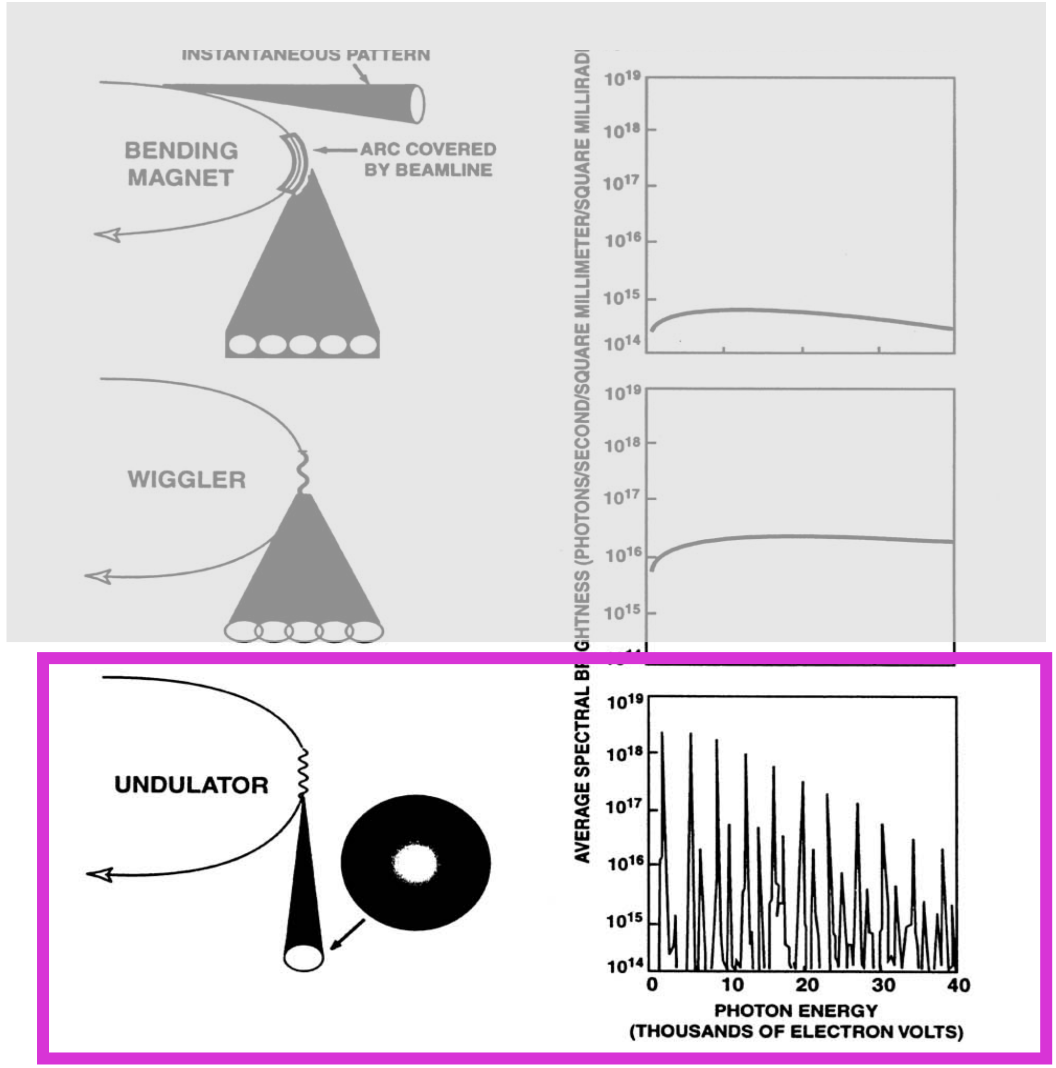
Undulator Radiation Spectra

Undulator Radiation

- undulators defined as IDs with horizontal deflection angle ≈ $1/\gamma$, i.e., K ≈1
- spectrum peaked but peaks are tunable by varying K (K = 0.94 B[T] λ_{ID}[cm])
- at the peaks (harmonics) the horizontal and vertical opening angles of the radiation is given by:
  - $\Delta\phi = 1/\gamma\ (1 / N)^{1/2}$

  where N = number of periods [typically 100]
- to get the true opening angle, need to consider the opening angle of the emitting particles (- more later on!)

See Appendix 4 for more details



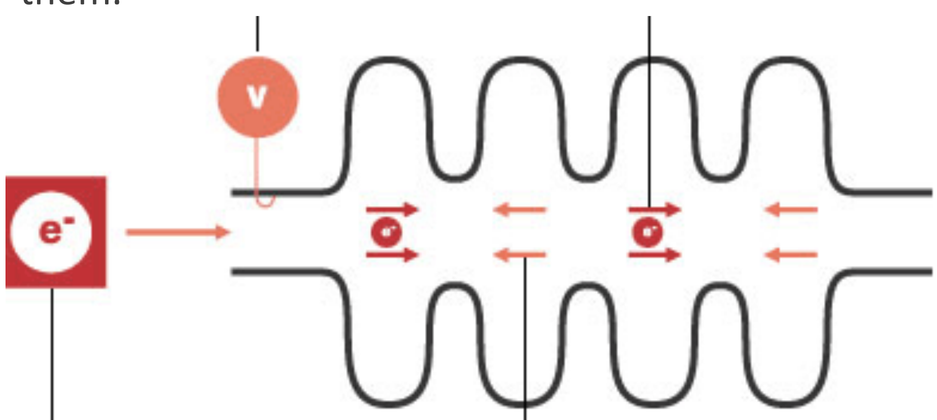
Time Structure of the Radiation

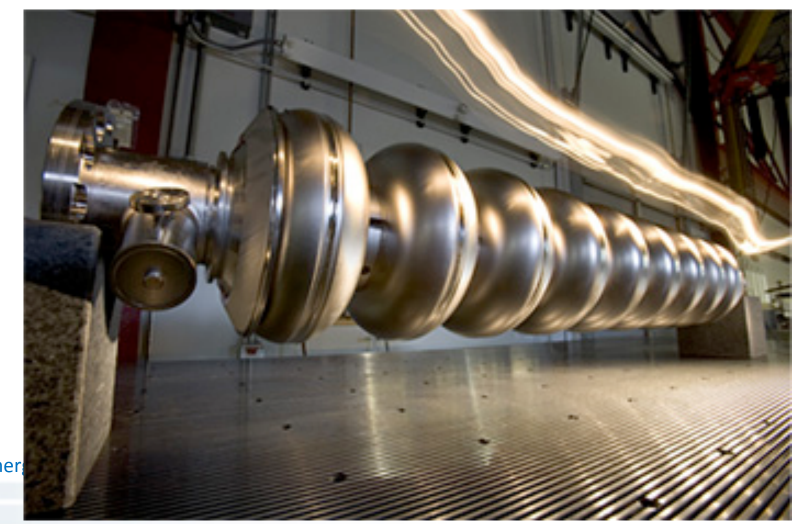
- Because the electrons are radiating x-rays, they are constantly losing energy.

- To restore the energy loss on each revolution, radio frequency (RF) cavities are installed in the storage ring to replenish the radiative energy losses.

- Particles are grouped together by the action of the radio frequency (RF) cavities into bunches. At the APS:
  - typically about 100 psec FWHM (about 3 cm in length)
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced “RF buckets” (stable orbit positions” around the ring)
  - minimum spacing is 2.8 nsec between bunches (determined by the RF frequency- 352MHz)

- Details of the time structure depends on the fill pattern, i.e. which RF buckets have electrons in them.





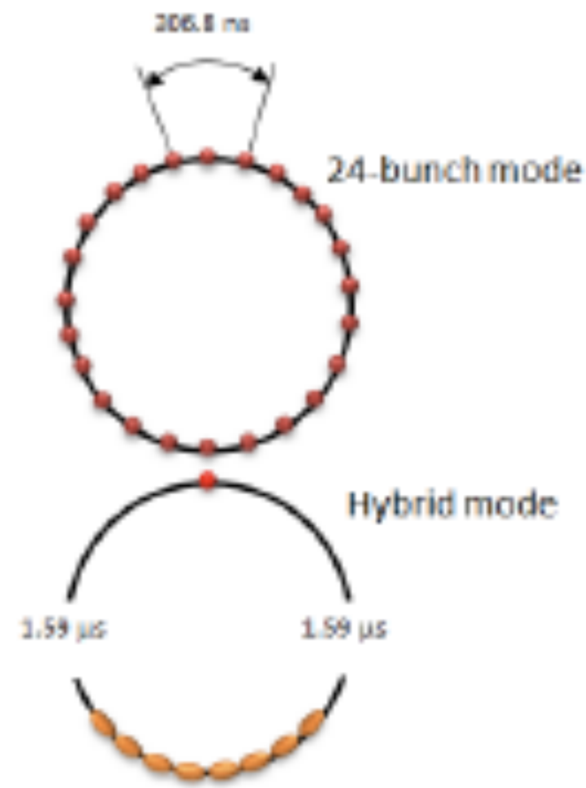
Timing and APS Filling Patterns

- The time-structure of the radiation depends on how the electron bunches are positioned around the ring.

- 324 equally spaced bunches
  - approximately 11 nsec between bunches
  - approximates a quasi-continuous source

- 24 equally spaced bunches
  - approximately 154 nsec between bunches
  - compromise between quasi-continuous source and pulsed source

- 1 + 7x8 (hybrid mode)
  - a single bunch followed by 8 groups of 7 bunches
  - Used for timing experiments (pump/probe)



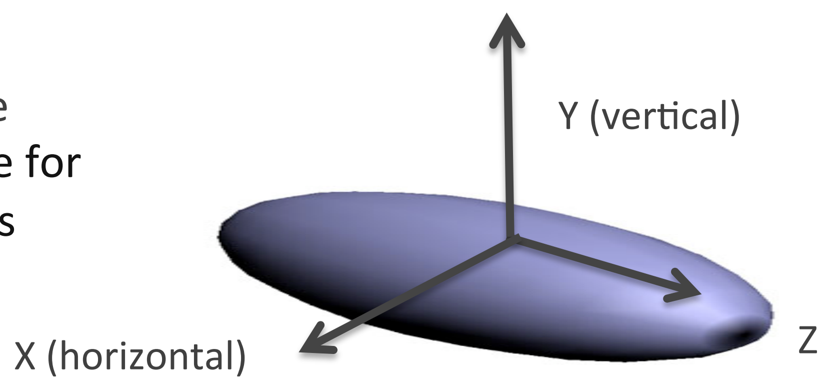
Transverse Properties of Particle Beams

- Up until now, we have calculated the radiation properties from a single electron, however in a storage ring, the radiation is emitted from an ensemble of electrons with some finite size and divergence distribution.

- Both the transverse and longitudinal properties of the particle beam in a storage ring are the equilibrium properties of the particle beam, but here we are interested in the ***transverse*** properties.

- The product of the particle beam size and divergence in the horizontal (X) direction is (proportional) to a parameter of the beam called the horizontal emittance or ε_H (units are length x angle). Similarly for the vertical (Y).

- The horizontal and vertical emittance is a constant around the length of the storage ring, although one can trade off beam size for divergence as long as the product remains constant.



X, Y are transverse directions
Z is the longitudinal direction

Why do we need to know about the transverse particle beam properties?

- Although the flux from undulators can be determined without detailed knowledge of the source size and divergence, one very important characteristic of the x-ray beam, namely **brightness,** requires a more detailed knowledge of the particle beam's size and divergence.

Recall, Brightness has units of: photons/sec/0.1% BW/source area/source solid angle

$$\text{Flux}/4\pi^2 \, \Sigma_h \, \Sigma_v \, \Sigma_h' \, \Sigma_v'$$

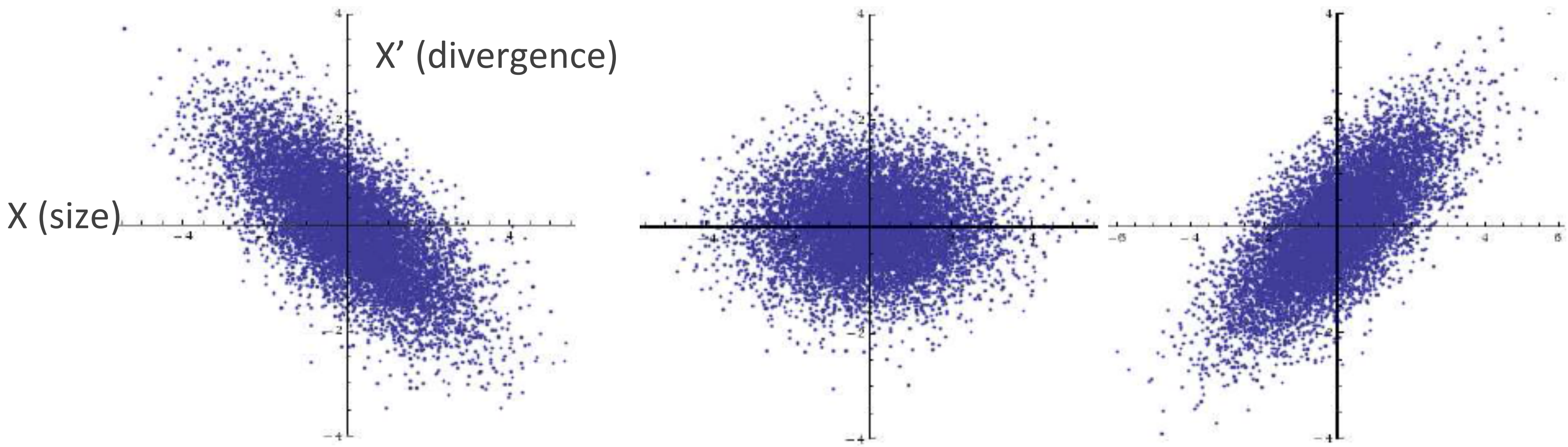
See Appendix 5 for more details

where Σ_i (Σ_i') is the effective one sigma value of the source size (divergence) in the i^{th} direction. If Gaussian distributions are assumed for both the particle beam and the radiation itself, the resultant source size and divergence is the quadrature sum of the two components, namely:

$$\Sigma_i = [\sigma_r^2 + \sigma_i^2]^{1/2} \quad \text{and} \quad \Sigma_i' = [\sigma_r'^2 + \sigma_i'^2]^{1/2}.$$

- When the electron emittance ↓ then brightness ↑

Shape of the Particle's Phase Space Varies Around the Ring but not the Area



For ease of visualization we will consider two-dimensional projections of the six-dimensional space. In this case, it could be X (size) and X' (divergence).

Diffraction Limited Sources and Coherence

- The Heisenberg Uncertainty Principle sets a lower limit for the emittance of radiation. Recall:

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\frac{p_x}{p_z} = \phi_x \text{ or } \frac{\Delta p_x}{p_z} = \Delta \phi_x \text{ and } p_z = \hbar k = \hbar (2\pi / \lambda)$$

$$so : \Delta x \Delta p_x = \Delta x \Delta \phi_x p_z = \Delta x \Delta \phi_x [\hbar (2\pi / \lambda)] \geq \hbar / 2$$

$$\Delta x \Delta \phi_x \geq \lambda / 4\pi$$

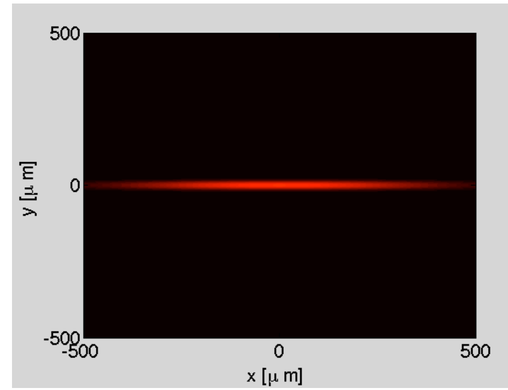
- So once the particle beam emittance for a given x-ray wavelength λ is less than $\lambda/4\pi$, the source is said to be at the diffraction limit in that direction. If both the horizontal and vertical emittance of the beam is less than $\lambda/4\pi$, then the radiation emitted is fully coherent. For 1Å (12 keV) x-rays, the particle beam emittance would have to be less than:

1Å $/4\pi$ = 10^{-10} meters $/4\pi \approx 8 \times 10^{-12}$ m or 8 picometers – radian (radians are dimensionless)

- Coherence can be an important parameter in some experiments such as photon correlation spectroscopy, x-ray holography, imaging, etc.

Partial Coherent Sources

For 1Å (12 keV) x-rays → 8 picometers – radian for fully coherent beam.



- APS operates with:

  ε_H = 3 x 10^{-9} m-rad or 3000 picometer-radian

  ε_V = 0.025 x 10^{-9} m-rad or 25 picometer-radian

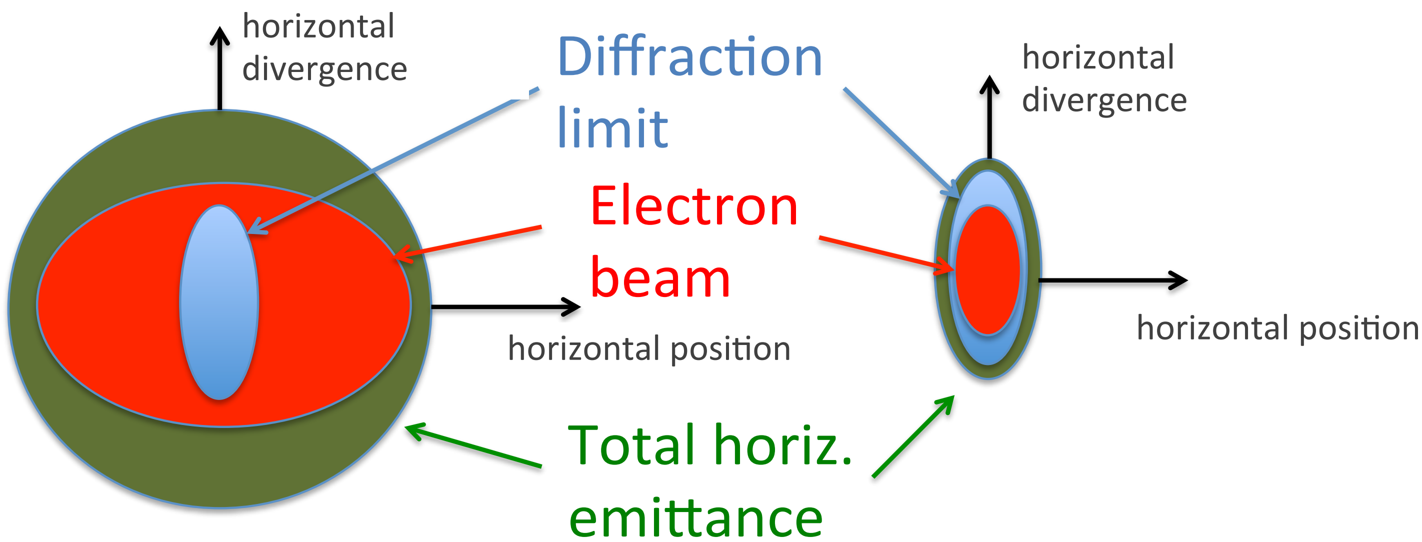
- Hence the APS is a partially coherent source at 1 Å.
- Partially coherent sources are sometimes characterized by the coherent fraction.

  Coherent fraction = ratio of diffraction-limited emittance to total emittance,
  or the the fraction of the x-ray flux that is coherent.

- For the APS at 1Å, the coherent fraction is ≈ 10^{-3}.
- So there is a general trend to try to reduce the particle beam emittance to increase coherence.

Coherence and Coherent Fraction

So once the particle beam emittance for a given x-ray wavelength λ is less than $\lambda/4\pi$, the source is said to be at the diffraction limit in that direction. For 1Å (12 keV) x-rays, the particle beam emittance would have to be less than:



Coherent fraction = ratio of diffraction-limited emittance to total emittance

$$f_{coh} = \frac{F_{\mathrm{coh,T}}(\lambda)}{F(\lambda)} = \frac{\sigma_\gamma \sigma'_\gamma}{\sigma_{\mathrm{Tx}}\sigma_{\mathrm{Tx'}}} \frac{\sigma_\gamma \sigma'_\gamma}{\sigma_{\mathrm{Ty}}\sigma_{\mathrm{Ty'}}}$$

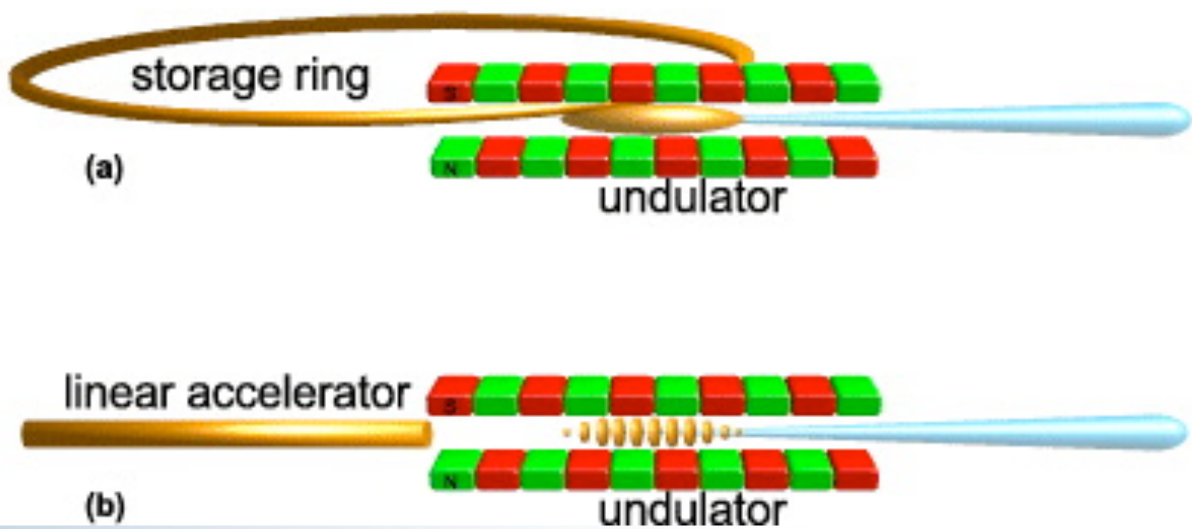
$$\sigma_\gamma \sigma'_\gamma = \frac{\lambda}{4\pi}$$

X-ray Free Electron Lasers (X-FELs)

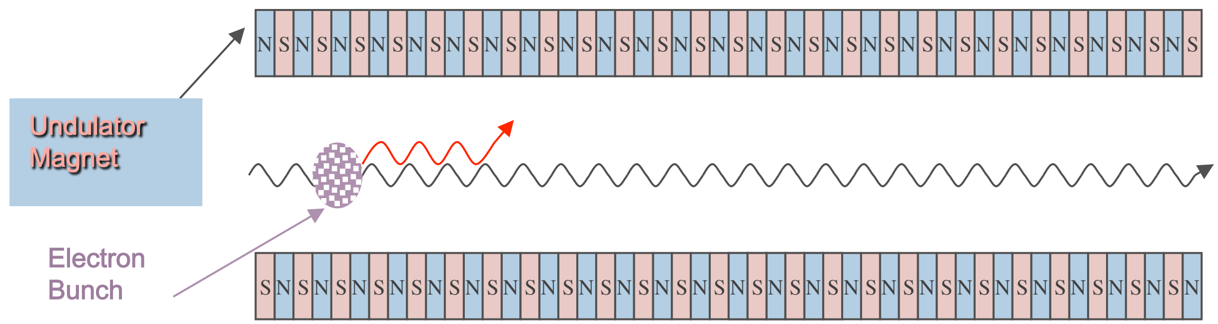
- One way to reduce the particle beam emittance is through linac-based x-ray free electron lasers (XFELs)
  - Full transverse (spatial) coherence
  - Femtosecond pulse lengths!

- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an emittance-preserving linac.

- The gain in the laser is obtained through a process called **Self-Amplified Spontaneous Emission** *or* **SASE.**

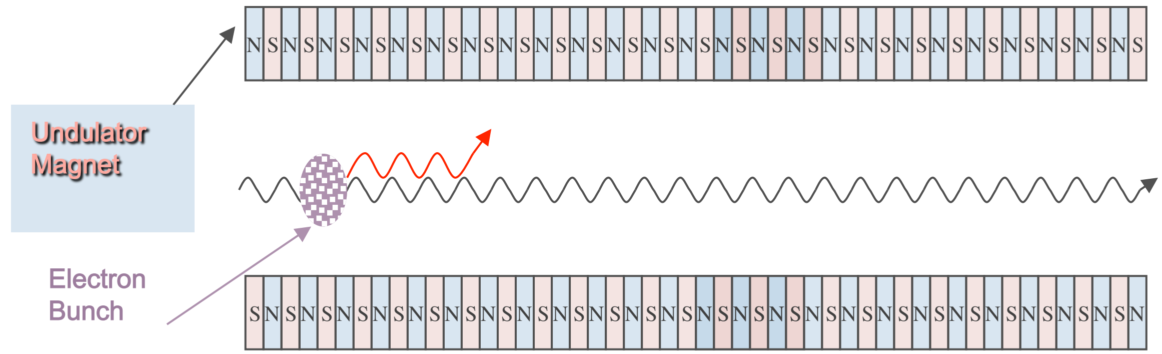


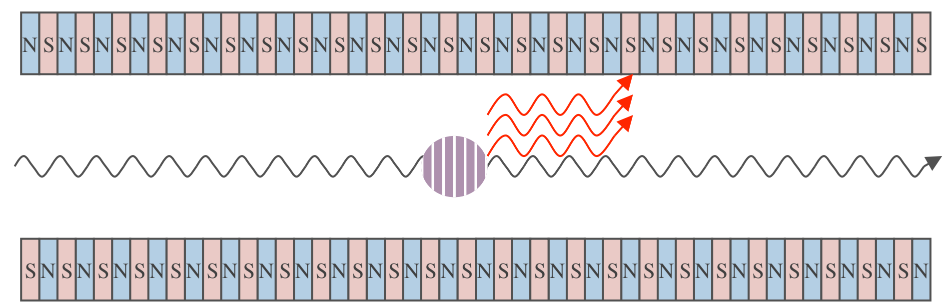
Self-Amplified Spontaneous Emission (SASE)



- The LCLS produces extraordinarily bright pulses of synchrotron radiation in a process called “self-amplified spontaneous emission” (SASE).
- In this process, an intense and highly collimated electron beam travels through an undulator magnet emitting synchrotron radiation as it goes.

Micro-bunching



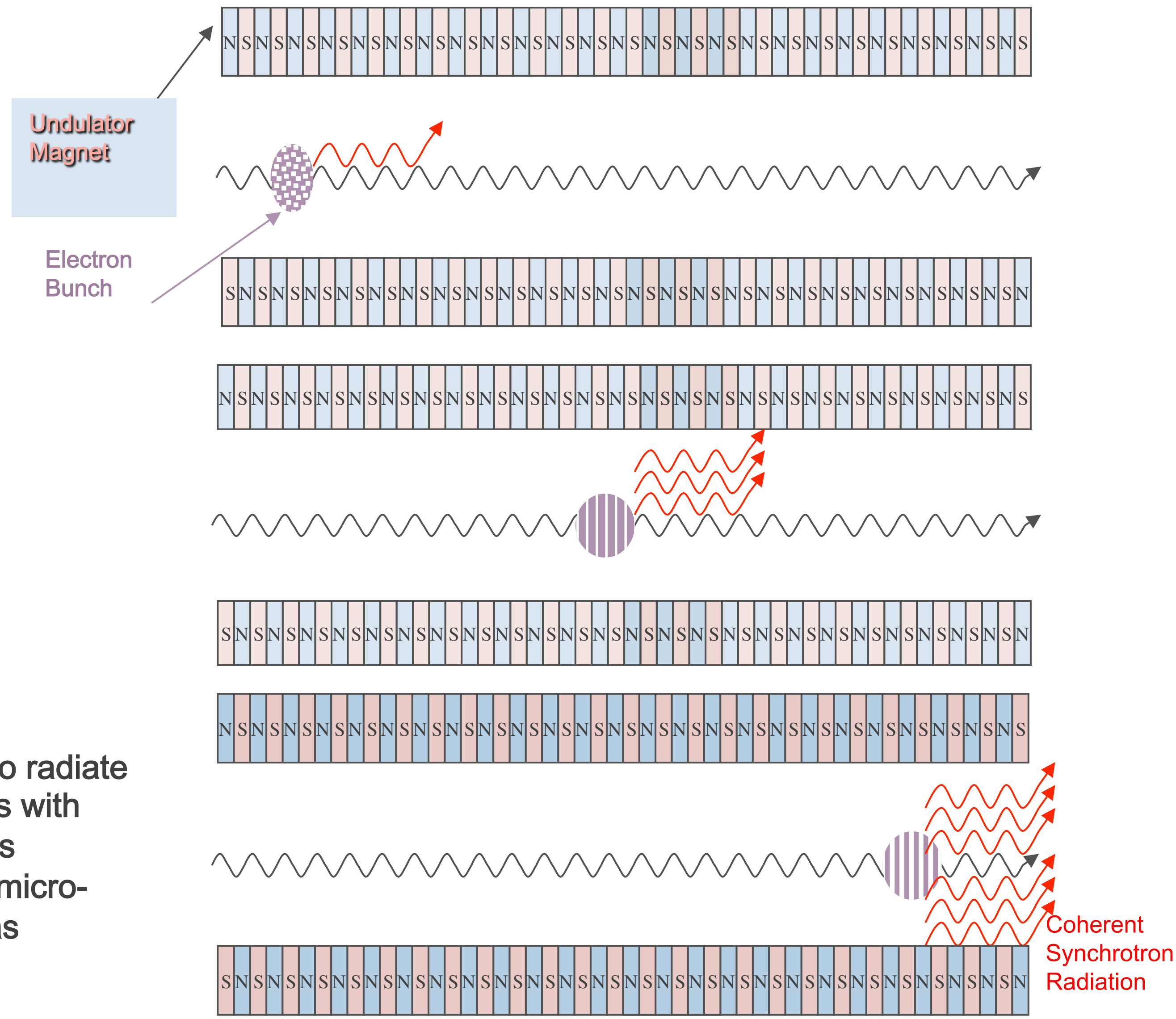


- The electron beam and its synchrotron radiation are so intense that the electron motion is modified by the electric and magnetic fields of its own emitted synchrotron light. Under the influence of both the undulator magnet and its own synchrotron radiation, the electron beam is forced to form micro-bunches,

separated by a distance equal to the wavelength of the emitted radiation.

John Galayda, LCLS

Spontaneous Emission



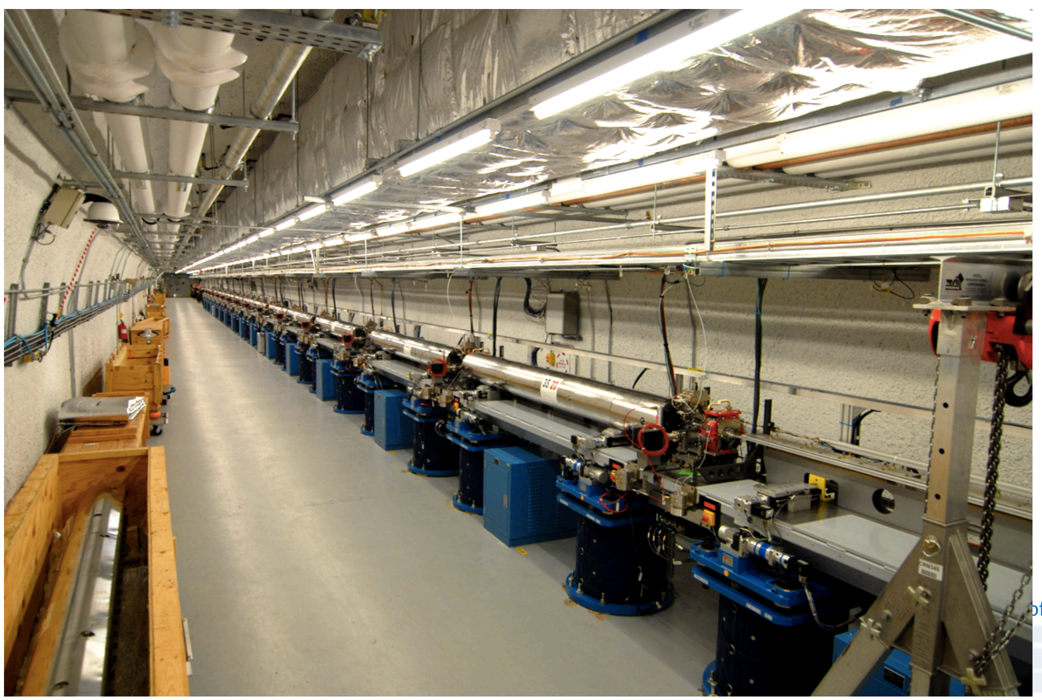
- These micro-bunches begin to radiate as if they were single particles with immense charge. The process reaches ***saturation*** when the micro-bunching process has gone as far as it can go.

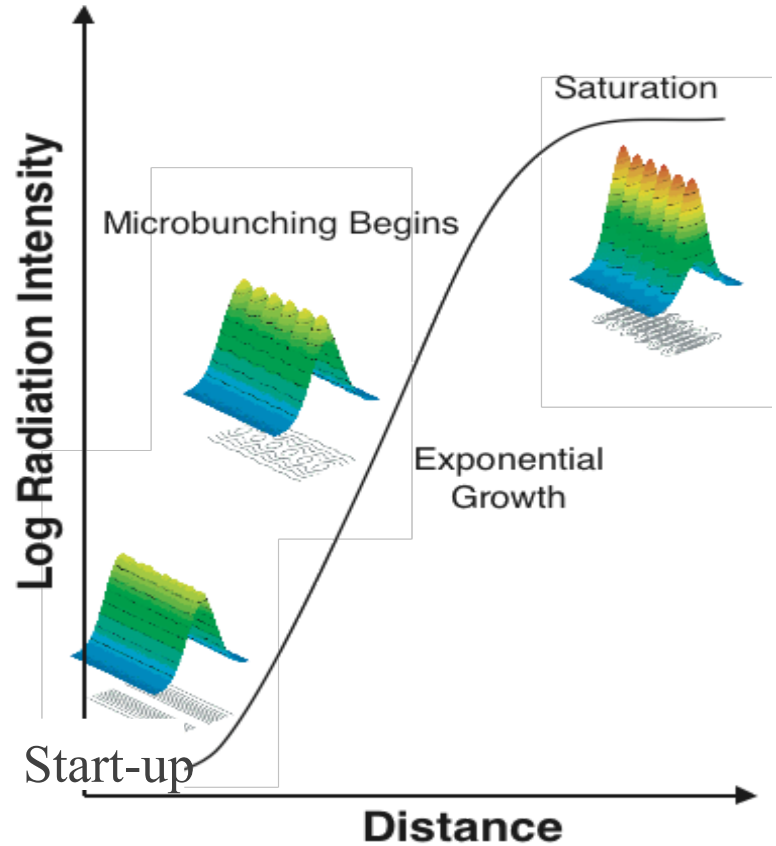
Free Electron Lasers (FELs)

Start-up stage

- External signal or spontaneous radiation interacts with the e-beam resonantly at undulator λ

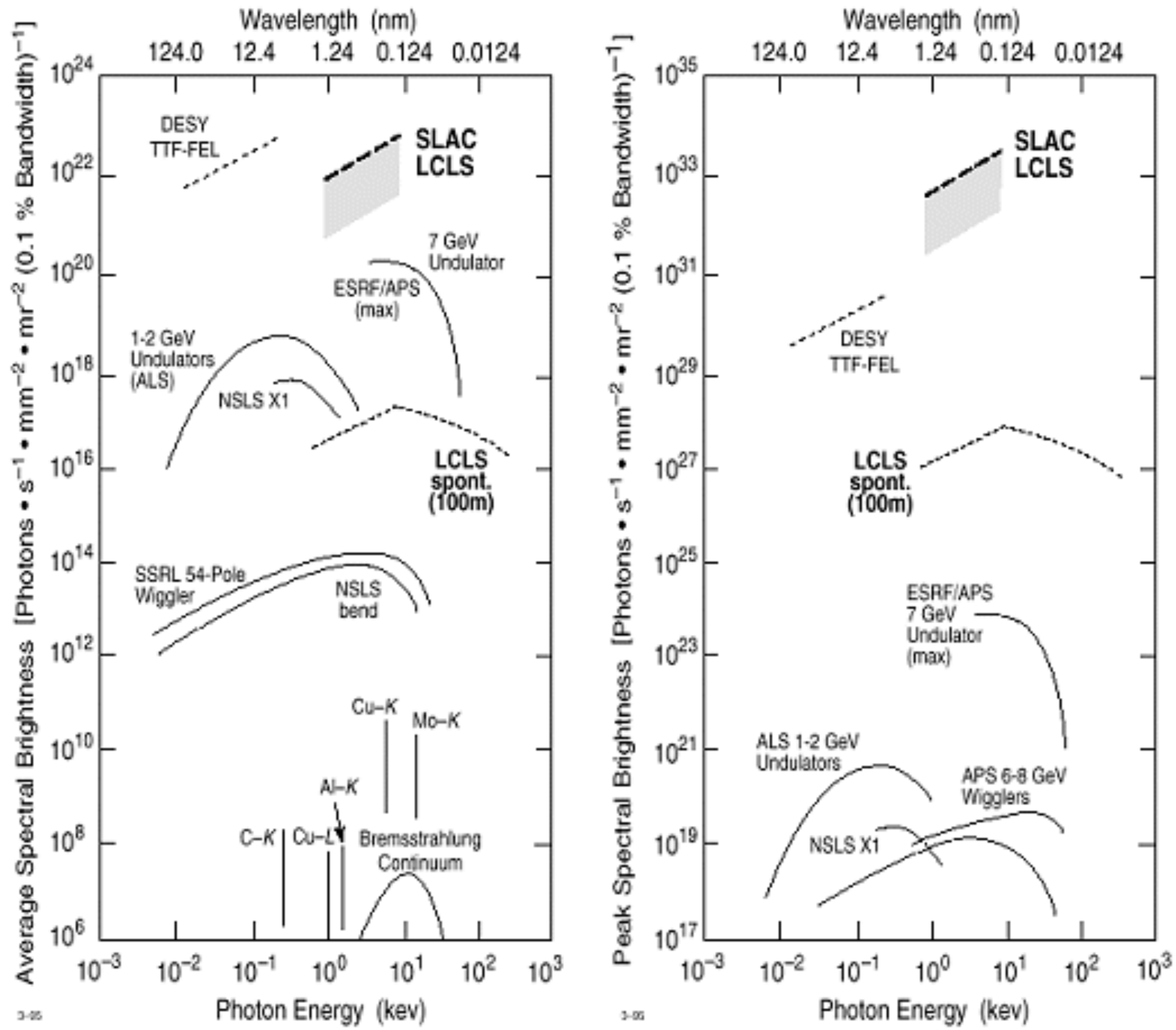
The Undulator Hall at the Linac Coherent Light Sources (LCLS) at Stanford.



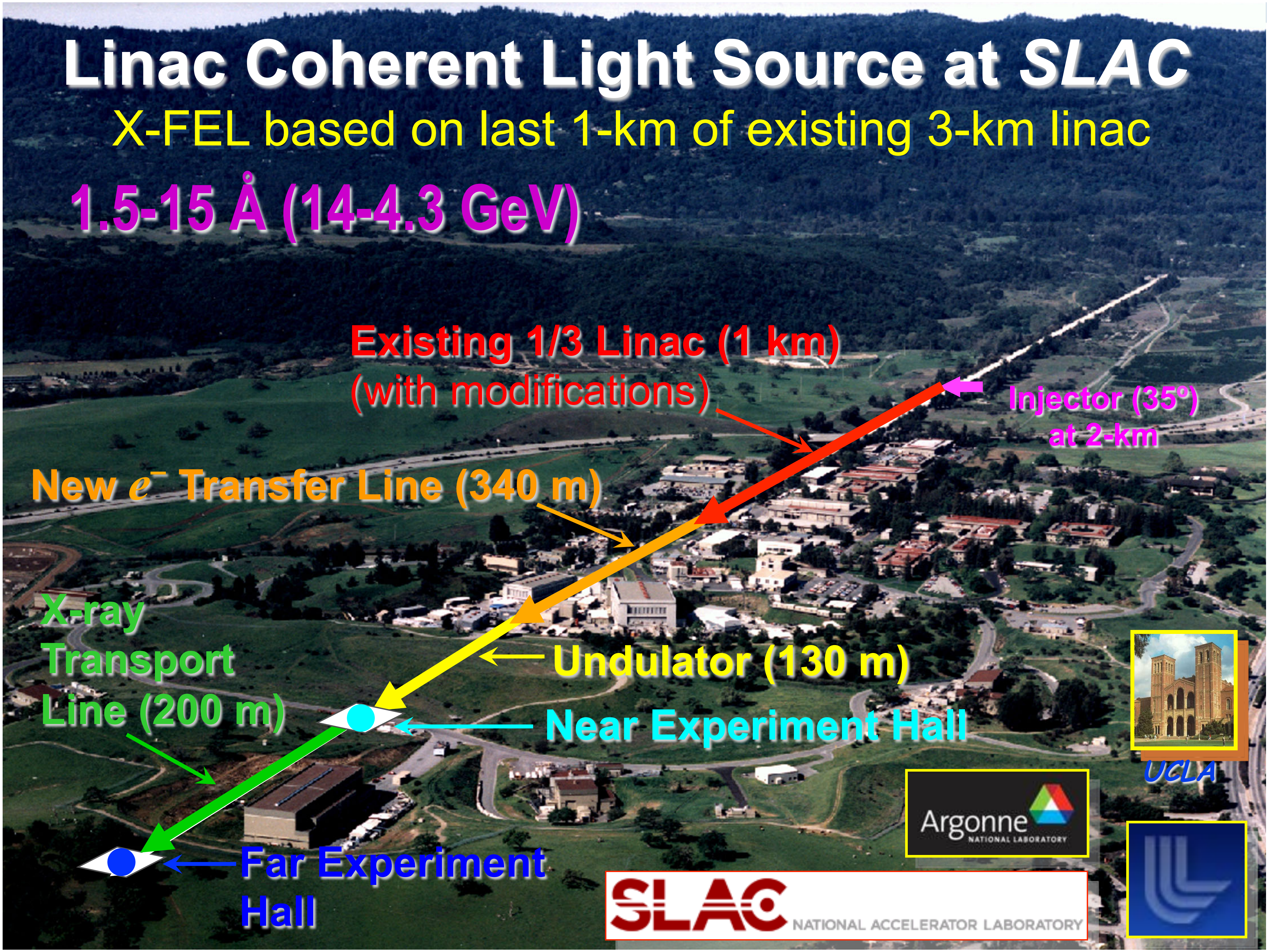


Energy modulation ➔
density modulation (microbunching) ➔
coherent radiation at l ➔
exponential growth of radiated x-rays

Spectral Properties for X-ray Free Electron Lasers



Average and peak brightness calculated for the LCLS and for other facilities operating or under construction.


Linac Coherent Light Source at SLAC
X-FEL based on last 1-km of existing 3-km linac
1.5-15 Å (14-4.3 GeV)
Existing 1/3 Linac (1 km)
(with modifications)
Injector (35°)
at 2-km
New e^- Transfer Line (340 m)
X-ray
Transport
Line (200 m)
Undulator (130 m)
Near Experiment Hall
Far Experiment
Hall
UCLA
Argonne
NATIONAL LABORATORY
SLAC
NATIONAL ACCELERATOR LABORATORY

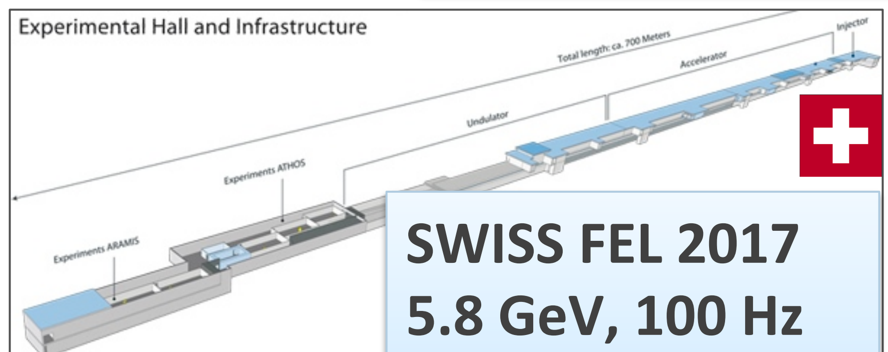
Hard X-Ray FELs in Operation & Under

LCLS-I, II 2009, 2018
14.5 GeV, 120 Hz NC

SACLA 2011
8.5 GeV, 60 Hz NC

PAL XFEL 2015
10 GeV, 100 Hz NC

XFEL 2015
17.5 GeV, 3000 x 10 Hz SC

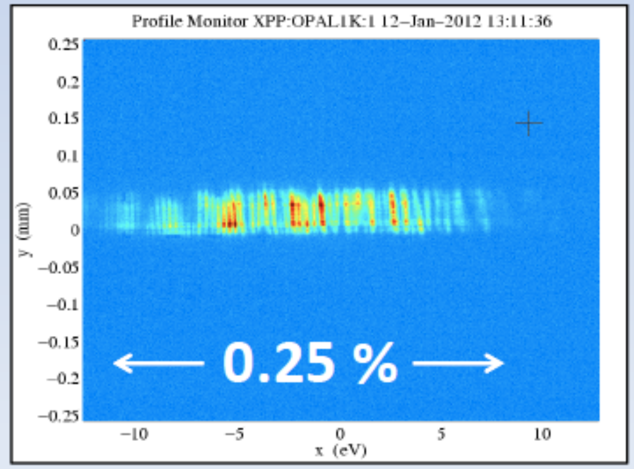


SWISS FEL 2017
5.8 GeV, 100 Hz NC

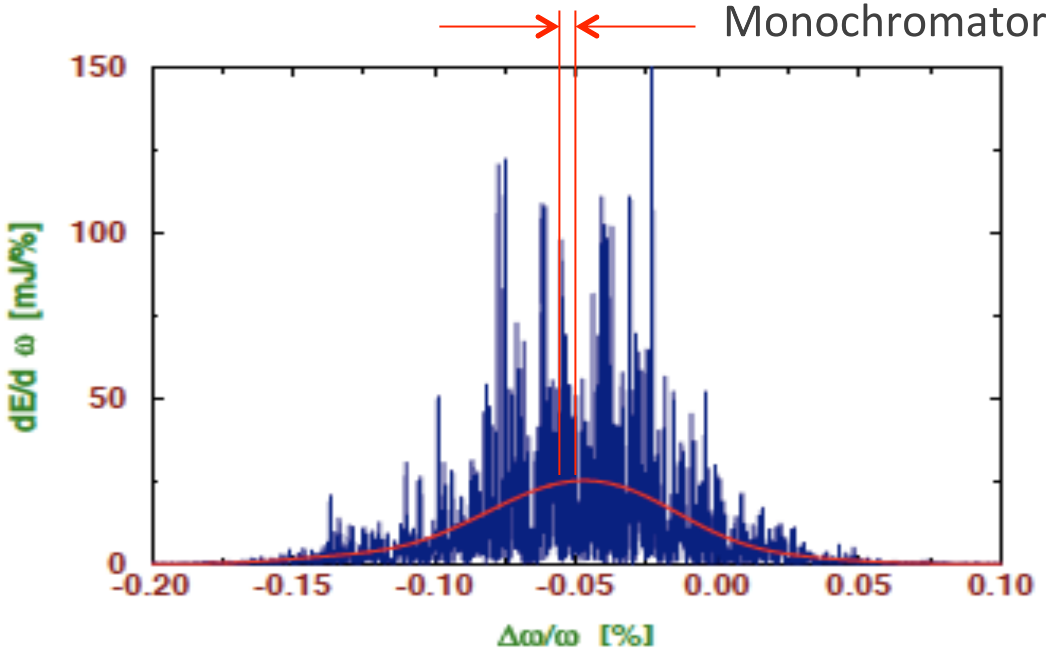
The Self-Amplified Spontaneous Emission (SASE) Process

- The lasing starts up from the random microbunching (i.e., shot noise) on the electron beam instead of being coherently produced by an input "seed" source.

Measured SASE spectrum (LCLS)



Monochromator acceptance



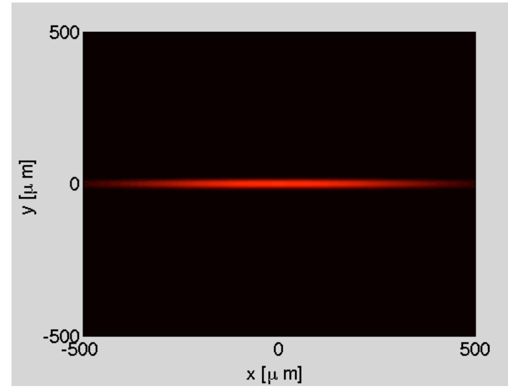
High flux on sample

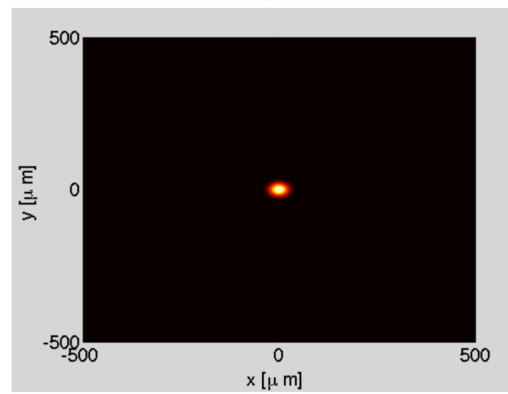
Beyond the Capabilities of Today's Storage Rings

- What about storage rings - can we reduce the the emittance of storage rings significantly? **YES!**

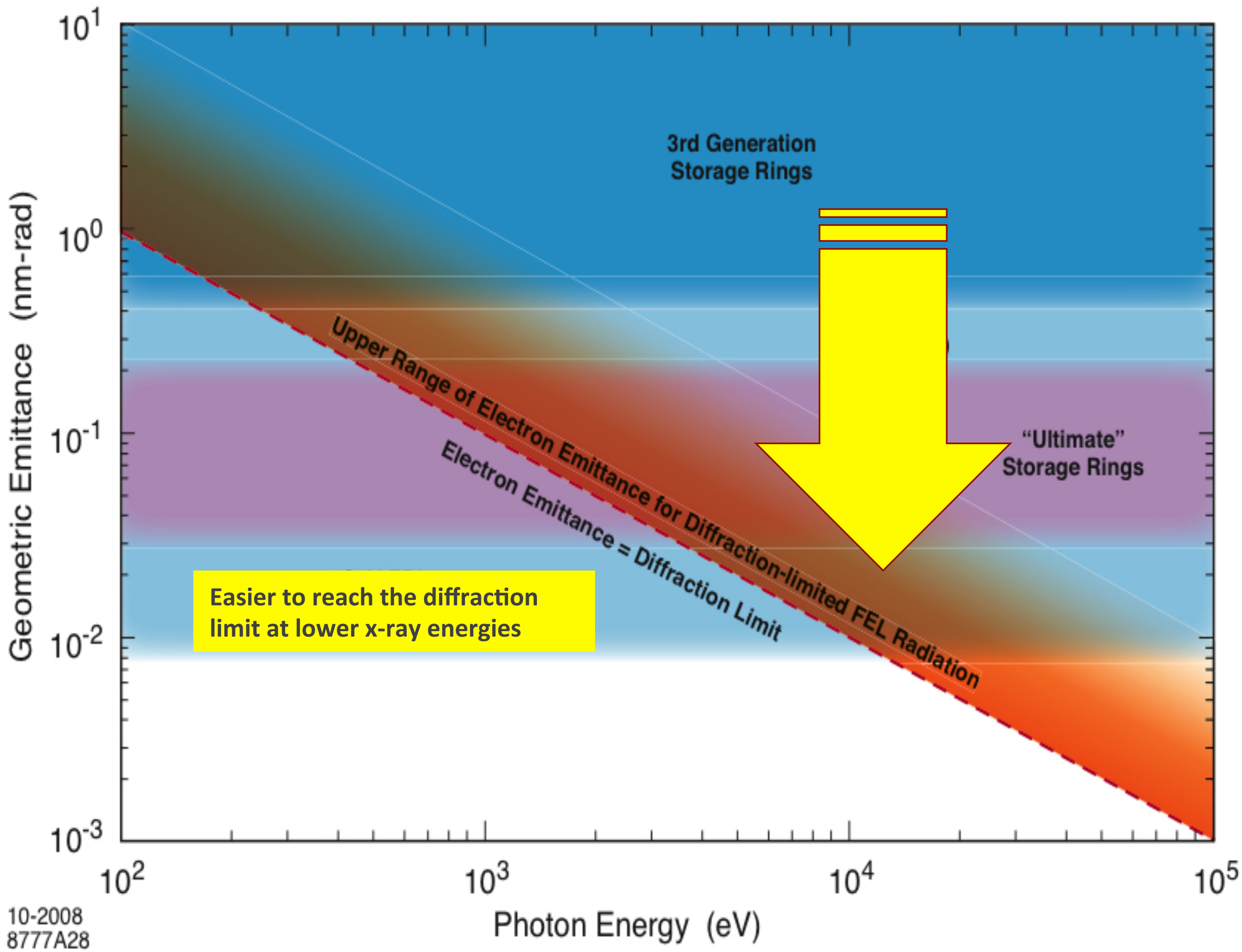
- Broad consensus in the accelerator community that much lower emittance is possible
  - Advances in simulation fidelity and sophistication
  - Demonstration of few-picometer vertical emittances
  - Success of top-up allows short beam lifetime
  - Advances in beam diagnostics and machine correction
  - Demonstration of sub-micron and sub-microradian beam stability

- World-wide activity to develop ultra-bright storage ring x-ray sources based on MBA lattices
  - MAX-IV (3 GeV, 200 pm-rad in Sweden) and SIRIUS (3 GeV, 280 pm-rad Brazil) under construction





The Goal of New Storage Ring Sources is to Reach the Diffraction Limit and Hence Full Coherence



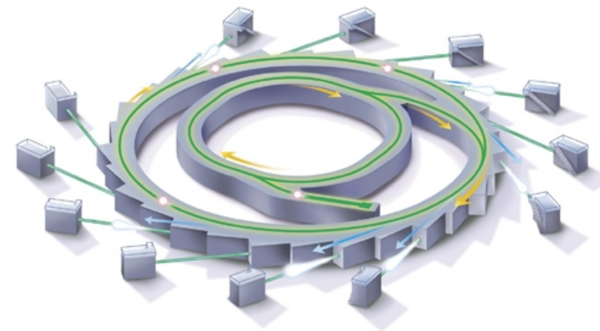
ANL-08/39
BNL-81895-2008
LBNL-1090E-2009
SLAC-R-917

Low Emittance Storage Rings

- In July 2013 the DOE's Basic Energy Sciences (BES) convened a Subcommittee of the BES Advisory Committee (BESAC) to provide them with *"advice on the future of photon sources and science."*

- *Diffraction Limited Storage Rings and U.S. Storage Ring Upgrades:*
  - *At best the present plans for upgrades of U.S. storage rings will leave the U.S.* ***behind the international community*** *in this area of x-ray science. The Office of Basic Energy Sciences should ensure that U.S. storage ring x-ray sources reclaim their world leadership position.* ***This will require a careful evaluation of present upgrade plans to determine paths forward that will guarantee that U.S. facilities remain at the cutting edge of x-ray storage ring science.***

Grand Challenge Science on Diffraction-Limited Storage Rings



A consensus report on future opportunities from scientists at

ALS, LBNL

APS, ANL

NSLS-II, BNL

SSRL, SLAC

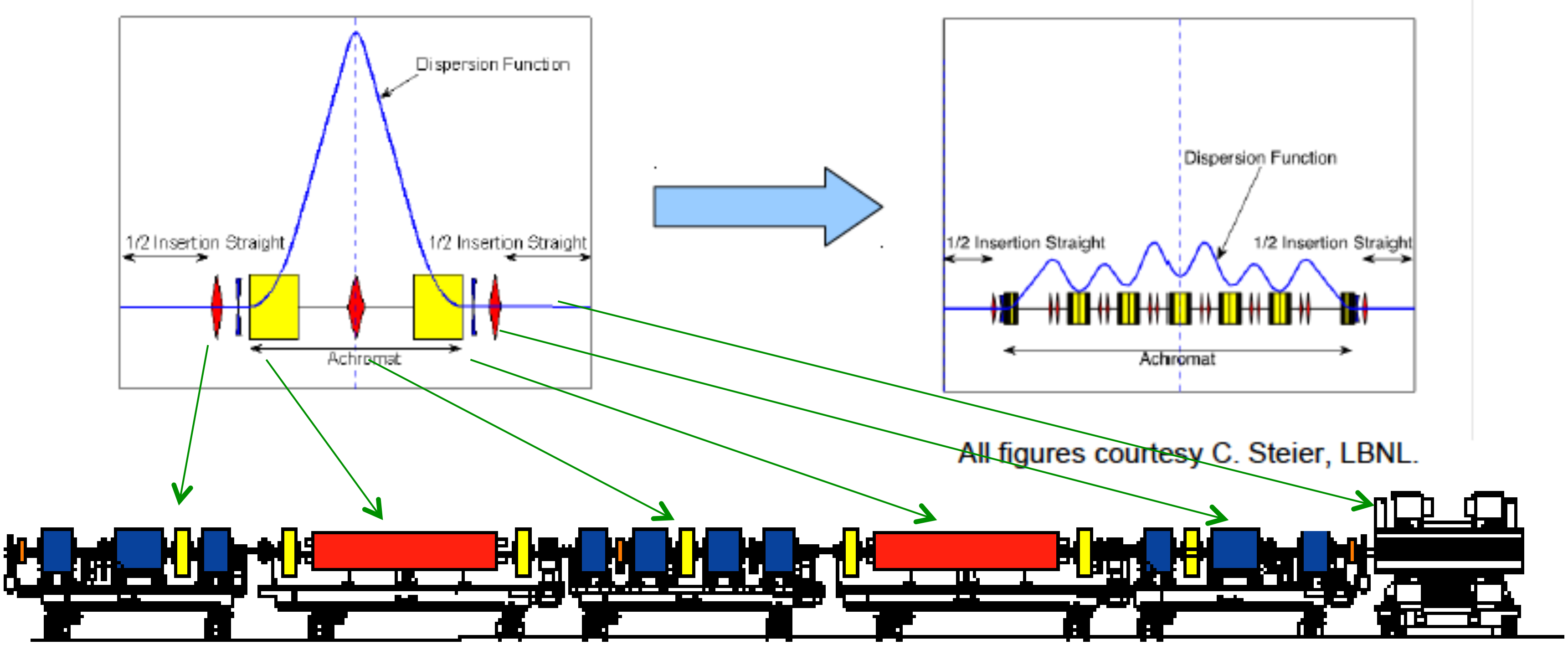
together with a broad community of scientists at laboratories and universities.

BESAC Subcommittee on Future Light Sources

July 10-12, 2013

Reducing the Beam Emittance in Storage Rings with Multi-Bend Achromat (MBA) Magnetic Lattices

The particle beam emittance scales as: $$\varepsilon_0 \propto \frac{E^2}{N_D}$$ Where E is the particle energy and N_D is the number of dipole magnets in the ring

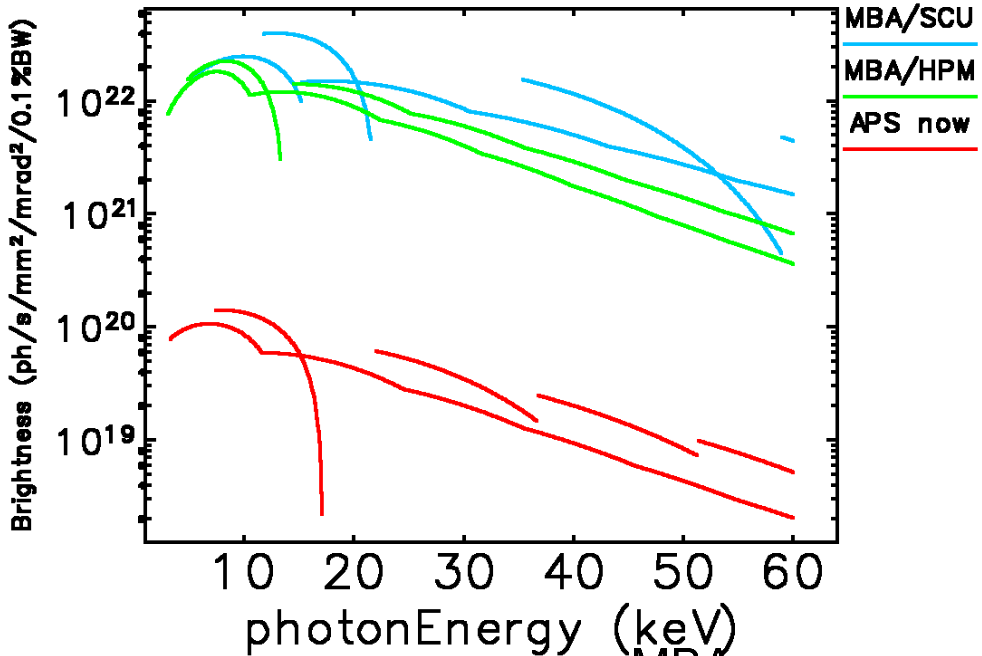


All figures courtesy C. Steier, LBNL.

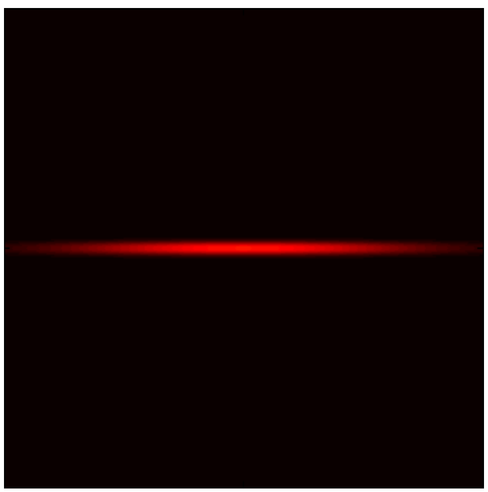
- 7 bending magnets in one sector (a seven bend achromat or 7BA) at the APS would have ~40x lower emittance than today's 2B(A) lattice.

An MBA lattice at APS: a New Generation of SR Sources

Dramatically enhance the Performance of the APS as a hard x-ray source

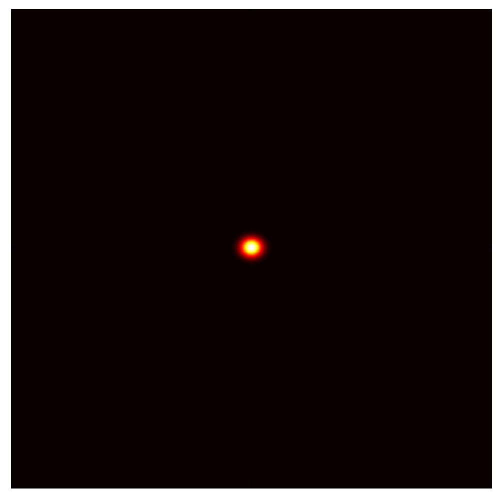


APS Now



Particle Beam Profiles

MBA



1 mm

*Courtesy Lahsen Assoufid, APS-XSD

Parameters comparisons: APS today vs APS MBA Upgrade

Quantity	Symbol	Units	APS Now	APS MBA Timing Mode	APS MBA Bright. Mode
Beam Energy	E	**GeV**	**7**	**6**	**6**
Beam current	I	**mA**	**100**	**200**	**200**
Number of bunches	N_b		**24**	**48**	**324**
Bunch duration	σ_t	**ps**	**34**	**70**	**68**
Bunch spacing	T_b	**ns**	**153**	**77**	**11**
Bunch rep. rate	f_b	**MHz**	**6.5**	**13**	**88**
Emittance ratio	$K = \epsilon_y / \epsilon_x$		**0.016**	**1.0**	**0.1**
Horizontal emittance	ϵ_x	**pm**	**3100**	**42**	**60**
Horizontal beam size	σ_x	**μm**	**275**	**7.4**	**8.8**
Horizontal beam divergence	σ_x'	**μrad**	**11**	**5.7**	**6.8**
Vertical emittance	ϵ_y	**pm**	**40**	**42**	**6**
Vertical beam size	σ_y	**μm**	**10**	**10.9**	**4.1**
Vertical beam divergence	σ_y'	**μrad**	**3.5**	**3.8**	**1.4**

Summary

- Synchrotron radiation continues to be an important tool for researchers across all scientific and engineering disciplines.
- There is a strong science case for a new generation of sources such as:
  - X-ray free electron lasers
  - Low-emittance storage rings

See Appendix 6 for a comparison of FEL and low emittance SR properties

- FELs can provide very short pulses (femtoseconds) of high peak intensity, coherent x-rays but the repetition rate is currently limited to hundreds of hertz.
  - The Linac Coherent Light Source II (LCLS II), under construction at Stanford, will increase that to the 10's of kilohertz.
- Storage rings can produce partially coherent x-ray beams with megahertz repetition rates but with lower peak power.
  - The APS is working to incorporate a low emittance lattice into the proposed upgrade of the facility (APS-U) that will produce beams of high coherence at megahertz rates.
- Both LCLS II and APS-U will keep US SR facilities at the cutting edge to produce world class science in the years to come.

Appendix 1a: Radiated Power from Charges at Relativistic Velocities

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3}a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o}\frac{dp}{d\tau} = \frac{1}{m_o}\gamma\frac{d\gamma m_o v}{dt} = \gamma^2\frac{dv}{dt} = \gamma^2\frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_oc^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

Appendix 1b: Dependence on Mass and Energy of Radiated Power

$$P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

Appendix 2: BM Spectral Distribution

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c.

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and ρ ισ the radius of curvature of the trajectory. Note that the critical energy scales as γ^3. In practical units, the critical energy can be written as:

$$E_c[\mathrm{keV}] = 2.218\ E^3[\mathrm{GeV}]/\ \rho[\mathrm{m}] = 0.06651\ B[\mathrm{kG}]\ E^2[\mathrm{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at E = 7 GeV . The critical energy of the radiation emitted from the BM is:

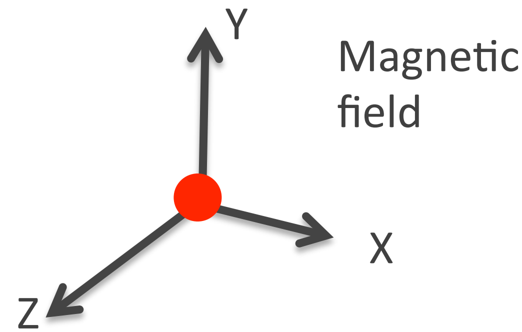
$$E_c[\mathrm{keV}] = 0.06651\ B[\mathrm{kG}]\ E^2[\mathrm{GeV}]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5\ \mathrm{keV}}\ \text{or}\ \underline{0.64\ \text{Å}}.$$

Appendix 3: Where did "K" come from?

Equation of motion for a relativistic charged particle in a magnetic field



$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi c}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0}\frac{\lambda_{ID}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \quad \textit{and} \quad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \quad \textit{where} \quad K = \left[\frac{eB_0}{2\pi}\frac{\lambda_{ID}}{m_0 c}\right]$$

Appendix 4: How Do You Get 1 Å X-rays from a 3 cm Period Magnetic Field?

Where does the $1/\gamma^2$ come from in the equation: $\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$?

1) Consider the electron in its rest frame:
   - It does not see a static magnetic field from the undulator, but rather a time-varying B-field and associated E-field (due to the relativistic transformation of the magnetic field of the device).
   - The period of the E and B field are Lorentz contracted so that: $\lambda_{e\text{-}frame} = \lambda_{ID} / \gamma$ and so the electron oscillates (and hence radiates) with that same period driven by the EM fields.

2) Back in the lab frame:
   - Due to the fact that the electron is traveling towards us, the radiation emitted by the electron is Doppler shifted to higher frequencies (shorter wavelengths). The relativistic Doppler shift goes as $\sqrt{1-\beta} / \sqrt{1+\beta} \approx 1/2\gamma$, and so the wavelength observed in the lab is:

$$\lambda_{lab} \approx (\lambda_{ID}/\gamma)(1/2\gamma) = (\lambda_{ID}/2\gamma^2)$$

Appendix 5: Calculating X-ray Beam Brightness

APS Electron Beam Parameters

APS operates with $\varepsilon_H = 3 \times 10^{-9}$ m-rad and a coupling (ratio of vertical emittance to horizontal emittance) of 0.9% , therefore

$$\varepsilon_V = 0.025 \times 10^{-9} \text{ m-rad.}$$

The particle beam source size and divergence at the locations of the IDs are:

$$\sigma_H = 270 \text{ microns}$$
$$\sigma'_H = 11 \text{ microradians}$$
$$\sigma_V = 9 \text{ microns}$$
$$\sigma'_V = 3 \text{ microradians}$$

APS Undulator Beam Parameters

APS Undulator A has a length of 2.4 meters. For 1Å radiation the natural opening angle is:

$$\sigma'_{x\text{-ray}} = 1/\gamma \, (1/N)^{1/2}$$

$$= \sqrt{[\lambda/2L]} = 4.5 \times 10^{-6} \text{ rad}$$

The corresponding source size of the radiation is:

$$\sigma_{x\text{-ray}} = \sqrt{[\lambda L/8\pi^2]} = 1.7 \text{ microns.}$$

For source size $\Sigma = [\sigma_{x\text{-ray}}{}^2 + \sigma_{electron}{}^2]^{1/2}$ and source divergence $\Sigma' = [\sigma'_{x\text{-ray}}{}^2 + \sigma'_{electron}{}^2]^{1/2}$

$$\mathcal{B} = \text{Flux}/4\pi^2 \, \Sigma_H \, \Sigma_V \, \Sigma_H' \, \Sigma_V'$$

Appendix 6: How do Storage Rings and FELs Compare?

Parameter	Storage Rings	X-ray FEL
Wavelength Range	2-3+ decades typically	1+ decades (multiple undulators)
Peak Brightness (ph/s/mr^2/mm^2/0.1%BW)	$10^{22} – 10^{24}$	$10^{31} – 10^{33}$ (10^9 times higher than SR)
Average Brightness (ph/s/mr^2/mm^2/0.1%BW)	$10^{19}– 10^{21}$	$10^{20}– 10^{22}$
Minimum Pulse Width (fs)	~10,000	~5
Coherence	Limited transverse spatial coherence	Transverse spatial coherence, limited temporal coherence without seeding
Stability: Energy Position Time	<.01% (with ~0.1% energy spread) < 0.1 σ (~10 μm H, ~0.3 μm V) < 0.1 σ (~1 ps, ~0.2 ps low α)	0.01-0.03% wo / self seeding ~0.1 σ ~100 fs
Number of Beamlines	Large (~30-60)	Limited (6 endstations per undulator)