Inelastic x-ray scattering, IXS

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Inelastic X-Ray Scattering & Spectroscopy @ APS



- Nuclear Resonant Inelastic X-Ray scattering, NFS, NRIXS: Sectors 3, 16, (30)
- Momentum Resolved High Energy Resolution IXS (HERIX) Sectors 3, 30
- Resonant Inelastic X-Ray Scattering, RIXS (MERIX): Sectors 9, 30 --> 27
- X-Ray Raman Scattering, XRS (LERIX): Sectors 13, 16, 20
- X-Ray Emission Spectroscopy, XES (MINIX): Sectors 13, 16, 17

IXS: Inelastic X-Ray Scattering

A set of **vastly different** techniques based on measuring exact:

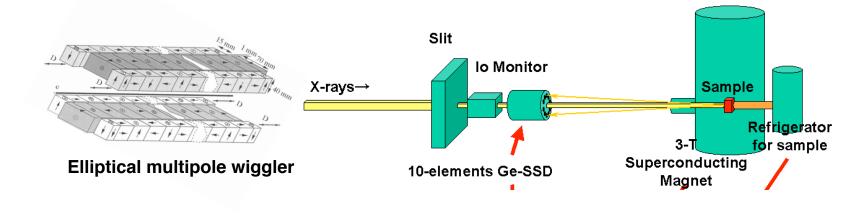
- i) energy, and
- ii) momentum transfer in a scattering experiment.

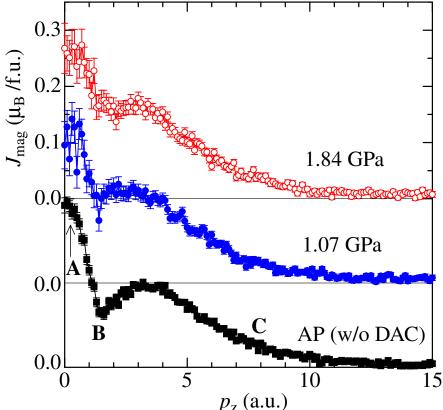
It provides thermodynamic, elastic, electronic and chemical information about the scattering system.

Since X-ray energies extend from a few eV to a few hundred keV, we need to measure energy loss or gain with a resolution changing from

```
nano-eV
meV,
eV, and
keV.
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Magnetic Compton Scattering under high-pressure: e.g. ErCo2

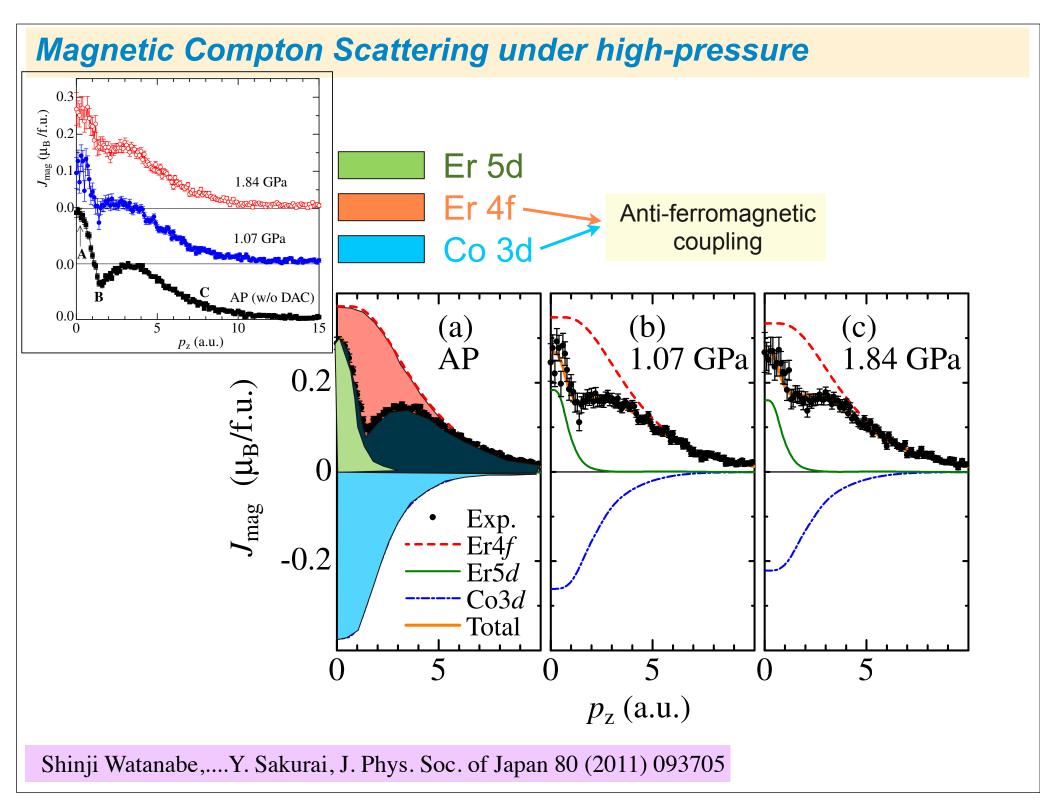




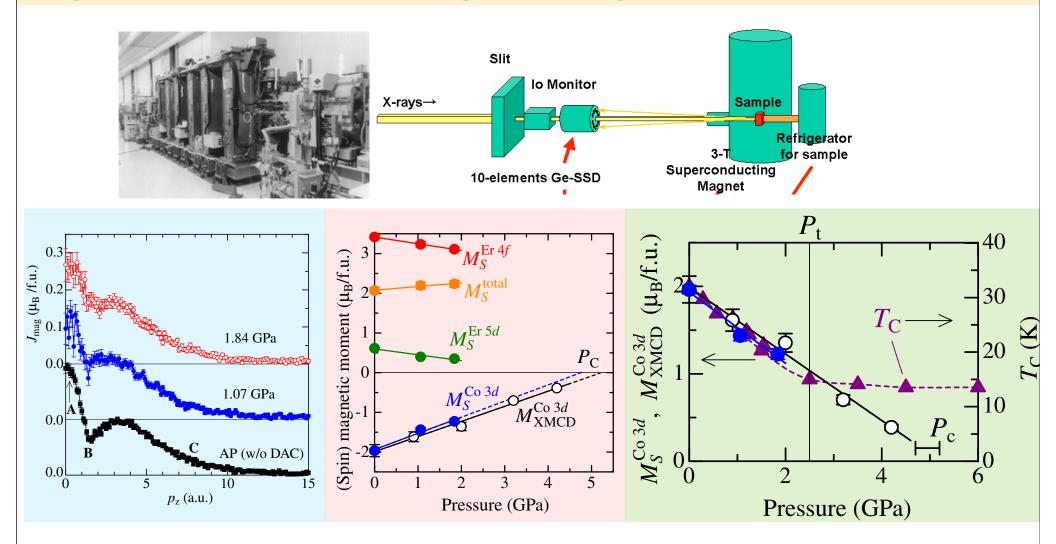
One dimensional projection of electron spin density

$$J_{\text{mag}}(\mathbf{p}_z) = \frac{1}{\mu} \iint_{\infty}^{\infty} (n_{\uparrow}(\mathbf{p}) - n_{\downarrow}(\mathbf{p})) d\mathbf{p}_x d\mathbf{p}_y$$

Shinji Watanabe,....Y. Sakurai, J. Phys. Soc. of Japan 80 (2011) 093705



Magnetic Compton Scattering under high-pressure: ErCo2



MCS can determine element specific band broadening, and magnetic moment reduction. MCS is a quantitative probe of for spin magnetic moments under high pressure.

IXS: Inelastic X-Ray Scattering

IXS can measure

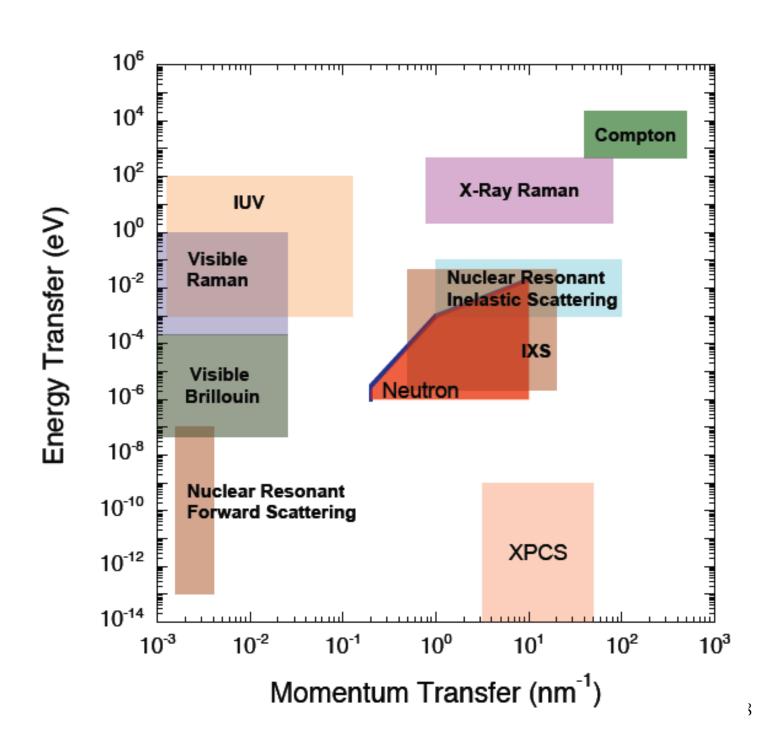
- nuclear hyperfine interactions (neV),
- collective excitations of atoms such as phonons (meV),
- electronic excitations like d-d excitations, charge-transfer energy, plasmons or magnons (eV),
- core-valence electron boundary to reconstruct Fermi surface (keV)
- determine orbital occupancies (keV)

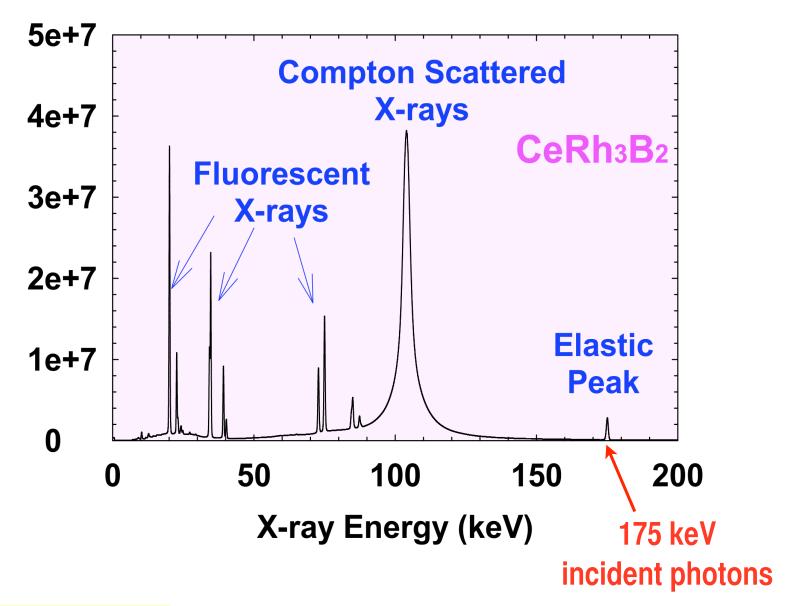
1920-1930: P. Debye, A. Compton and J. DuMond:

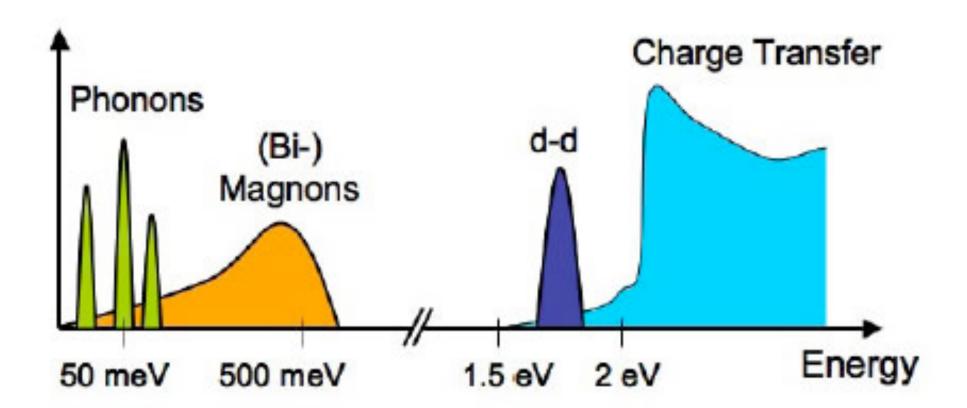
1960-1990: Development of

- i) pure silicon and germanium with $\Delta d/d \sim 10^{-9}$,
- ii) sophisticated high resolution monochromators,
- iii) low-noise detectors
- iv) crystal analyzers, and
- v) the third generation synchrotrons

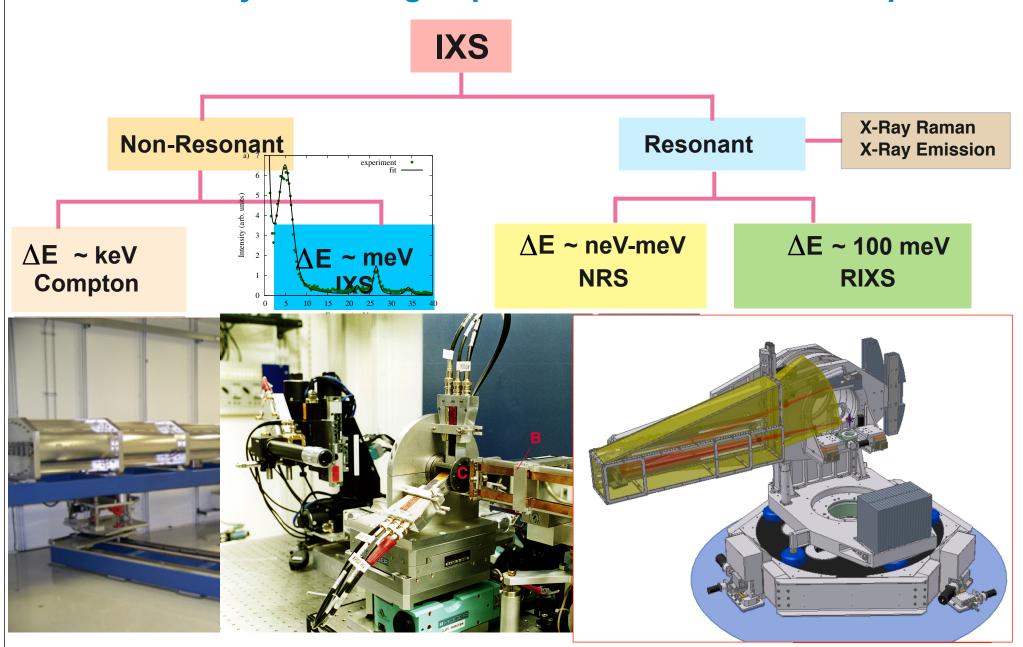
1990-present: More than a dozen new instruments around the world





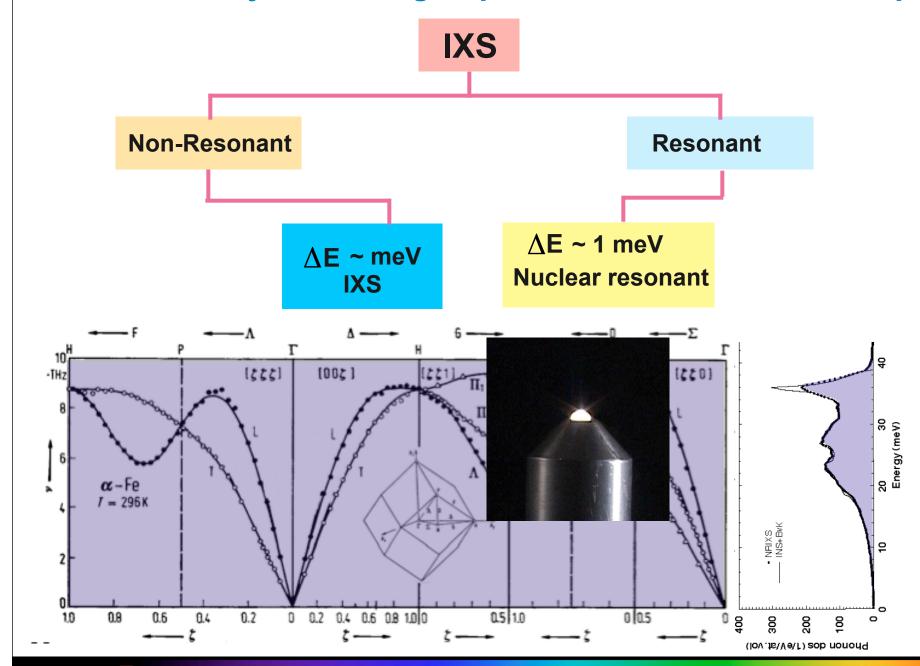


Inelastic X-Ray Scattering: A plethora of different techniques





Inelastic X-Ray Scattering: A plethora of different techniques



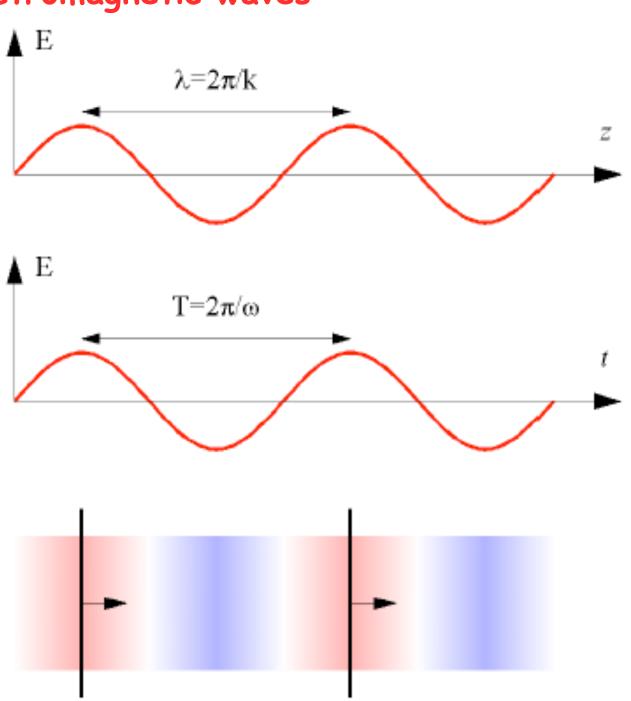


Electromagnetic waves

Spatial variation k= wave number $\lambda=$ wavelength

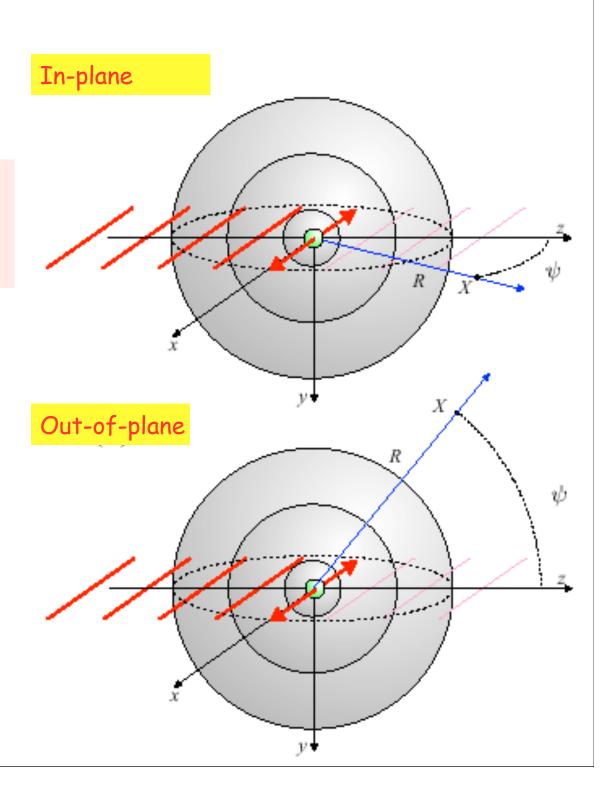
Temporal variation ω = angular frequency

Top view showing high and low field amplitudes



Classical description of scattering of radiation by a charged particle

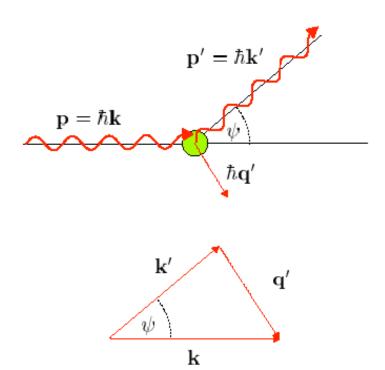
The incident plane wave incident upon an electron sets the electron in oscillation. The oscillating electron then radiates, experiencing a phase shift of π .



First order Born approximation

For weakly scattering media, it is possible to obtain solution to the integral equation by a perturbation approach, provided that the scattering medium is weakly interaction with the probe of x-rays.

The first order Born approximation states that amplitude of the scattered wave far away from the scatterer depends entirely on one and only one Fourier component of the scattering potential, namely the one that corresponds to the transferred momentum $K=k(s-s_0)$.



Conservation of momentum has a correspondence between classical and quantum mechanical treatment:

$$p = hk$$

 $\Delta p = p - p' = hk'$

If a plane wave is incident on the scatterer in the direction of s, the Fourier component of the scattering potential can be determined.

And if one has the ability to vary the amount of momentum transfer at will, then, the scattering potential can be reconstructed.

This is the essence of x-ray scattering experiments.

What is being measured?

$$\frac{d^2\sigma}{d\Omega d\omega} = r_0^2 \frac{\omega_f}{\omega_i} |\mathbf{e}_i \cdot \mathbf{e}_f| N \sum_{i,f} \left| \langle i | \sum_{i} e^{i\mathbf{Q}\mathbf{r}_j} | f \rangle \right|^2 \delta(E_f - E_i - \mathbf{h}\omega)$$

Thomson cross section

Dynamical structure factor S(Q,w)

$$S(\mathbf{Q},\omega) = \frac{1}{2\pi} \int dt \ e^{-i\omega t} \left\langle \phi_i \left| \sum_{ll'} f_l(\mathbf{Q}) e^{-i\mathbf{Q}\cdot\mathbf{r}_l(t)} f_{l'}(\mathbf{Q}) e^{i\mathbf{Q}\cdot\mathbf{r}_{l'}(0)} \right| \phi_i \right\rangle$$

Density-density correlations

$$f(Q) = f_{ion}(Q) + f_{valence}(Q)$$
 Atomic form factor



→Sum over phonon branch j at reduced momentum transfer, q

Sum over different atoms in the unit cell

Atomic form factor for each atom

scaling with square root of mass

→ Debye-Waller factor to account for bond strength

phonon occupation probability

$$S(Q,\omega) = \sum_{q,j} \left| \sum_{s} f_{s}(Q) \frac{\hbar}{\sqrt{2m_{s}}} e^{-W_{s}} e^{i\vec{Q} \cdot \vec{R}_{s}} \left[\vec{Q} \cdot \vec{e}(q,s,j) \right] \right|^{2} \frac{\left(\frac{1}{e^{\hbar\omega/kT} - 1} + \frac{1}{2} \pm \frac{1}{2} \right)}{\omega_{q,j}} \delta(\omega \pm \omega_{q,j})$$

Phase of the scattering amplitude ←

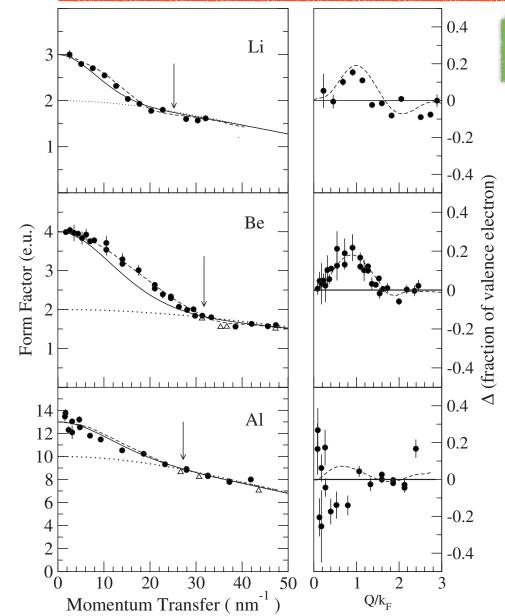
Polarization factor between momentum transfer and photon's electric field

phonon frequency

delta-function in ω

Atomic form-factor measurements in the low-momentum transfer region for Li, Be, and Al by inelastic x-ray scattering

A. Alatas,^{1,*} A. H. Said,^{1,2} H. Sinn,¹ G. Bortel,^{1,†} M. Y. Hu,^{1,‡} J. Zhao,¹ C. A. Burns,² E. Burkel,³ and E. E. Alp¹



$$f(Q) = f_{ion}(Q) + f_{valence}(Q)$$

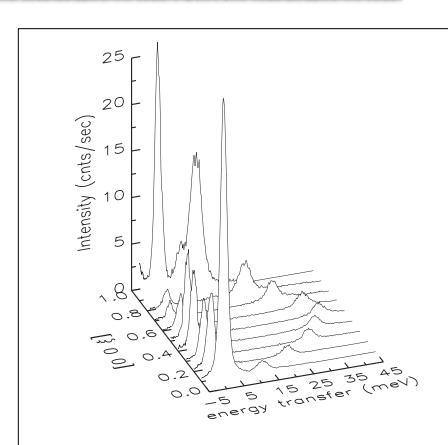


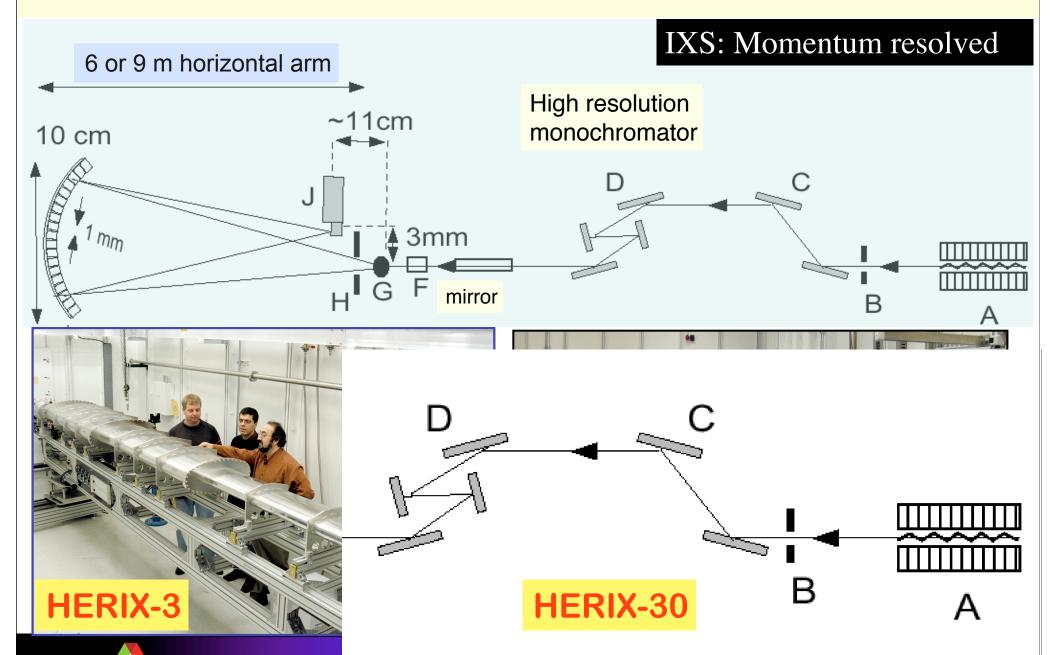
FIG. 2. Energy scans for lithium along the $[0\ \zeta\ \zeta]$ direction for longitudinal modes.

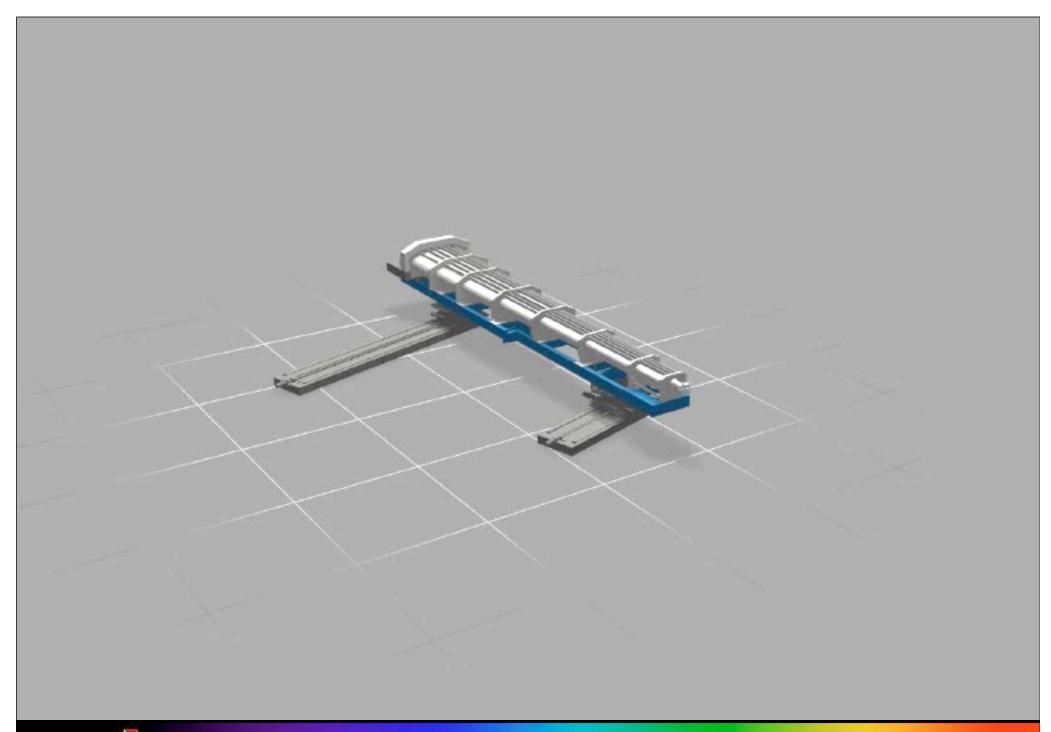
Scattering geometry and physics

The physical origin of the correlations depend on how $1/\mathbf{q}$ compares with the characteristic length, l_c , of the system, which is related to spatial inhomogeneity (due to thermal or concentrations fluctuations)

```
when \mathbf{q} \cdot l_c << 1 \Rightarrow Collective excitations when \mathbf{q} \cdot l_c >> 1 \Rightarrow Single particle excitations when \frac{1}{\mathbf{q}} \approx d and \omega \approx phonon frequency \Rightarrow Collective ion excitations (PHONON) when \frac{1}{\mathbf{q}} \approx r_c and \omega \approx plasma frequency \Rightarrow Valence electron excitations
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Inelastic % Ray Scattering two tapproaches at Sector 3 and Sector 30





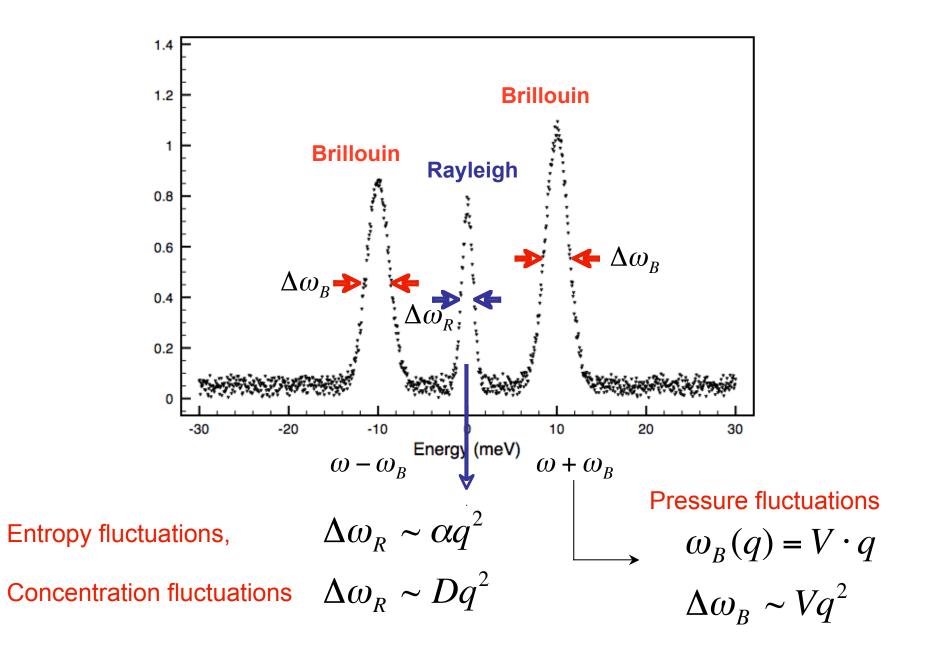


30-ID-C: HERIX Spectrometer



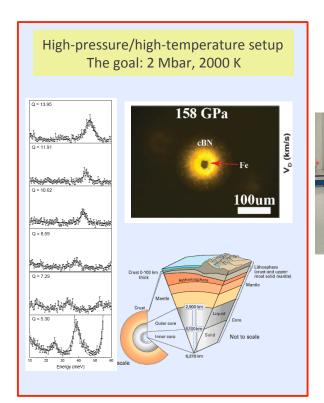
30-ID-C: HERIX Spectrometer





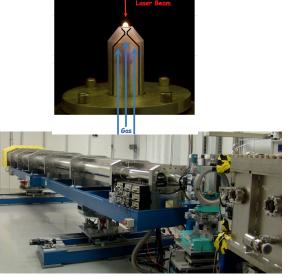
High Energy Resolution Inelastic X-ray Scattering

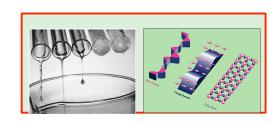
APS-U will provide two enhanced HERIX spectrometers optimized for high-pressure and high-resolution work at HERIX-3-ID and HERIX-30-ID, respectively.



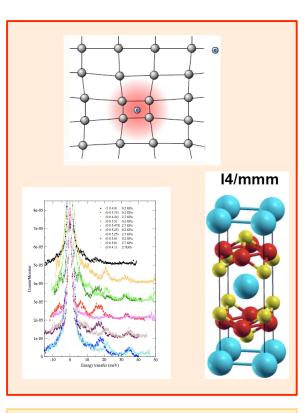
Mineral physics and earth sciences

Sound velocity, elastic constants





Dynamics of liquids and liquid crystals stripe or nematic phases



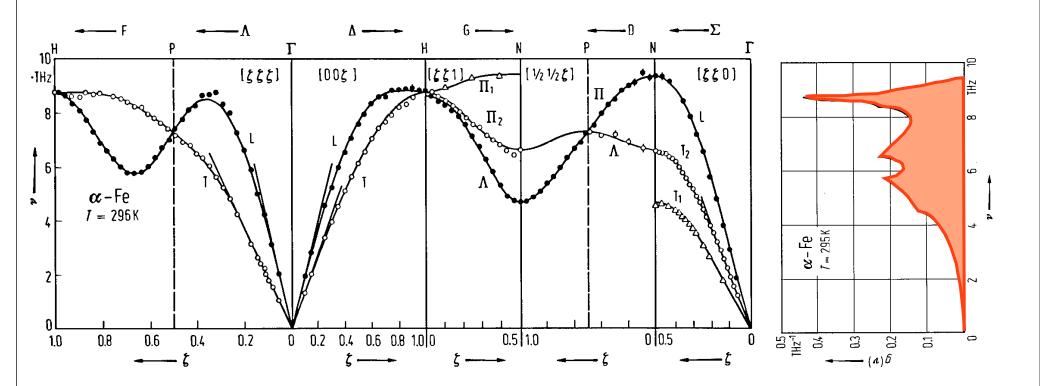
Role of phonons in pnictide superconductors



$\phi\omega\nu\dot{\eta}$ (phonē), sound

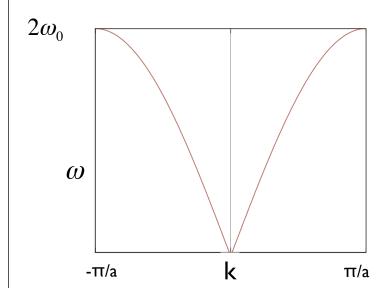
- Phonons are periodic oscillations in condensed systems.
- They are inherently involved in thermal and electrical conductivity.
- They can show anomalous (non-linear) behavior near a phase transition.
- They can carry sound (acoustic modes) or couple to electromagnetic radiation or neutrons (acoustical and optical).
- Have energy of $\hbar\omega$ as quanta of excitation of the lattice vibration mode of angular frequency ω . Since momentum, $\hbar k$, is exact, they are delocalized, collective excitations.
- Phonons are bosons, and they are not conserved. They can be created or annihilated during interactions with neutrons or photons.
- They can be detected by Brillouin scattering (acoustic), Raman scattering, FTIR (optical).
- Their dispersion throughout the BZ can ONLY be monitored with x-rays (IXS), or neutrons (INS).
- Accurate prediction of phonon dispersion require correct knowledge about the force constants: COMPUTATIONAL TECHNIQUES ARE ESSENTIAL.

Dispersion relations and phonon density of states lpha -iron (bcc)



V. J. Minkiewicz, G. Shirane, and R. Nathans, Phys. Rev. 162 (1967) 528, and Landolt-Börnstein, New Series, Group III, Vol 13, Eds. K.-H Hellwege, and J. L. Olsen, Springer Verlag, Berlin (1981) p. 53-56.

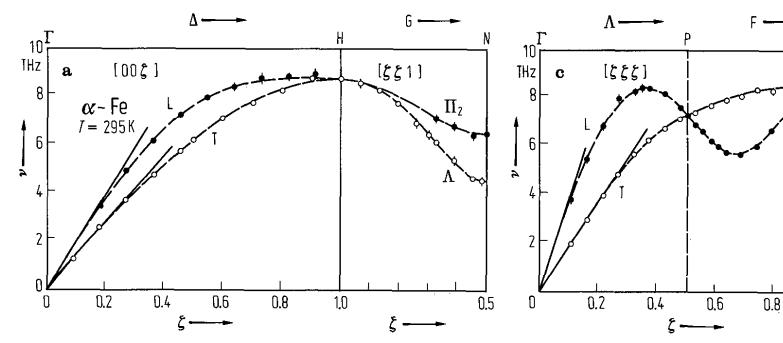
Dispersion relations



$$\omega(k) = 2\omega(\sin|ka/2|)$$

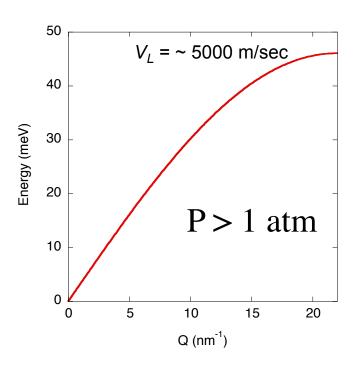
 $rac{\partial \omega_k}{\partial k}$: sound velocity

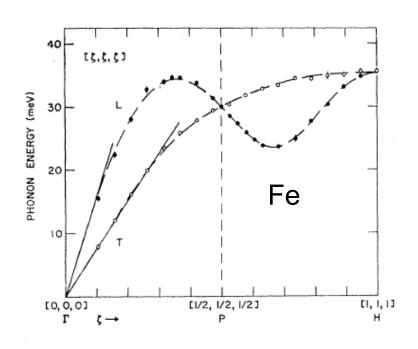
1.0



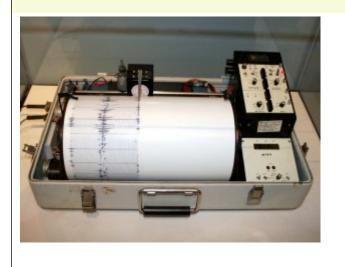
$$E = \frac{2\hbar}{\pi} V_L Q_{\text{max}} \sin \left(\frac{\pi}{2} \frac{Q}{Q_{\text{max}}} \right)$$

$$E(meV) = 4.192 \cdot 10^{-4} \cdot V_L(\text{m/sec})Q_{\text{max}}(\text{nm}^{-1}) \cdot \sin\left(\frac{\pi}{2} \frac{Q}{Q_{\text{max}}}\right)$$

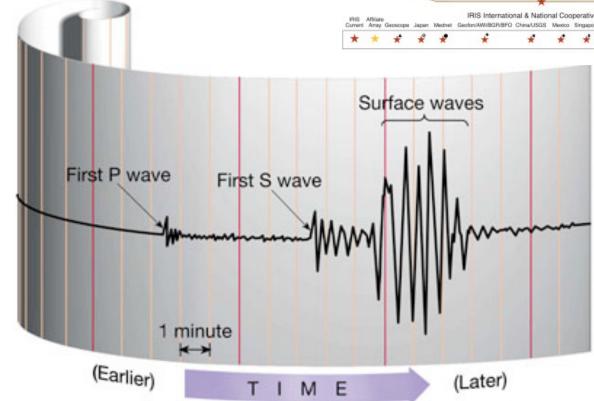




Seismic waves





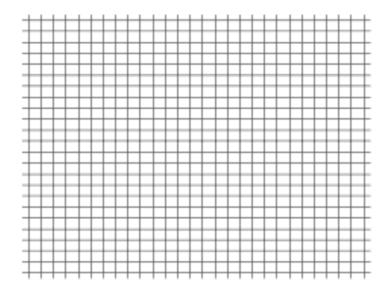


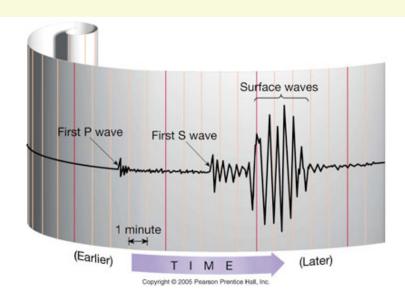
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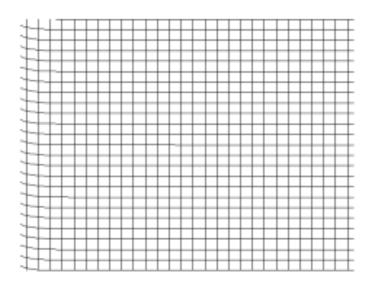
Seismic waves

Longitudinal --> Compression --> Primary: V_P

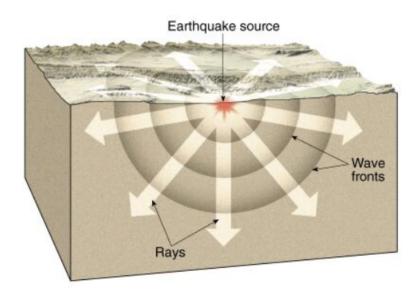




Transverse --> Shear --> Secondary: V_S



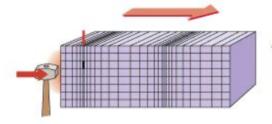
SEISMOLOGY

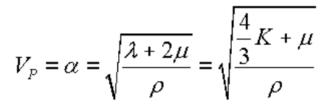


· Later, an Swave shadow zone was recognized, meaning no S waves were received at seismographs stations from 104° to 180° from an earthquake; the S wave shadow zone is caused by the outer core, which is liquid iron/nickel.

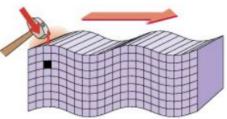
P Waves

Pressure or **Primary Waves**





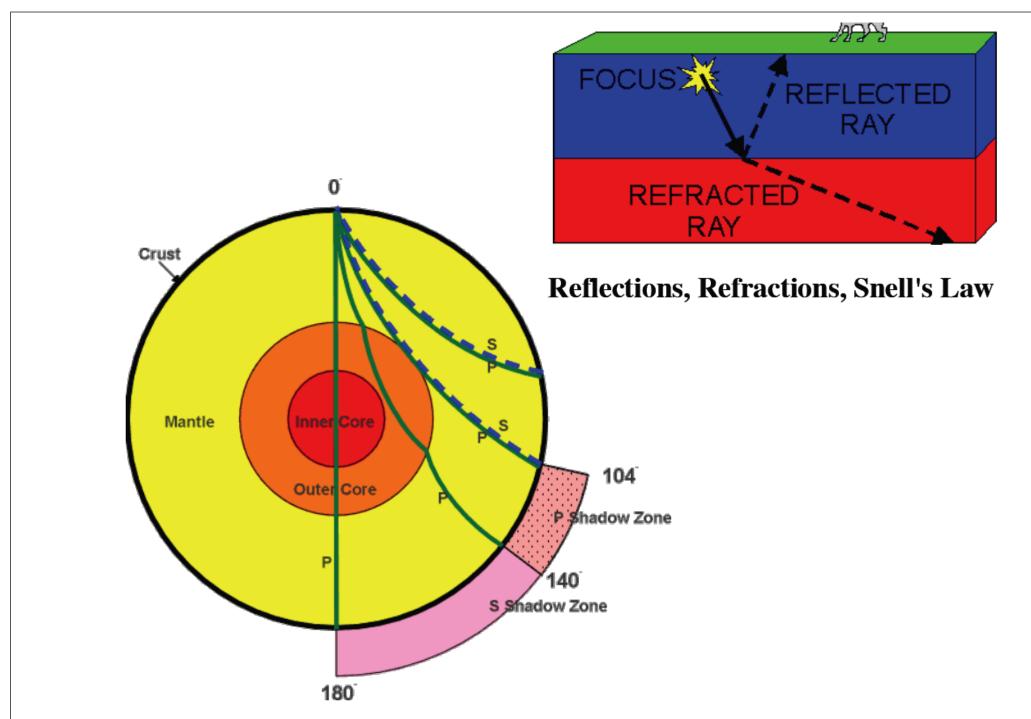
S-Waves

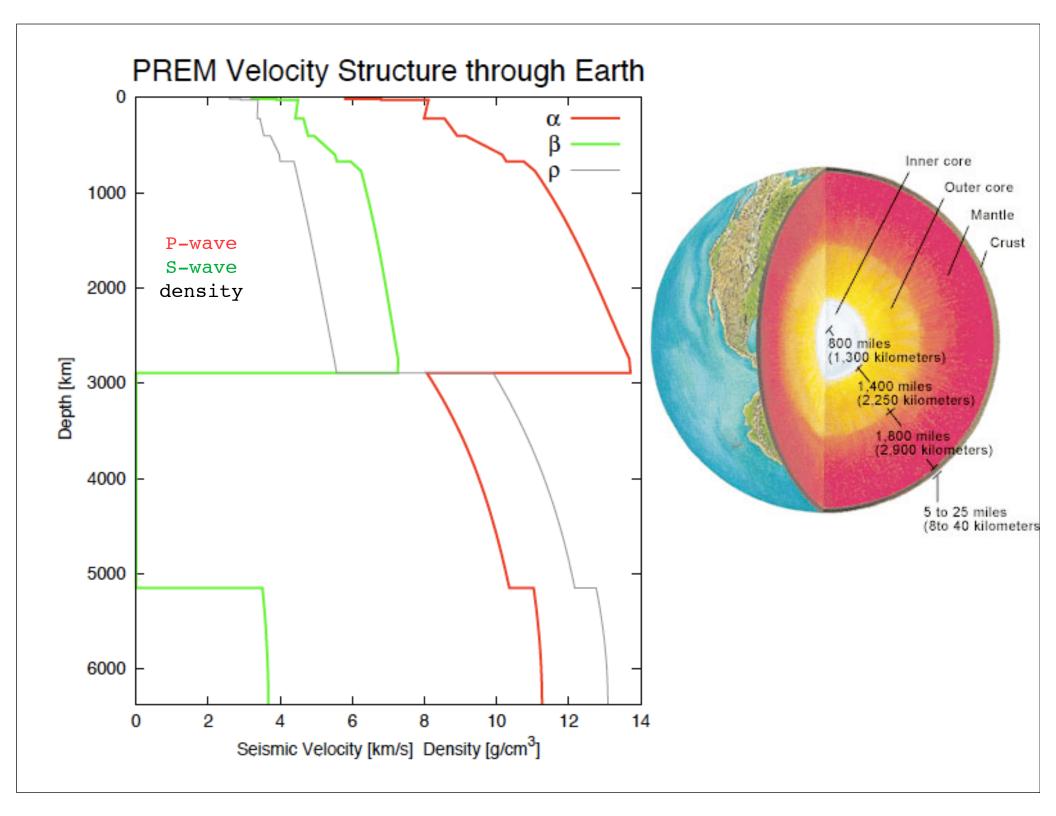


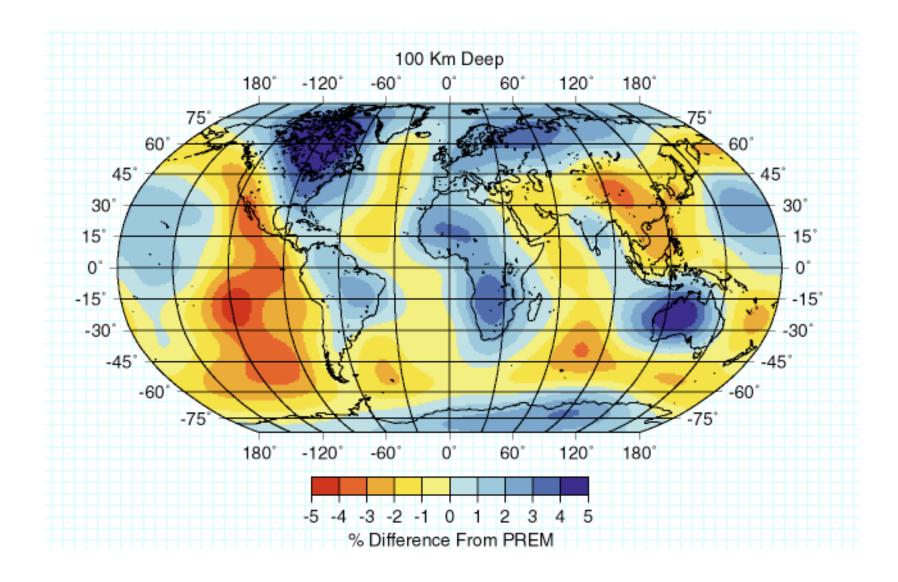
$$V_{\scriptscriptstyle S} = oldsymbol{eta} = \sqrt{rac{\mu}{
ho}}$$

Shear or **Secondary Waves**

 $V_S = \beta = \sqrt{\frac{\mu}{\rho}}$ Lamé's constants



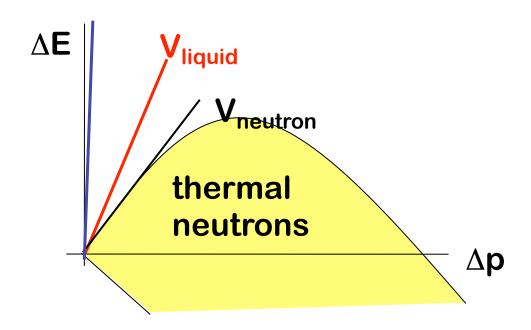




Map of the variations in seismic shear-wave speed with respect to the value in PREM at 100 km depth. The warm colors (red, orange, and yellow) show regions with slower than normal speeds, the darker regions are faster than normal.

(Model S12 WM13, from W.-J. Su, Journal of Geophysical Research, vol. 99(4) 4945-4980, 1994).

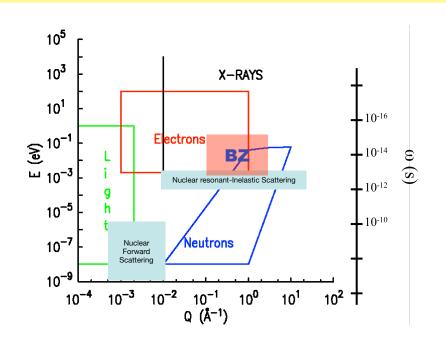
Why x-rays instead of neutrons or visible light?

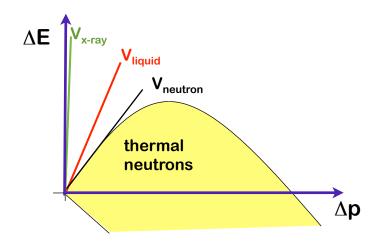


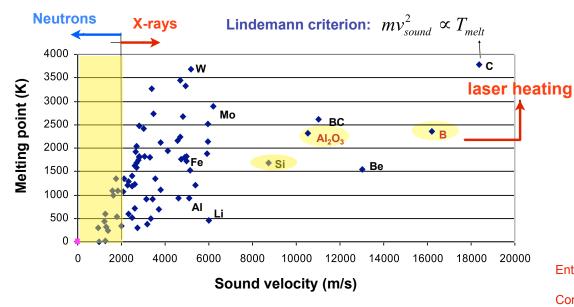
Limited momentum transfer capability of neutrons at low energies favor x-rays to study collective excitations with large dispersion, like sound modes.

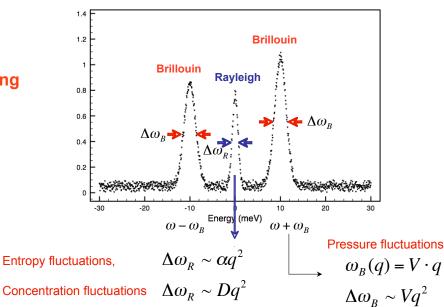
When the sound velocity exceeds that of neutrons in the liquid, x-rays become unique. The low-momentum/high-energy transfer region is only accessible by x-rays.

Why X-Rays?



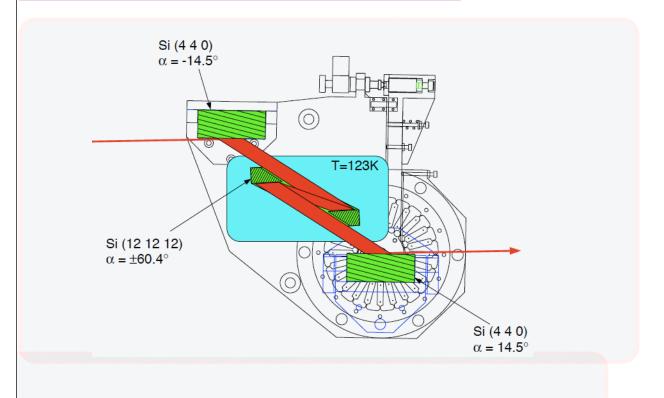




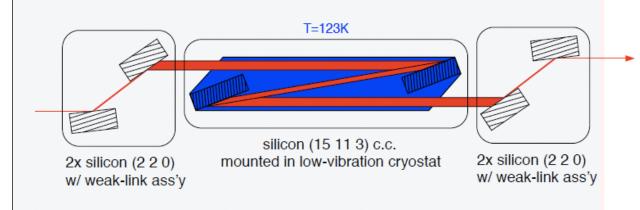


Methodology developments

Cryo-cooled monochromators



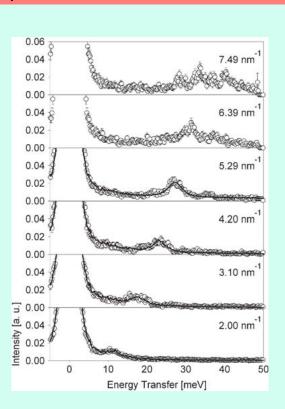
¹¹⁹Sn @ 14.4 keV ∂E = 1.3 meV



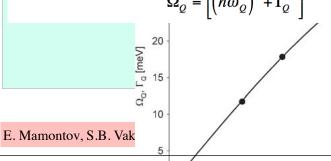
¹⁵¹Eu @ 21.5 keV ∂E = 1.4 meV

Acoustic phonons in chrysotile asbestos, Mg₃Si₂O₅(OH)₄

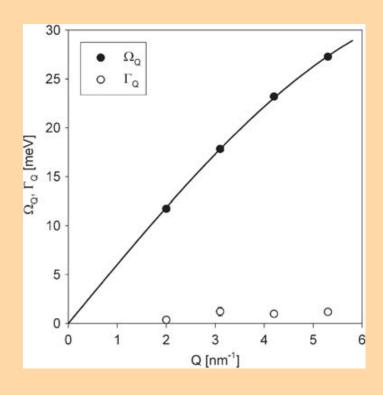
Damped harmonic oscillator model



$$\begin{split} S(\hbar\omega) &= \frac{1}{1-e^{-\hbar\omega/kT}} \left[I_c \frac{1}{\pi} \frac{\Gamma_c}{\left(\hbar\omega\right)^2 + \Gamma_c^2} + I_Q \frac{1}{\pi} \frac{4\Gamma_Q \left(\hbar\omega_Q\right) \left(\hbar\omega\right)}{\left(\left(\hbar\omega\right)^2 - \Omega_Q^2\right)^2 + 4\Gamma_Q^2 \left(\hbar\omega\right)} \right] \\ &\Omega_Q = \left[\left(\hbar\omega_Q\right)^2 + \Gamma_Q^2 \right]^{1/2} \end{split}$$



Longitudinal sound velocity

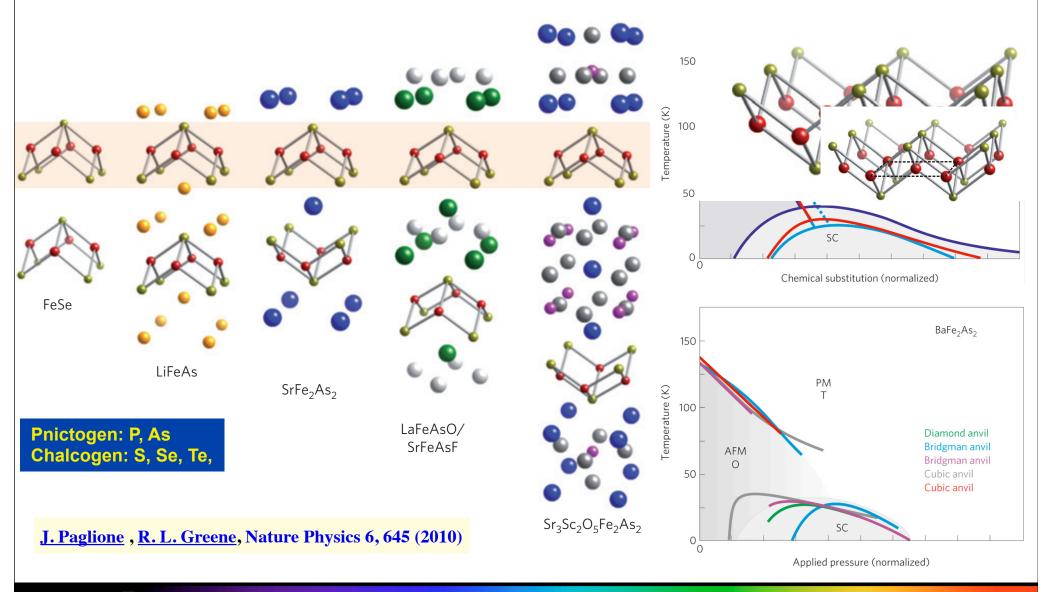


$$E = \frac{2V_L \hbar Q_{\text{max}}}{\pi} \sin \left(\frac{\pi}{2} \frac{Q}{Q_{\text{max}}} \right)$$

$$E \text{ (meV)} = 4.192 \cdot 10^4 V_L Q_{\text{max}} \sin \left(\frac{\pi}{2} \frac{Q}{Q_{\text{max}}} \right)$$

, Solid State Commun. 149 (15-16), 589-592 (2009).

Pnictides: A scientific opportunity for IXS:





Are pnictides BCS type electron-phonon superconductors? Is Migdal-Eliashberg theory obeyed?

$$T_{\rm c} = \frac{\omega_{\rm ln}}{1.2} \exp\left[-\frac{1.04(1+\lambda)}{\lambda-\mu^*(1+0.62\lambda)}\right], \qquad {\rm \sim 26~K,~SC~transition~temperature}$$

$$\alpha_{\mathrm{C}} = \frac{1}{2} \left[1 - \frac{1.04(1+\lambda)(1+0.62\lambda)\mu^{*2}}{[\lambda-\mu^*(1+0.62\lambda)]^2} \right], \quad \text{Isotope effect coefficient}$$

$$\frac{2\Delta}{T_{\rm c}} = 3.53 \left[1 + 12.5 \left(\frac{T_{\rm c}}{\omega_{\rm ln}} \right)^2 \ln \left(\frac{\omega_{\rm ln}}{2T_{\rm c}} \right) \right], \quad \text{SC energy gap / T }_{\rm c} \text{ ratio}$$

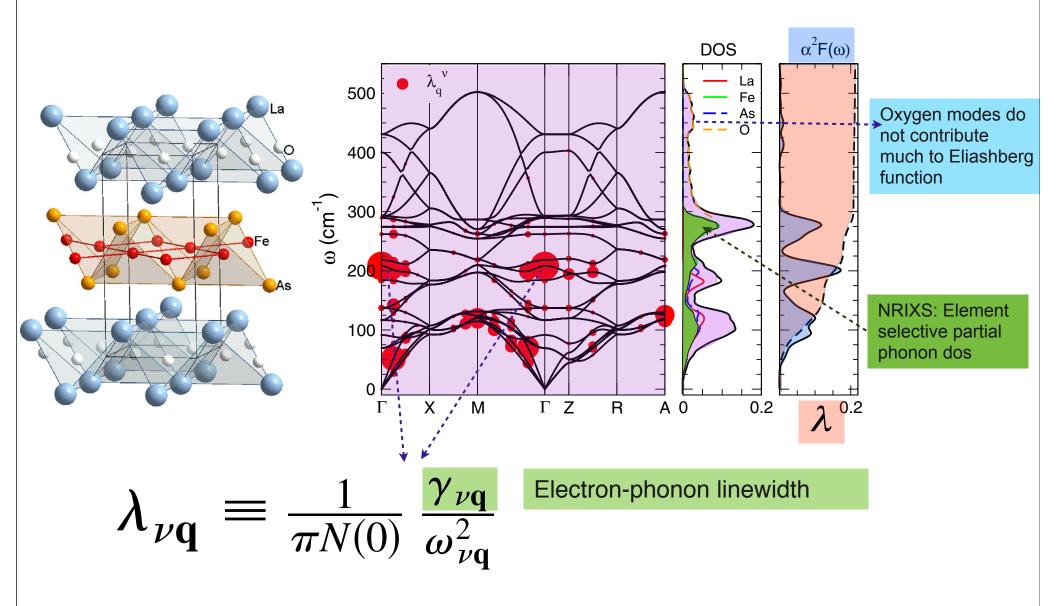
$$\lambda = 2 \int \alpha^2 F(\omega) d\omega / \omega$$

electron-phonon coupling constant

$$\ln \omega_{\ln} = (2/\lambda) \int \ln \omega \alpha^2 F(\omega) \frac{d\omega}{\omega}$$
 is the relevant phonon frequency

For pnictides values of λ is inconsistent with observed T_c. Estimated value of 0.2 is too small for the observed 26 K transition temperature.





Influence of magnetism on phonons in CaFe₂As₂ as seen via inelastic x-ray scattering

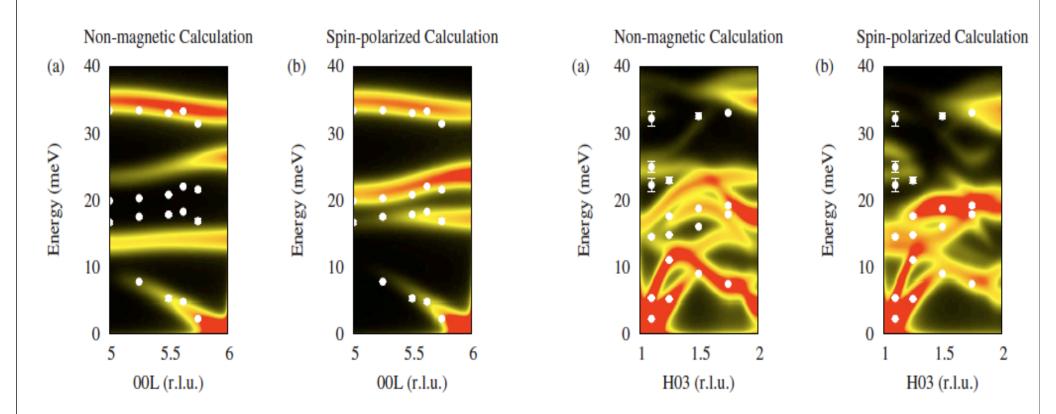
S. E. Hahn,* Y. Lee, N. Ni, P. C. Canfield, A. I. Goldman, R. J. McQueeney,† and B. N. Harmon Department of Physics and Astronomy and Ames Laboratory, Iowa State University, Ames, Iowa 50010, USA

A. Alatas, B. M. Leu, and E. E. Alp Advanced Photon Source, Argonne National Laboratory, Argonne, Illinois 60439, USA

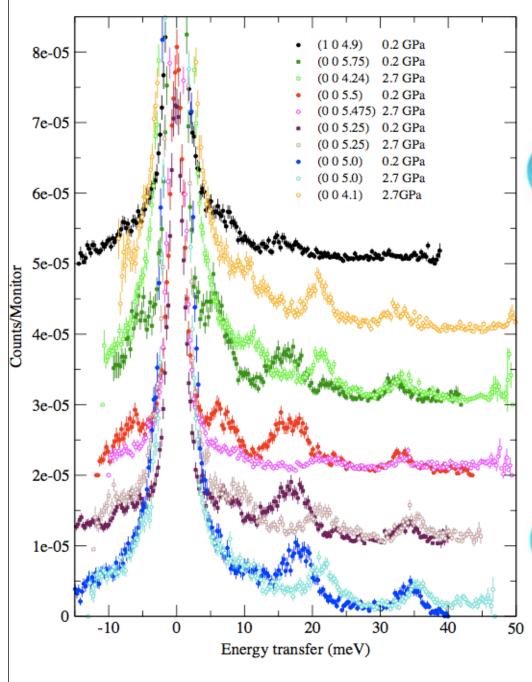
D. Y. Chung and I. S. Todorov Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA

M. G. Kanatzidis

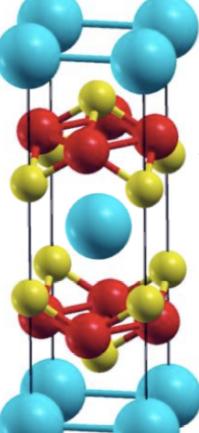
Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA and Department of Chemistry, Northwestern University, Evanston, Illinois 60208, USA

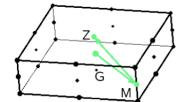


CaFe₂As₂ under pressure



14/mmm

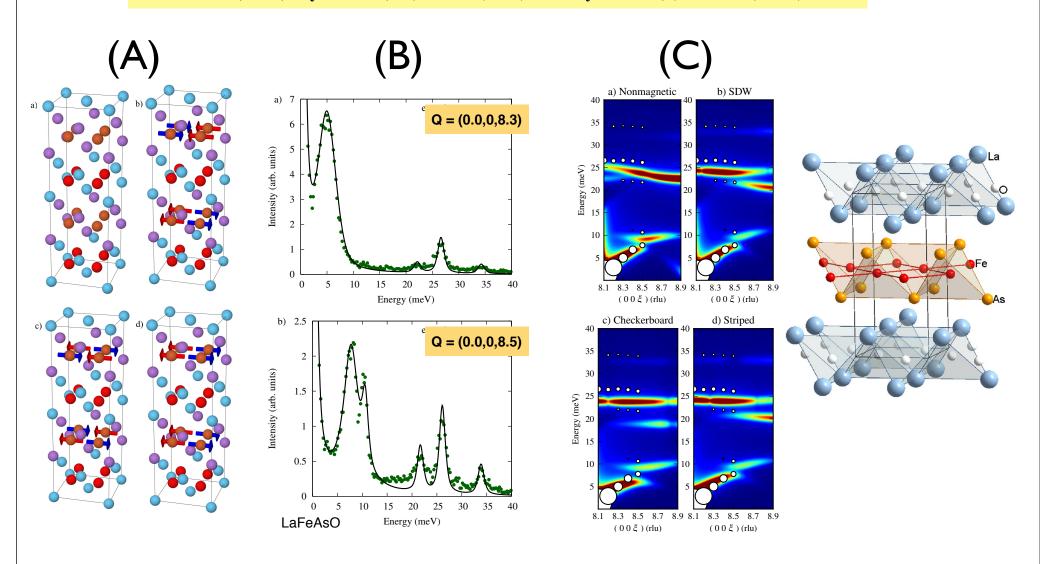




There is a phase transition from magnetically ordered orthorhombic phase to a nonmagnetic "collapsed" tetragonal phase, accompanied by a significant volume change at 0.3 GPa.

Pnictides: A scientific opportunity for IXS & NRVS:

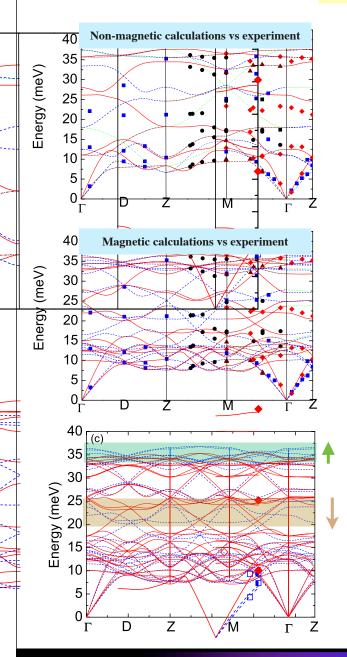
S. E. Hahn, et al, Phys. Rev. B, 79, 220511 (2009) and Phys. Rev. B, (submitted, 2012)

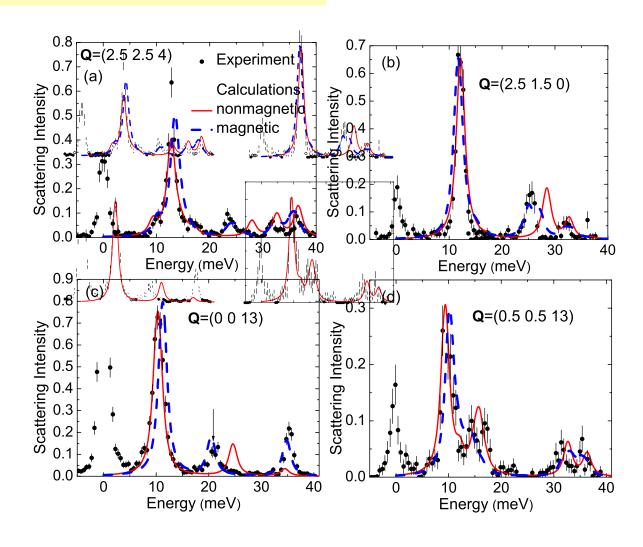




BaFe_{1.8}Co_{0.2}As₂

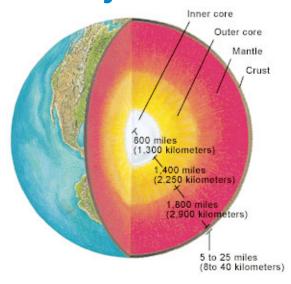
D. Reznik et al, *Phys. Rev B* **80**, 214534 (2009)

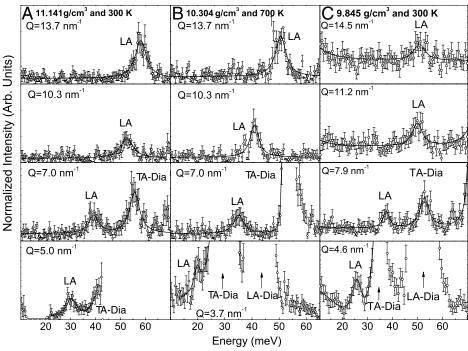


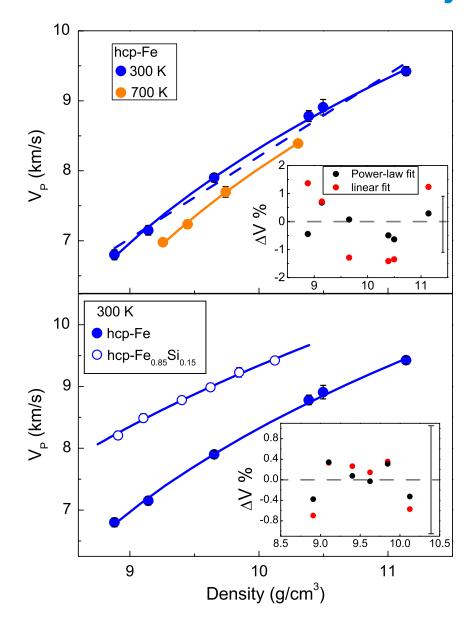




Sound velocity at the conditions of the Earth's core in iron alloys

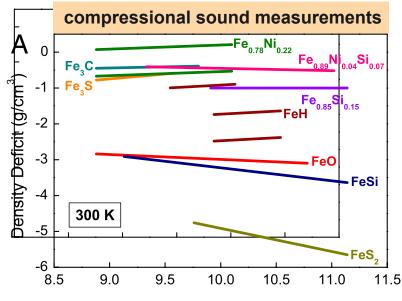


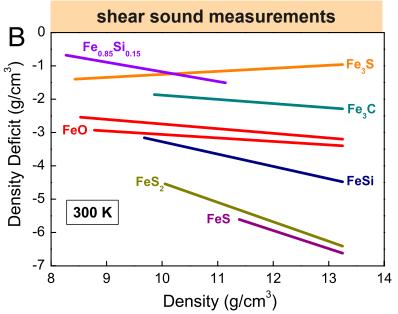


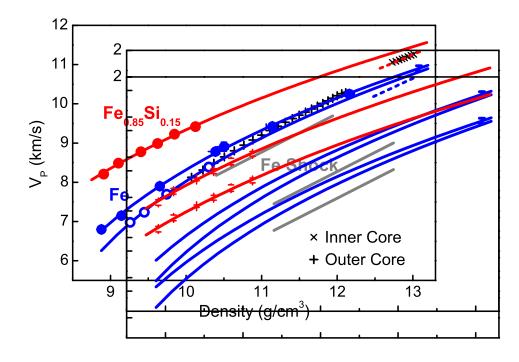




Sound velocity at the conditions of the Earth's core in iron alloys



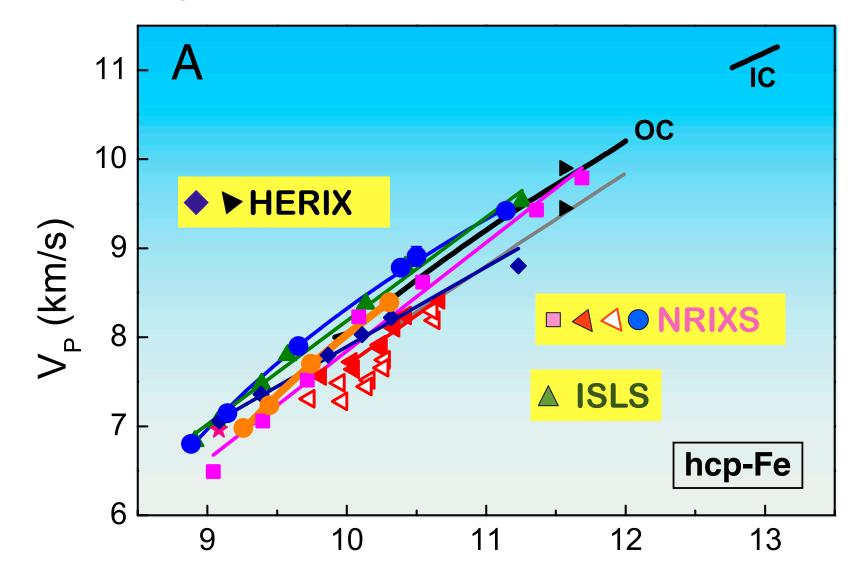




Direct measurements of the V_p relationship of Felight element alloys at relevant P-T conditions of the core now appear to be on the horizon, which in turn may eventually answer the longstanding question on the composition of the Earth's core.



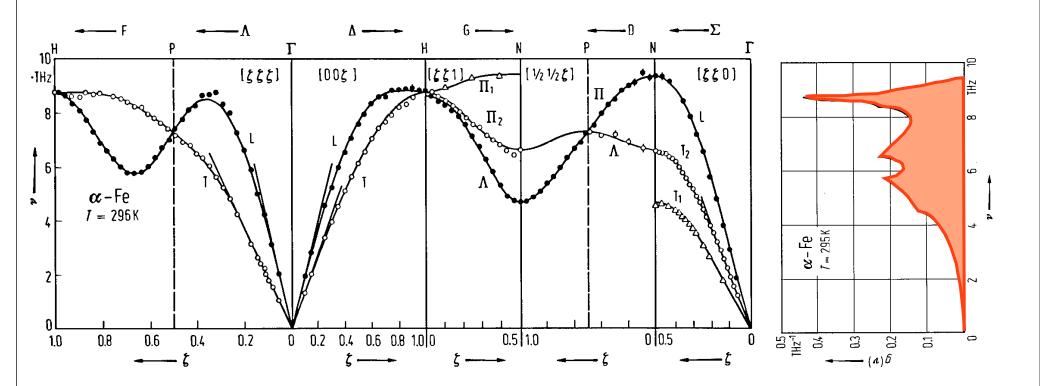
Sound velocity at the conditions of the Earth's core in iron alloys



ISLS: impulsive stimulated light scattering



Dispersion relations and phonon density of states lpha -iron (bcc)

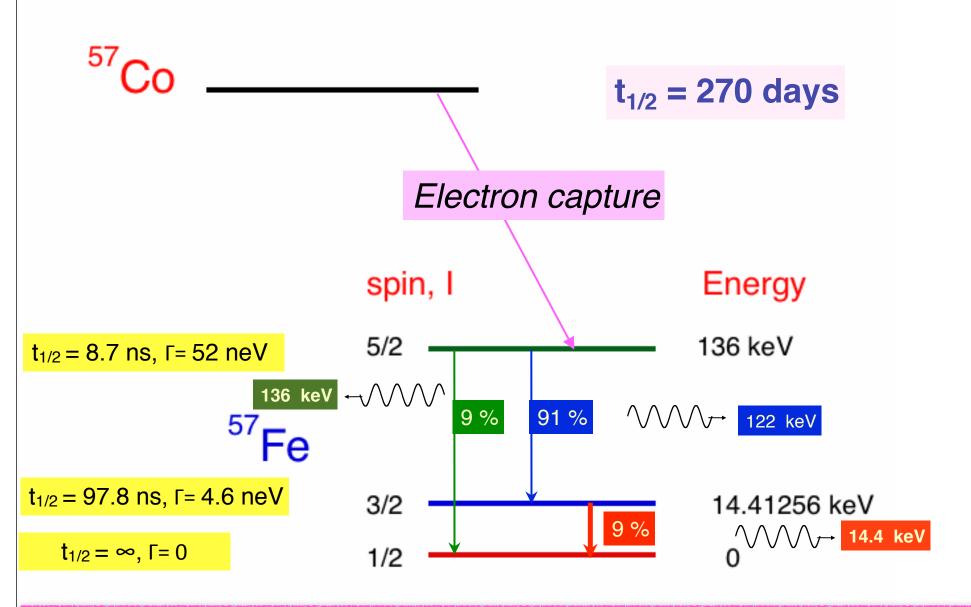


V. J. Minkiewicz, G. Shirane, and R. Nathans, Phys. Rev. 162 (1967) 528, and Landolt-Börnstein, New Series, Group III, Vol 13, Eds. K.-H Hellwege, and J. L. Olsen, Springer Verlag, Berlin (1981) p. 53-56.

a few questions

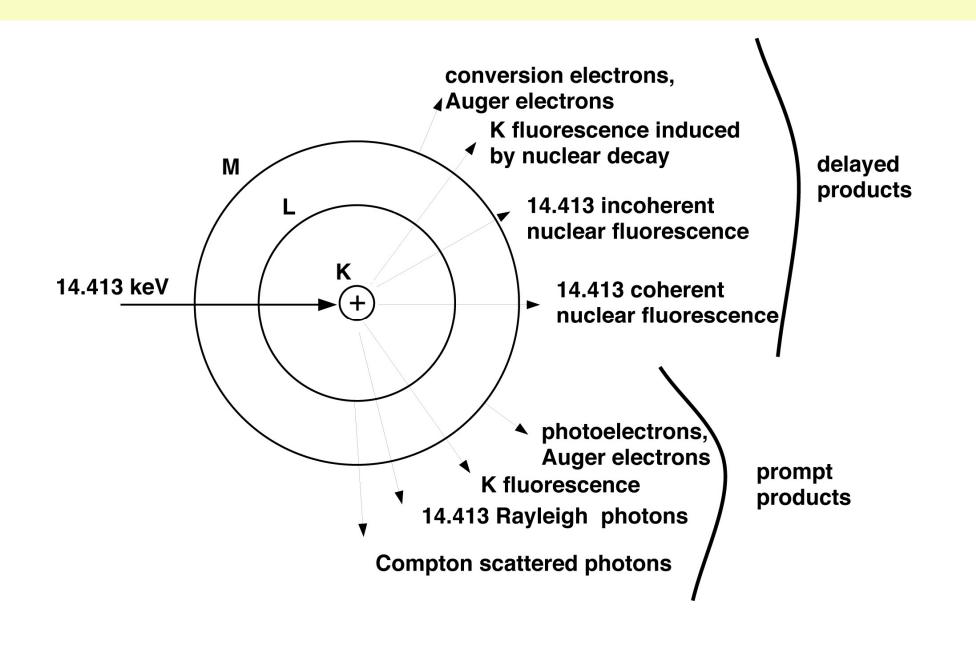
- Can one measure phonon dispersion and/or phonon dos
 - from a monolayer?
 - at a buried interface?
 - from nanosized particles on the surface?
 - at 3 Mbar and at 4 K to 5000K extreme conditions?
 - from a nanogram sample?
 - in a way that is element and isotope selective?
 - in a way that can be completely tested by DFT, i.e. both
 the frequency and amplitude of vibrations are determined

Characteristics of a Mössbauer nuclei

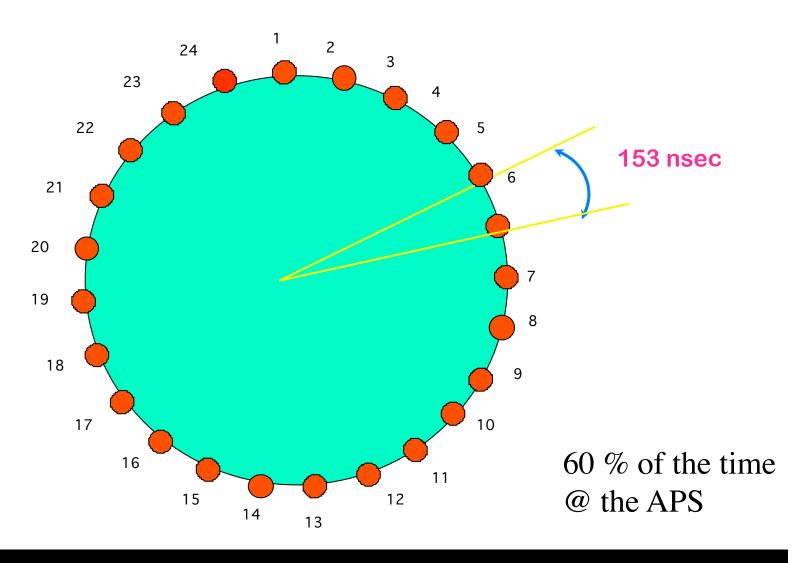


Transitions to ground state with a relatively low energy is what makes a nuclei a Mössbauer nuclei

Nuclear Resonance and Fallout in ⁵⁷Fe-decay

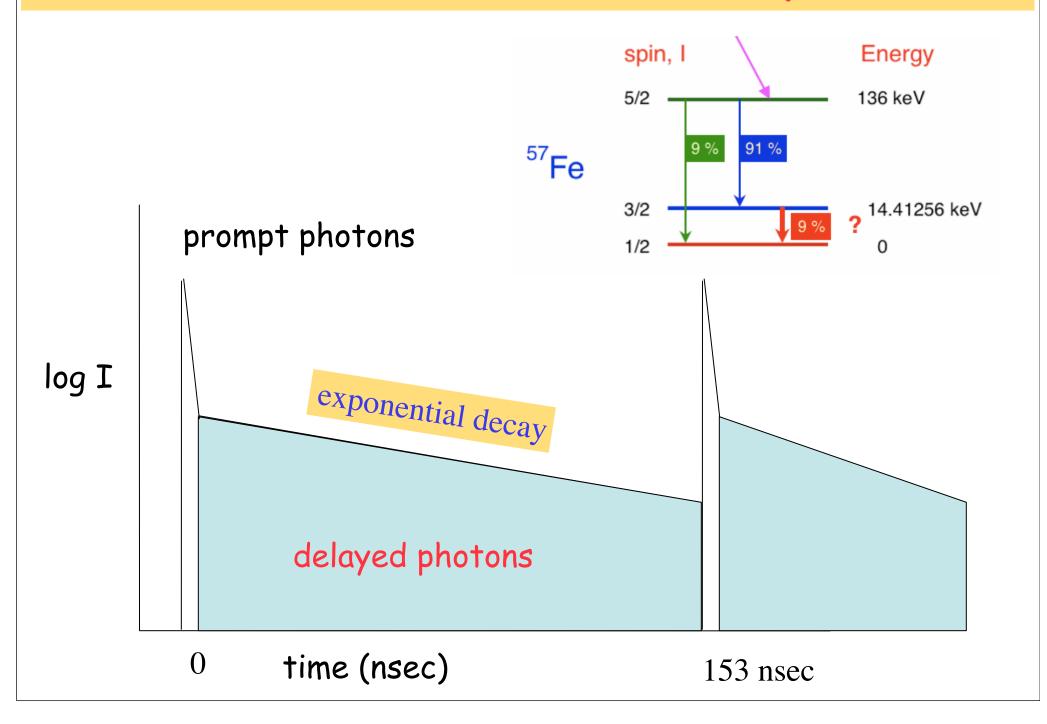


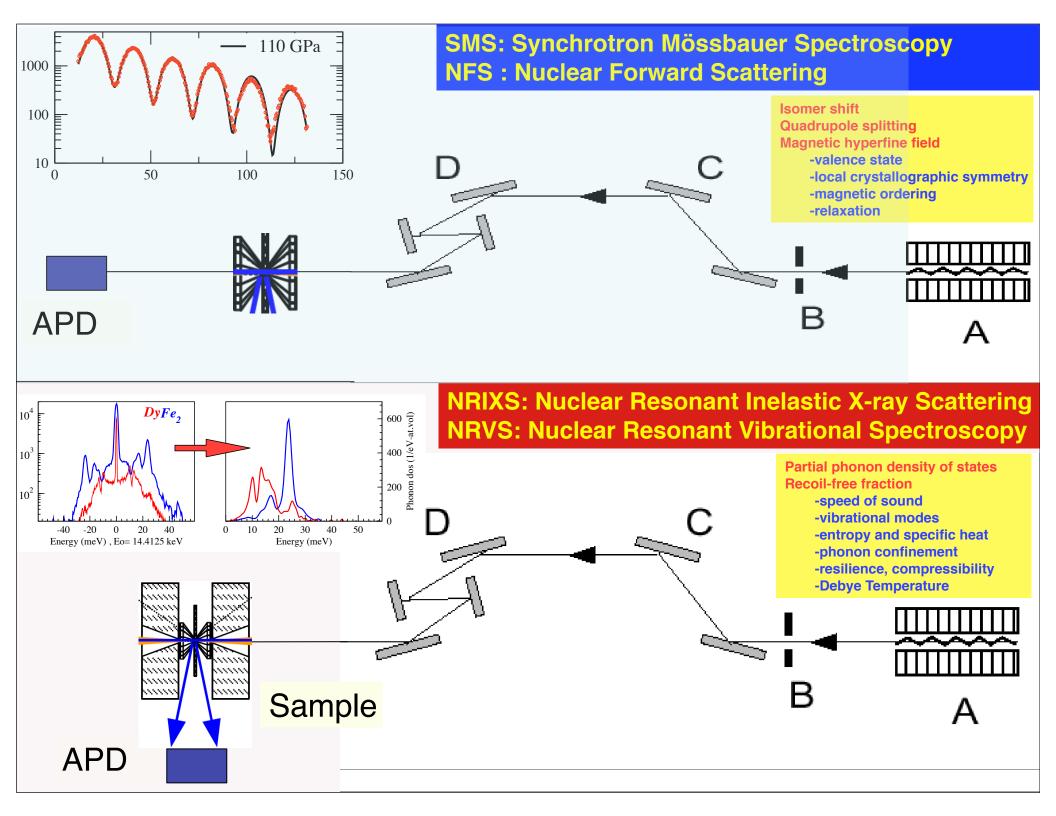
Standard Time structure @ APS

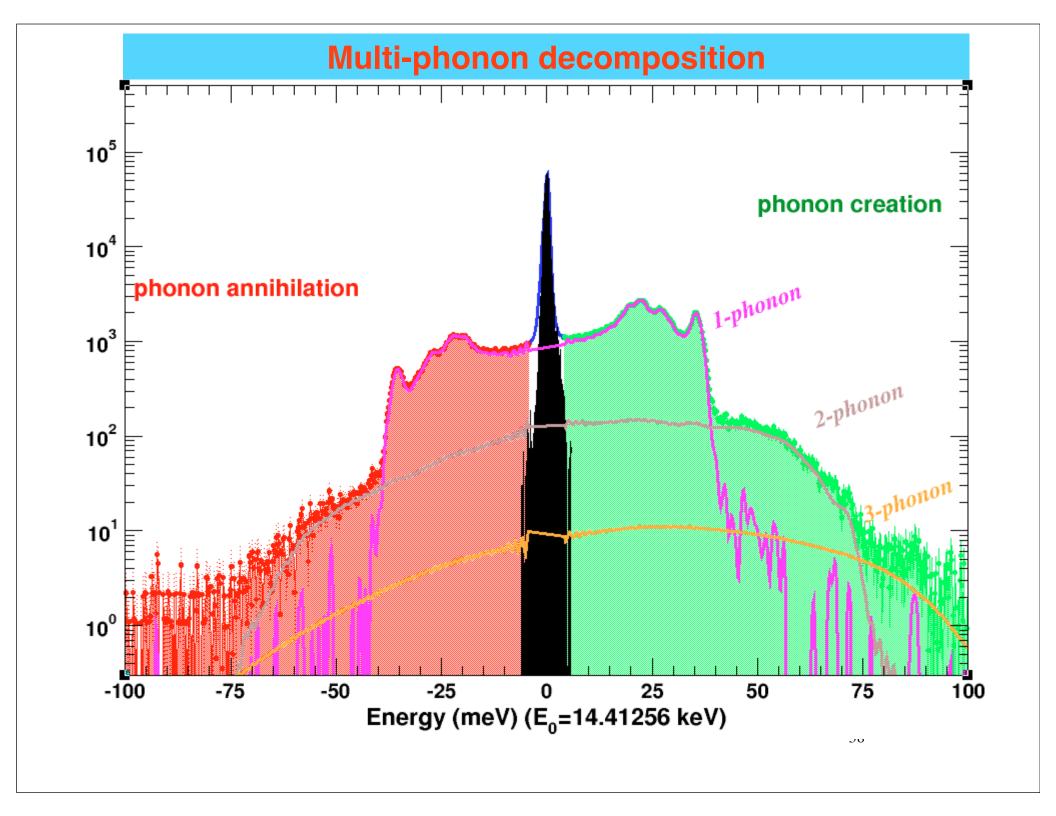


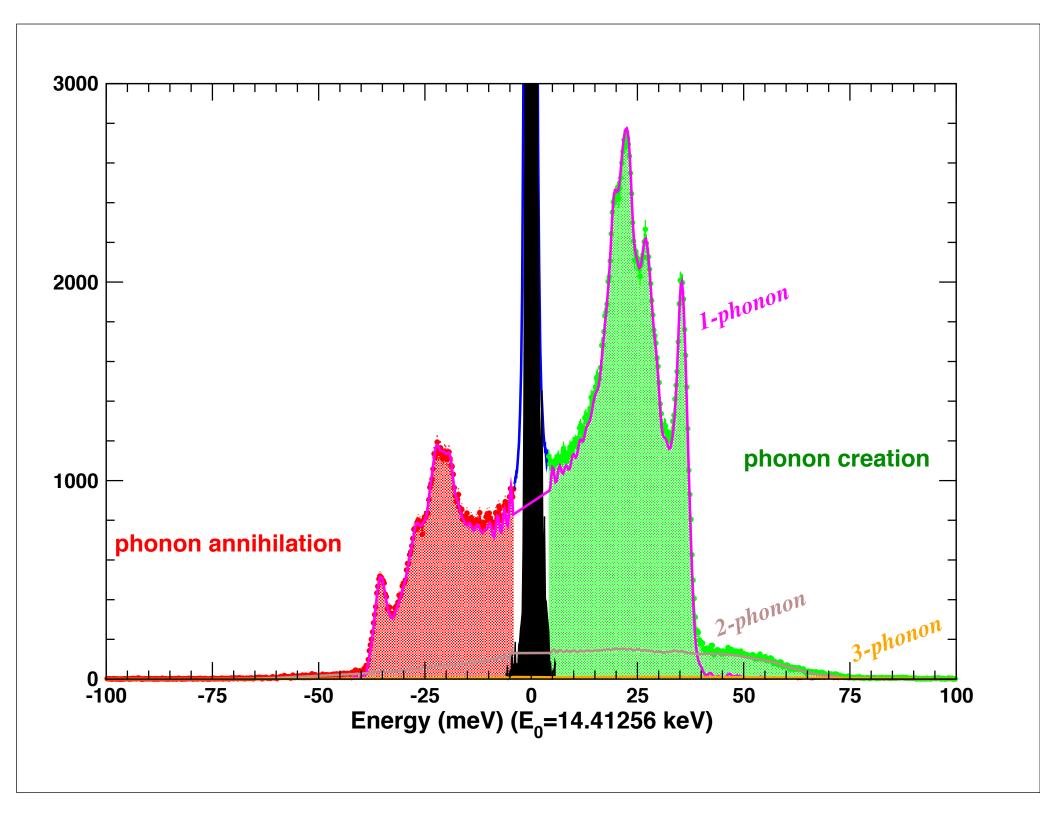
1 revolution=3.68 µsec =>1296 buckets

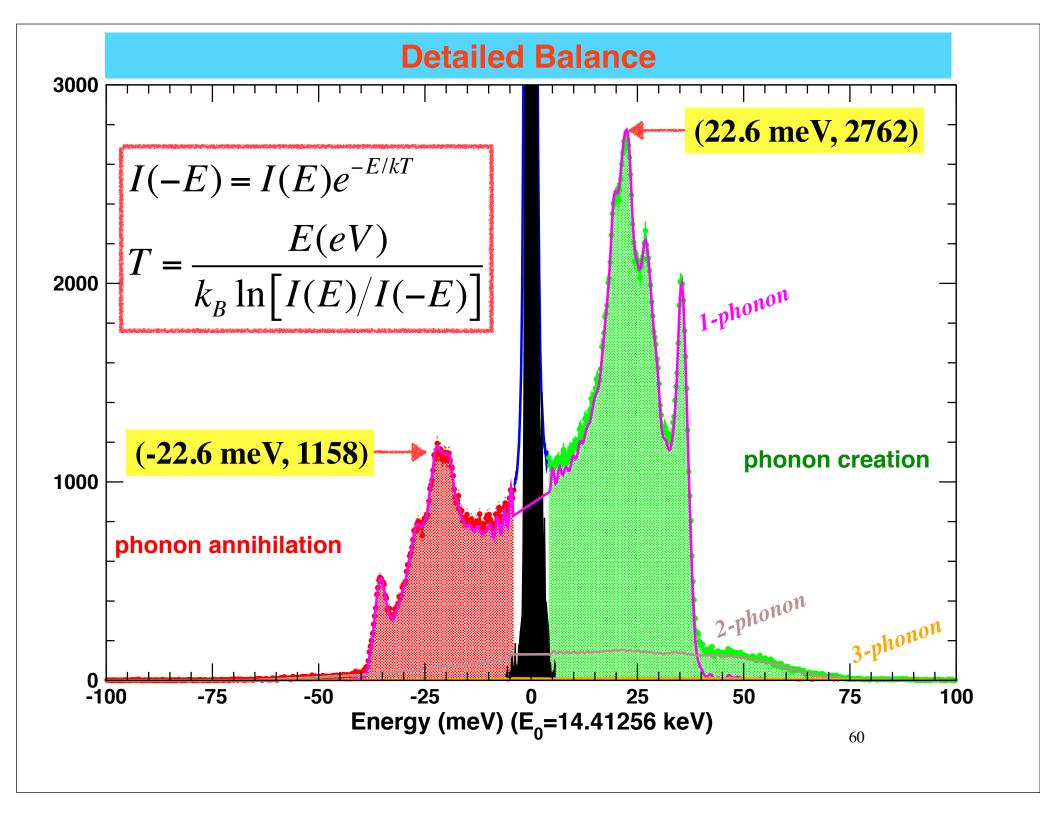
Detection of nuclear decay

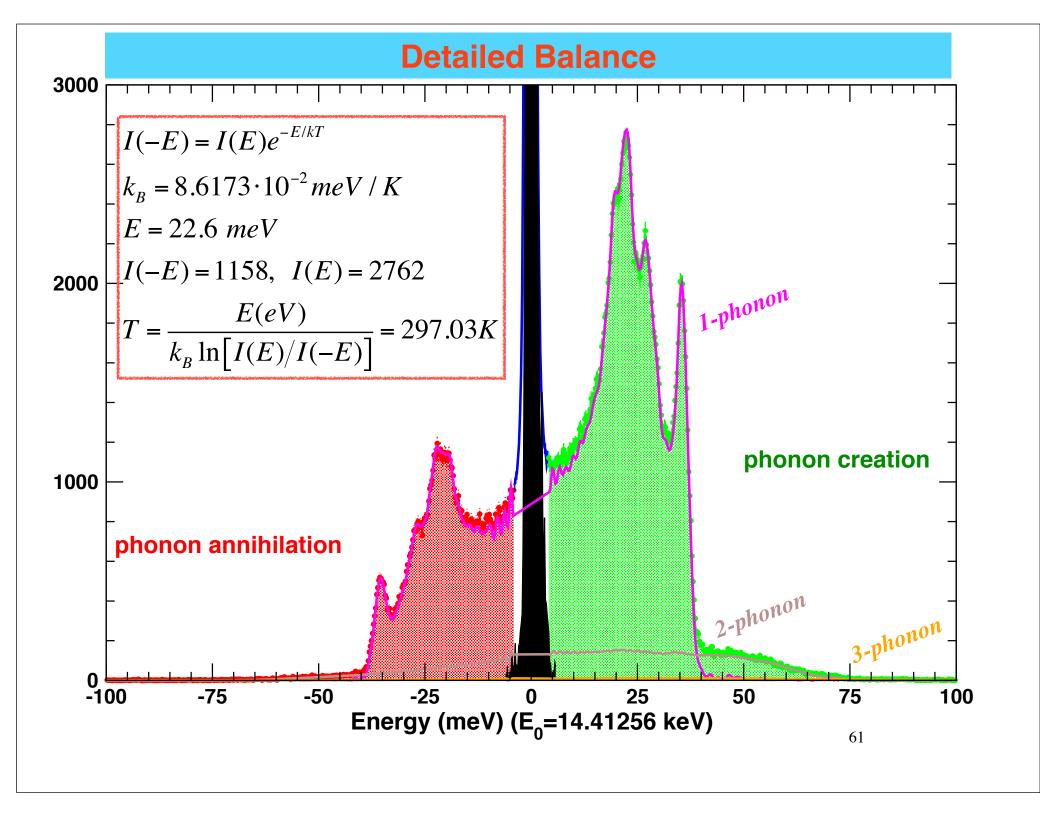






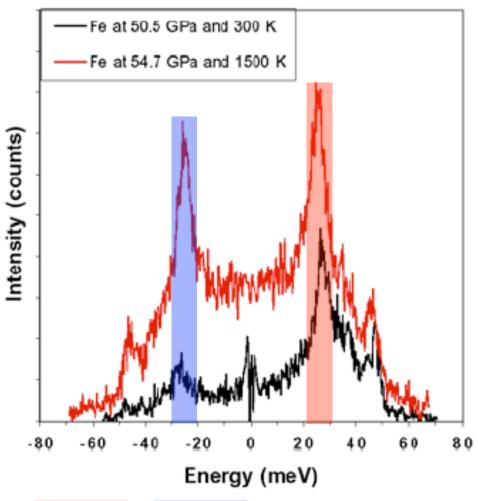






How to measure temperature in a DAC?

NRIXS of Fe⁵⁷ in a LHDAC

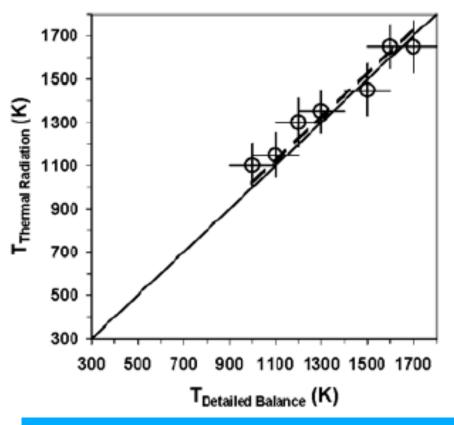


$$I(E) = I(-E)e^{(E/kT)}$$

$$I(E) = I(-E)e^{(E/kT)}$$

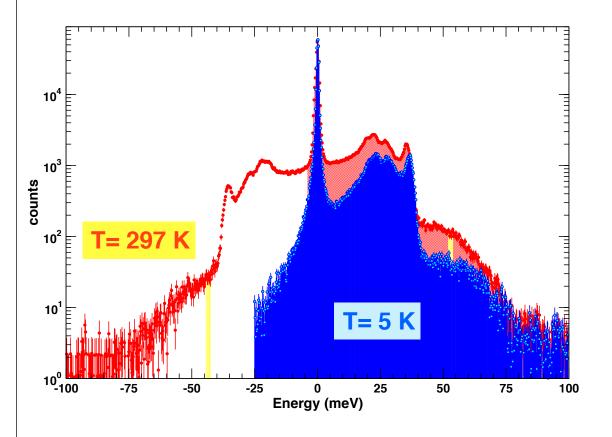
$$\int I(E)dE = \int I(-E)e^{(E/kT)}dE$$

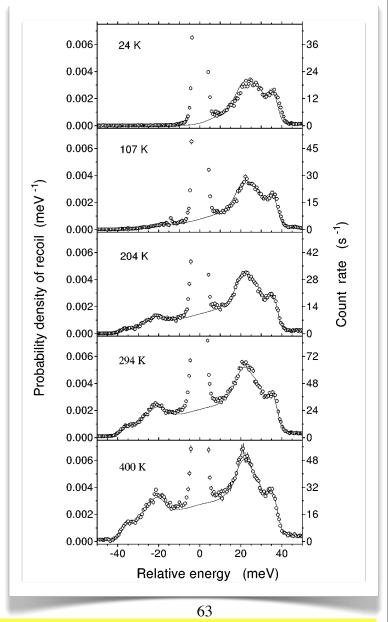
Spectroradiometry vs. detailed balance principle



J.F. Lin, et al, Geophys. Res. Lett., 31 (2004) L13611

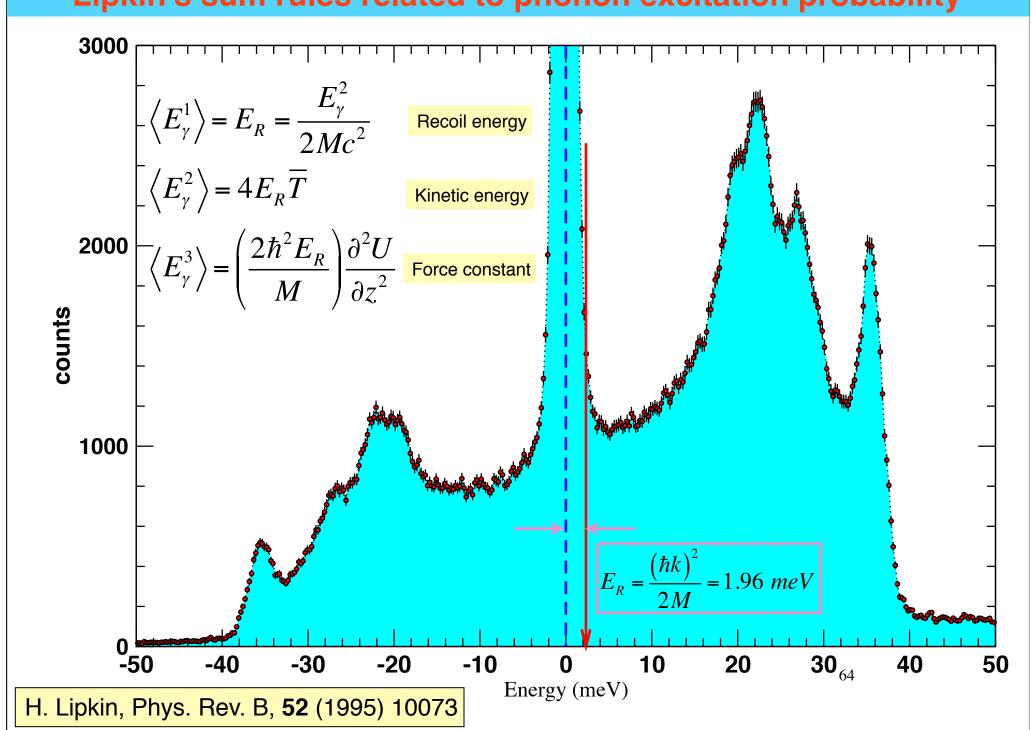
Temperature dependence of phonon excitation probability



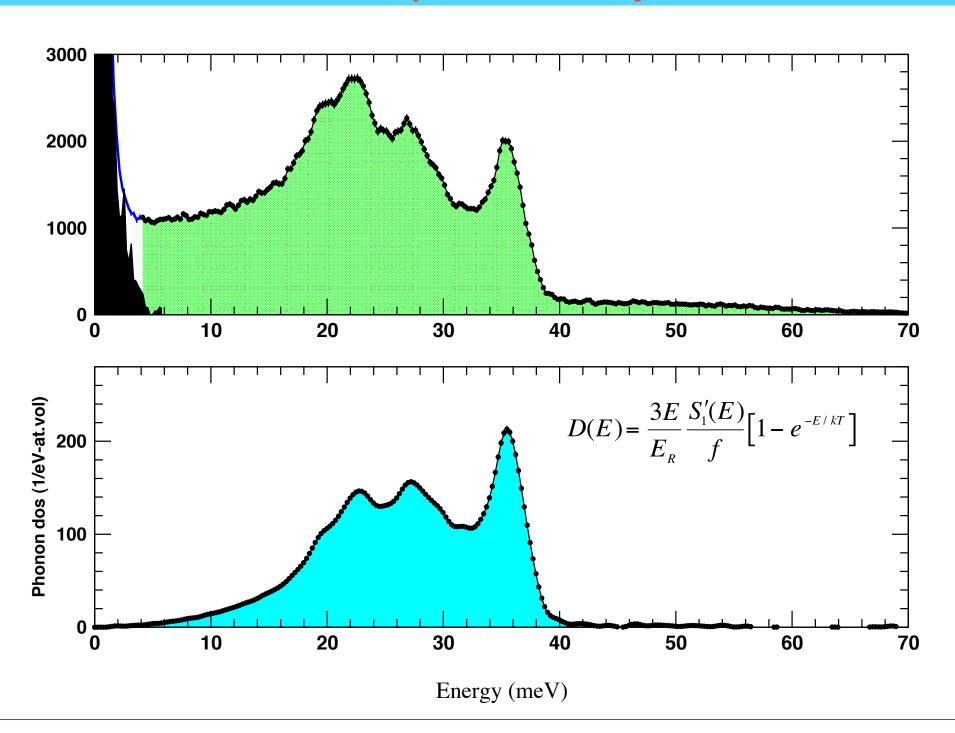


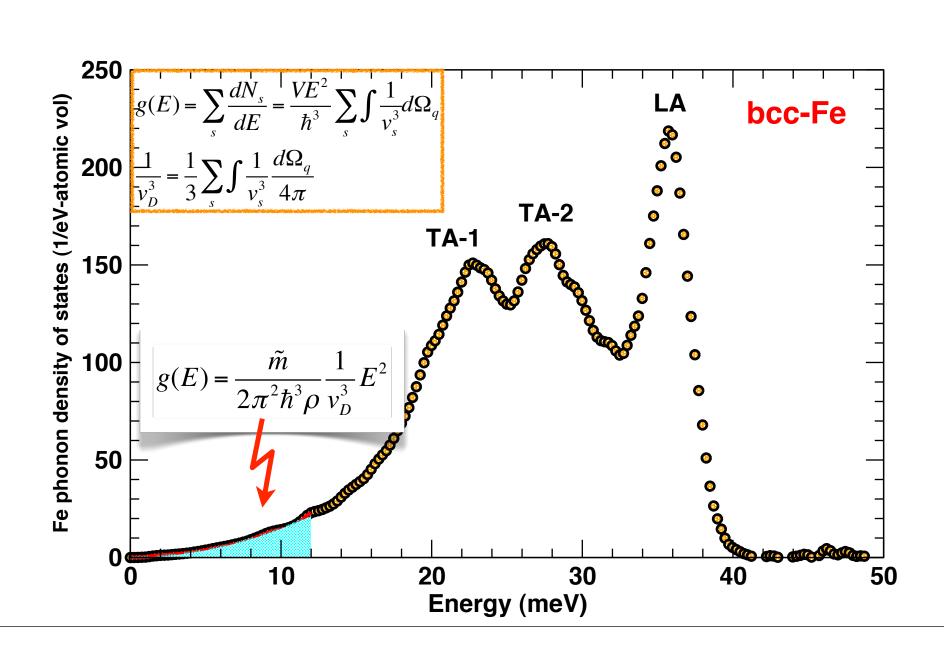
Chumakov, et al, Phys. Rev. B 54 (1996) 9596.

Lipkin's sum rules related to phonon excitation probability

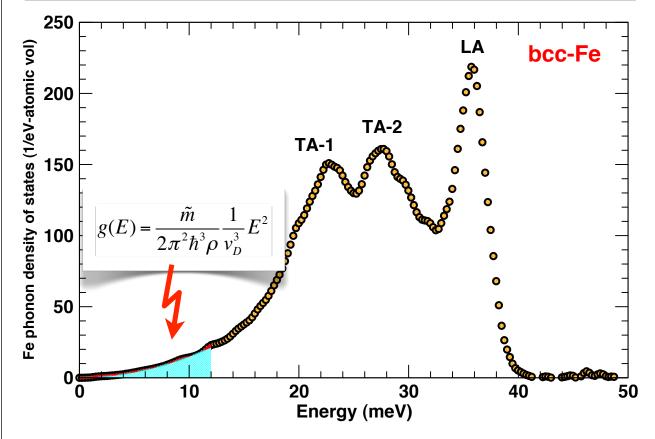


Extraction of phonon density of states





Measurement of v_D , Debye sound velocity allows to resolve longitudinal and shear sound velocity, provided that bulk modulus and density, is independently and simultaneously measured by x-ray diffraction.



K (GPa)	ρ (g/cc)	V_D (m/s)	$V_P(\mathbf{m/s})$	$V_S(m/s)$	G (GPa)
165 ± 1	8.01	3510 ± 12	5813 ± 13	3146 ± 11	79.3 ± 0.6

$$\frac{K_S}{\rho} = V_P^2 - \frac{4}{3}V_S^2$$

$$\frac{G}{\rho} = V_S^2$$

$$\frac{3}{V_D^3} = \frac{1}{V_P^3} + \frac{2}{V_S^3}$$

K_s: adiabatic bulk modulus

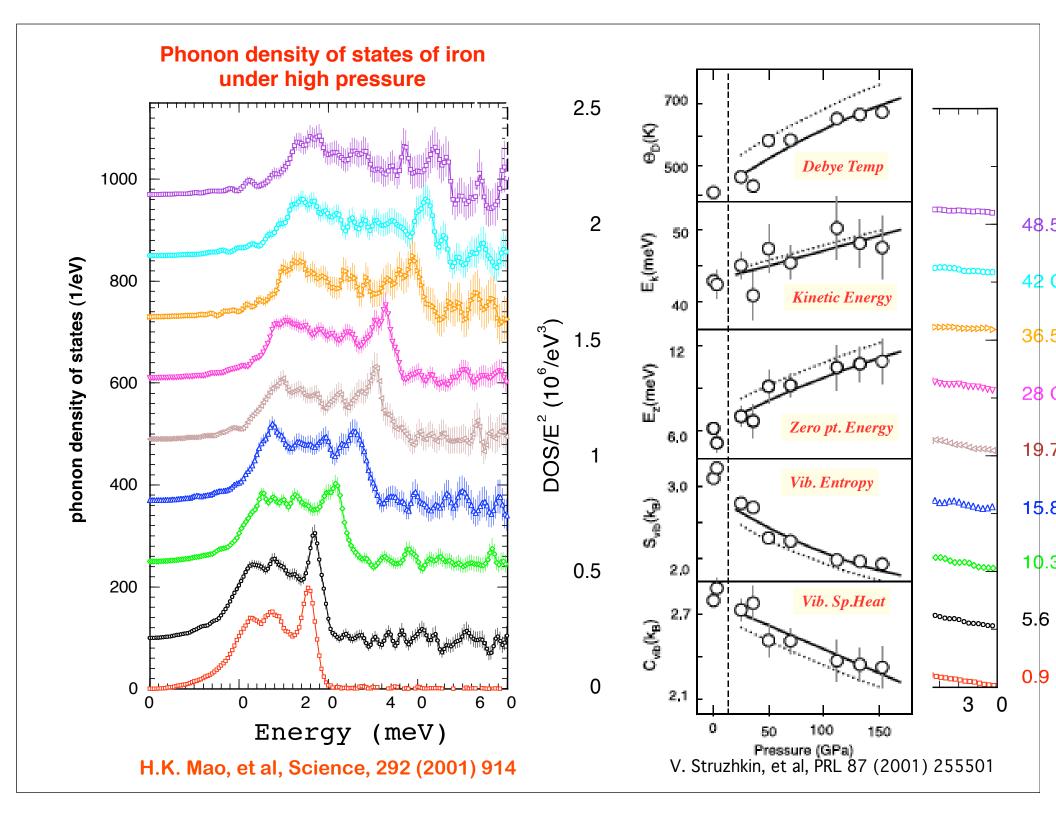
G: shear modulus

V_P: compression wave velocity

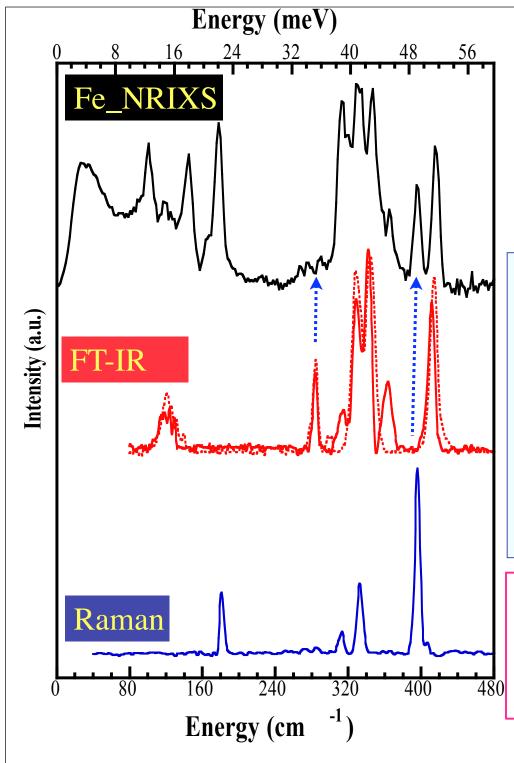
V_S: shear wave velocity

 V_D : Debye sound velocity

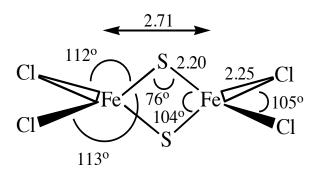
P: density



Property	Information content
Lamb-Mössbauer Factor, or recoil-free fraction	f_{LM} , recoil free fraction obtained from density of states, $g(E)$: $f_{LM} = \exp\left(-E_R \int \frac{g(E)}{E} \cdot \coth \frac{\beta E}{2} dE\right)$
Second order Doppler shift	$\delta_{SOD} = -E_0 \frac{\langle v^2 \rangle}{2c^2}$
Average kinetic energy	Extracted from second moment of energy spectrum: $T = \frac{1}{4E_R} \left\langle \left(E - E_R \right)^2 \right\rangle$
Average force constant	Extracted from third moment of energy spectrum: $\frac{\partial^2 U}{\partial z^2} = \frac{m}{2h^2} \langle E^3 \rangle$
Phonon density of states	Extracted one-phonon absorption probability, $S_I(E)$: $g(E) = \frac{E}{E_R} \tanh \left(\beta E / 2\right) \left(S_1(E) + S_1(-E)\right)$
Specific heat (vibrational part only)	$C_V = 3k_B \int_0^\infty (\beta E / 2)^2 \csc h(\beta E) g(E) dE$
Vibrational entropy	$S_V = 3k_B \int_0^\infty \left\{ \frac{\beta E}{2} \coth(\beta E) - \ln[2\sinh(\beta E)] \right\} g(E) dE$
Debye sound velocity (aggregate sound velocity)	From low-energy portion of the density of states: $g(E) = \frac{3V}{2\pi h^3 v_D^3} E^2$
Mode specific vibrational amplitude	Contribution of mode α of atom j to zero-point fluctuation [11,12]: $\left\langle r_{j\alpha}^{2}\right\rangle_{0} = \frac{h^{2}}{2m_{j}\omega_{\alpha}^{2}}e_{j\alpha}^{2}$
Mode specific Gruneisen constant	From pressure dependence of phonon frequencies ω_{α} of acoustic or optical modes: $\gamma_{\alpha} = -\frac{\partial \ln \omega_{\alpha}}{\partial \ln V}$
Temperature of the sample	From detailed balance between phonon occupation probability



$[NEt_4]_2[Fe_2S_2Cl_4].$

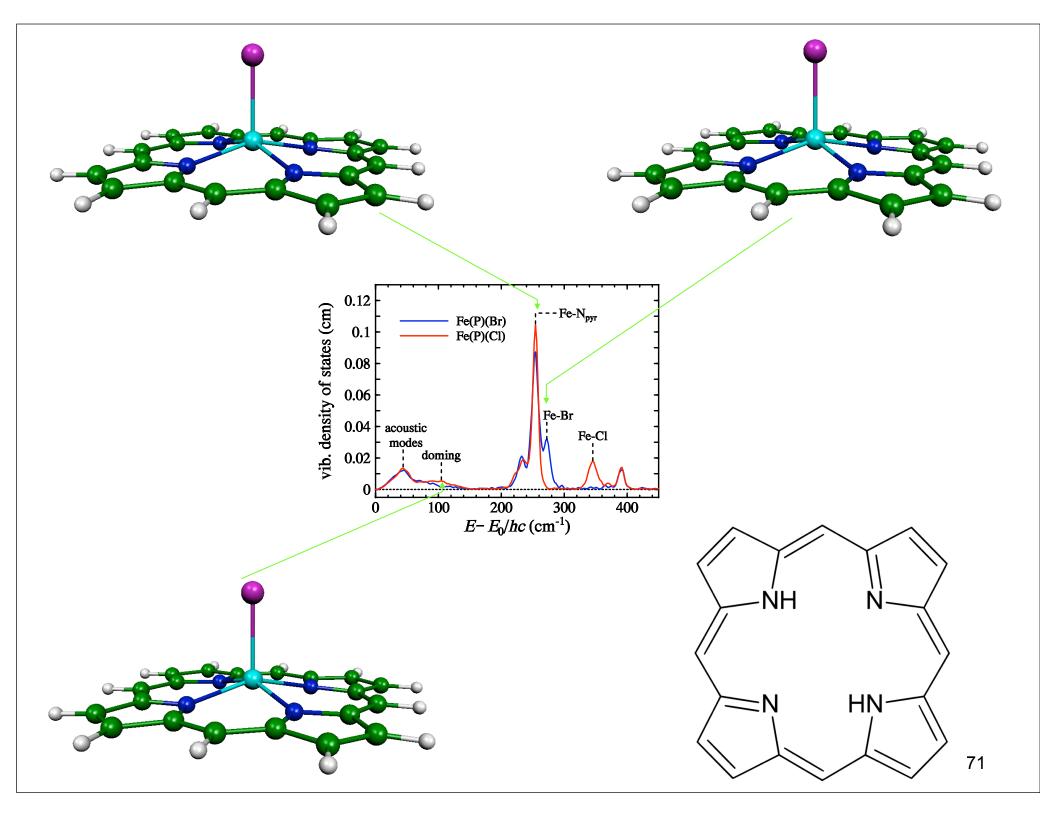


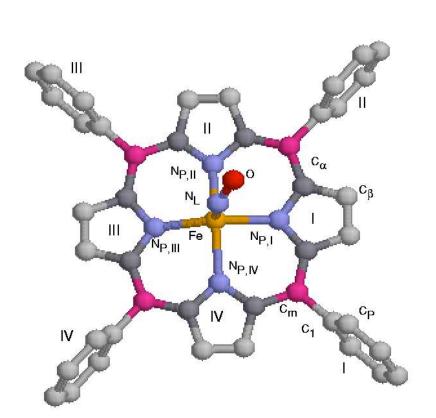
Some unique advantages of NRIXS

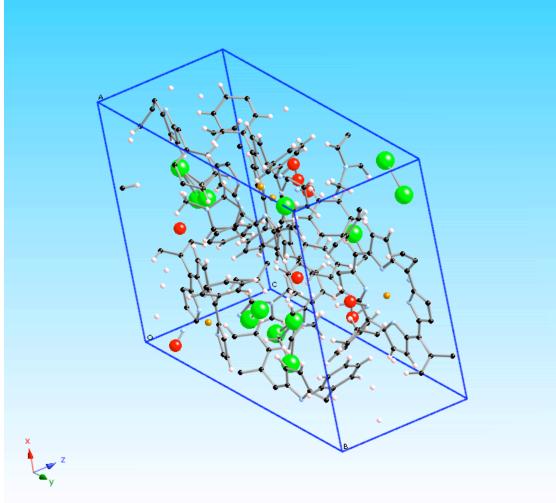
- 1. Low frequency motions: ~ total mass
- 2. No selection rule except motion of atoms along x-ray propagation
- 3. Peak intensity ~ mode participation ~ actual displacement
- 4. No matrix effects or limitations
- 5. Element and isotope selective
- 6. No unpredictable cancellations in scattering terms

$$\phi_{\alpha} = \frac{1}{3} \frac{\overline{v}_R}{\overline{v}_{\alpha}} e_{j\alpha}^2 (\overline{n}_{\alpha} + 1) f$$

Matt Smith, et al, Inorganic Chemistry, 2005, 44,5562

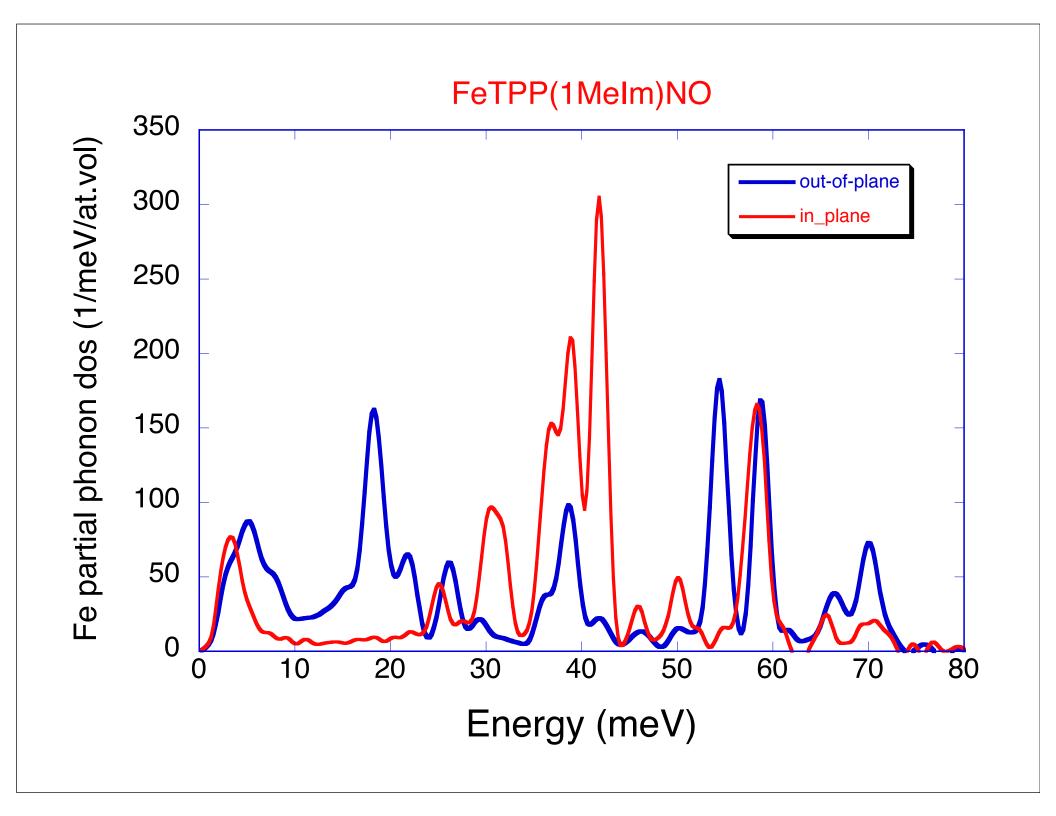






Porphyrins:

Tetraphenylporphyrin (TPP) Octaethylporphyrin (OEP) A B
Phenyl H
H Ethyl



Materials Science & Physics

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