Synchrotron Radiation: Production & Properties

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August 2013

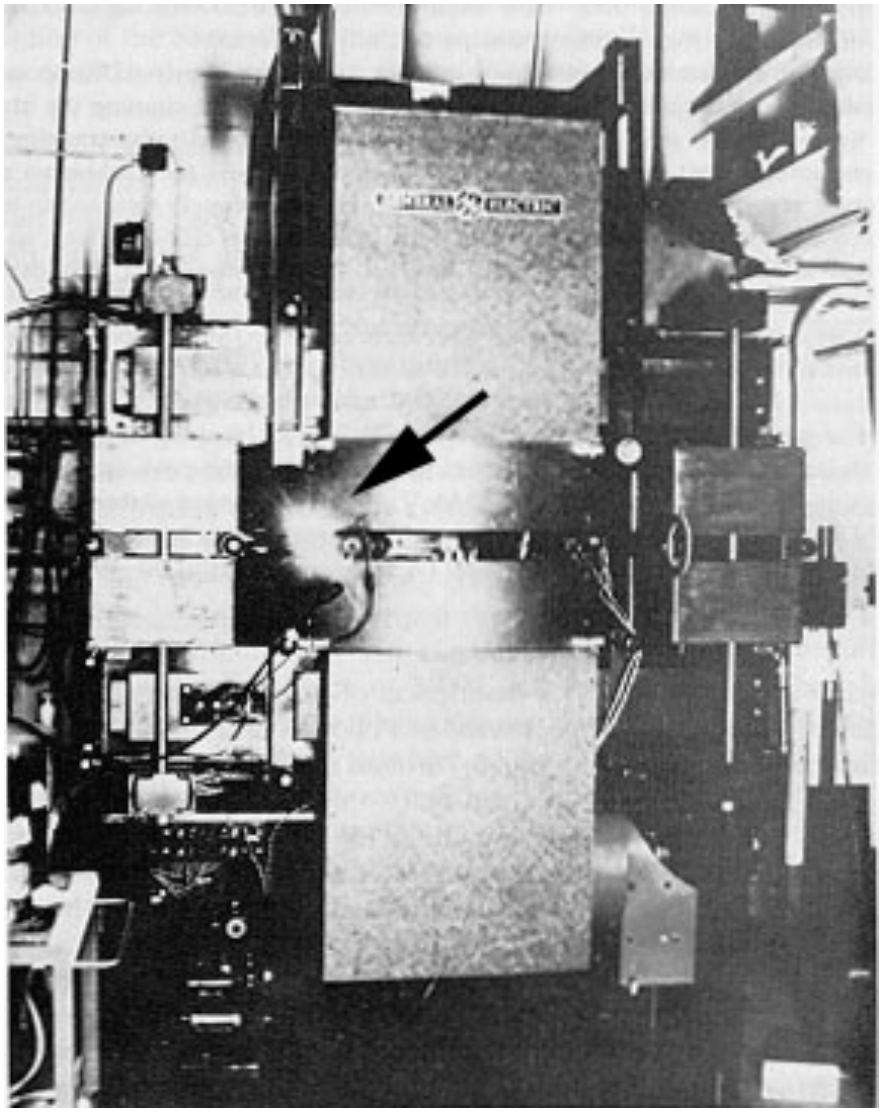
The Advanced Photon Source is an Office of Science User Facility operated for the U.S. Department of Energy Office of Science by Argonne National Laboratory

Synchrotron Radiation - Some Background

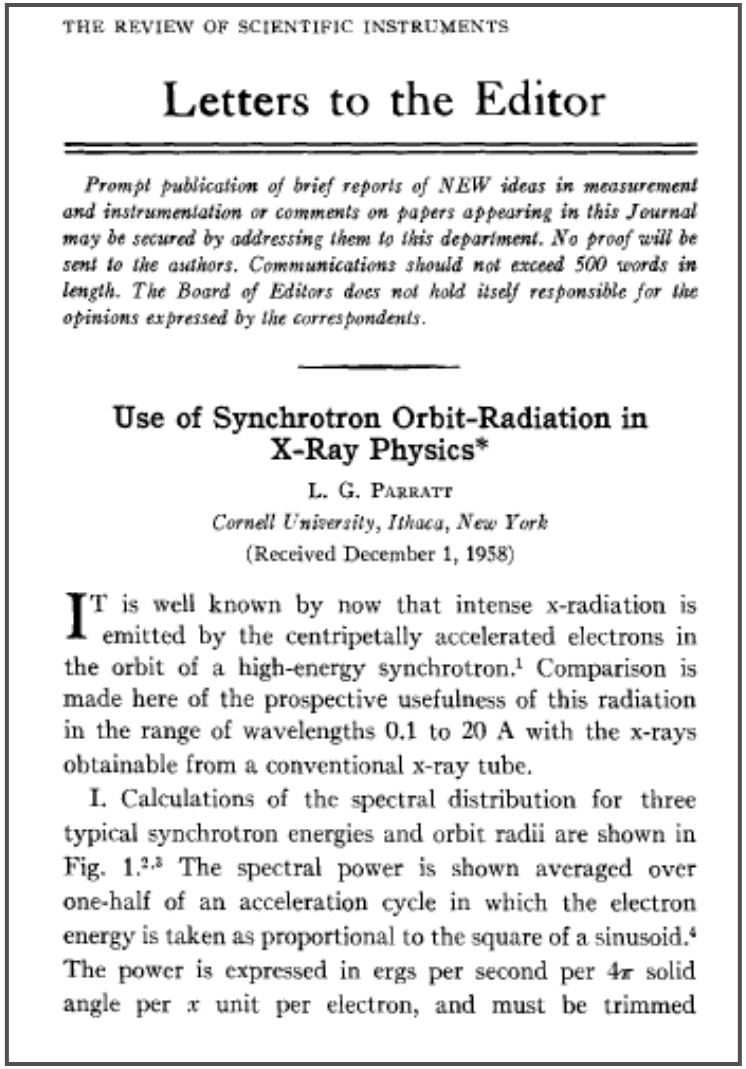
Synchrotron radiation (SR) - radiation from charged particles traveling in circular orbits - was first observed (accidentally) from a 70 MeV synchrotron 1947.

On April 24,[1947] Langmuir and I [Herbert Pollack] were running the machine and as usual were trying to push the electron gun and its associated pulse transformer to the limit. Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.

Excerpted from Handbook on Synchrotron Radiation, Volume 1a, Ernst-Eckhard Koch, Ed., North Holland, 1983.



From Synchrotrons to Storage Rings



THE REVIEW OF SCIENTIFIC INSTRUMENTS

Letters to the Editor

Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Use of Synchrotron Orbit-Radiation in X-Ray Physics*

L. G. Parratt

Cornell University, Ithaca, New York

(Received December 1, 1958)

It is well known by now that intense x-radiation is emitted by the centripetally accelerated electrons in the orbit of a high-energy synchrotron.[1] Comparison is made here of the prospective usefulness of this radiation in the range of wavelengths 0.1 to 20 A with the x-rays obtainable from a conventional x-ray tube.

I. Calculations of the spectral distribution for three typical synchrotron energies and orbit radii are shown in Fig. 1.[2,3] The spectral power is shown averaged over one-half of an acceleration cycle in which the electron energy is taken as proportional to the square of a sinusoid.[4] The power is expressed in ergs per second per 4π solid angle per x unit per electron, and must be trimmed

- Synchrotrons were first used as sources of SR. However, the particles' constantly changing energy was not attractive and the advent of storage rings provided a far more attractive source.
- We now use the name synchrotron radiation to describe radiation that is emitted from charged particles traveling at relativistic speeds, regardless of the accelerating mechanism and shape of the trajectory.
- Although synchrotron radiation can cover the entire electromagnetic spectrum, we are interested in radiation in the x-ray regime.

$$(\lambda[\text{Å}] = 12.4 / E[\text{keV}])$$

Square brackets indicate the units to be used in the calculation.

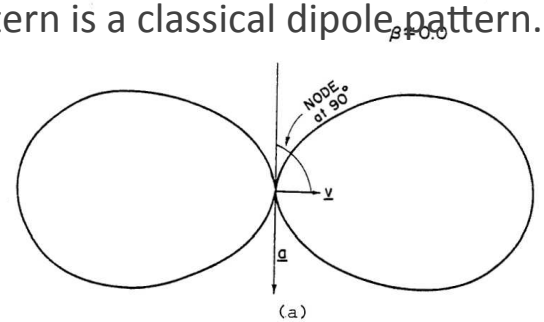
Radiation Patterns From Accelerating Charges

Definitions:

$$\beta = v/c$$

$$\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$$

When v << c, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.

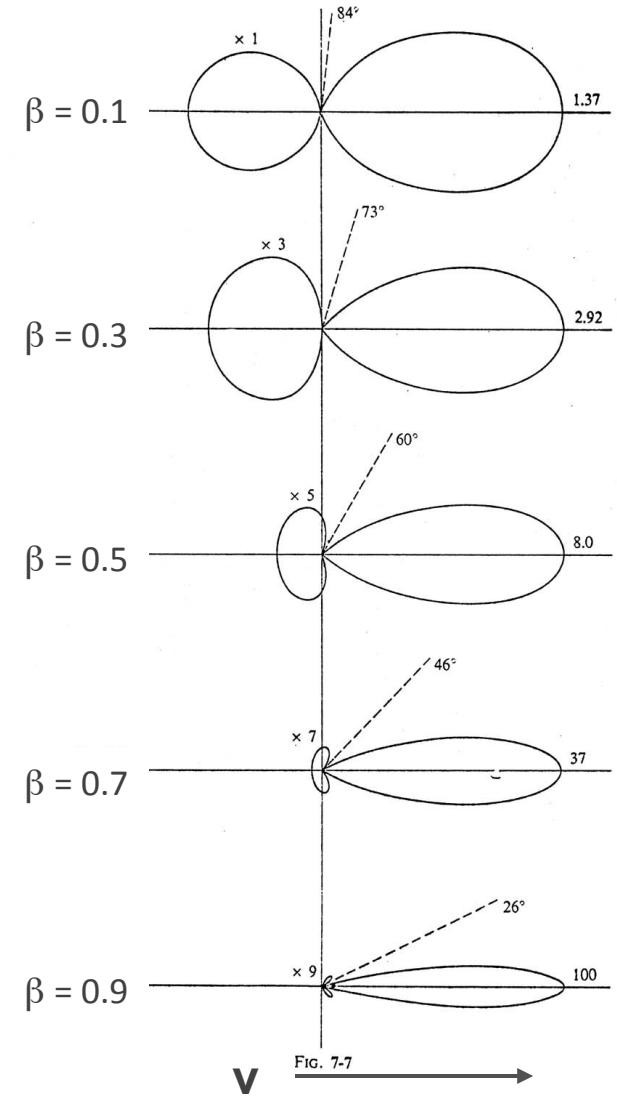


$\beta = 0$

But as β approaches 1 :

a. The shape of the radiation pattern changes; it is more forward directed

b. The intensity of the radiation pattern changes; it increases



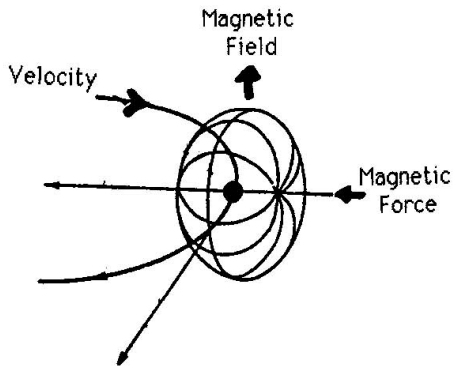
$$\tan\theta_{lab} = \frac{\sin\theta'}{\gamma\left(\cos\theta' + \beta\right)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$

Radiation from Highly-Relativistic Particles

RADIATION FROM CHARGED PARTICLE
(CONSTANT B-FIELD)

NON-RELATIVISTIC CASE: $v/c \ll 1$

Dipole Radiation

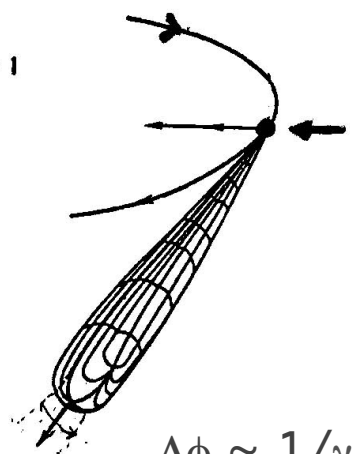


HIGHLY RELATIVISTIC CASE: $v/c = \beta \approx 1$

$\gamma = E/m_0c^2$

$m_0c^2 = 0.511\,\text{MeV}$

E(GeV)	γ	θ(μrad)
1	1957	511
3	5871	170
7	13699	73



$\Delta\phi \approx 1/\gamma,$

- In fact, the opening angle in both the horizontal and vertical directions, is given approximately by:

$$\Delta\phi_{vert} = \Delta\phi_{hor} \approx 1/\gamma,$$

when $\beta \approx 1$.

- Relativistic velocities are good!!
  - radiation forward directed
  - radiated power $\propto E^4$

At the APS with E = 7 GeV,

$\gamma = E/m_0c^2 = 7\ \text{GeV}/0.511\ \text{MeV}$

$\gamma = 1.4 \times 10^4$

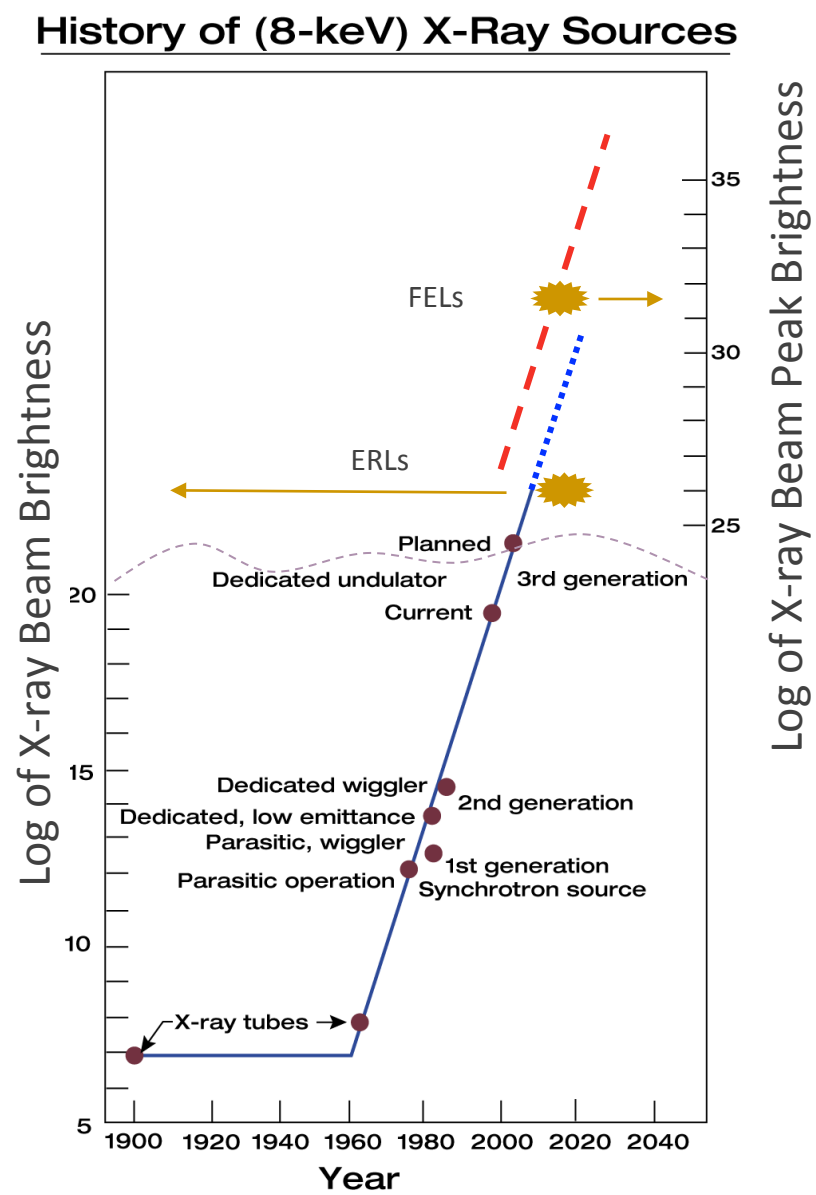
$1/\gamma = 73 \times 10^{-6}$

See Appendix 1 for more details

The Evolution of Brightness from Synchrotron Radiation Sources

Synchrotron radiation (from IR to hard X-rays) has been used a research tool for nearly 50 years.

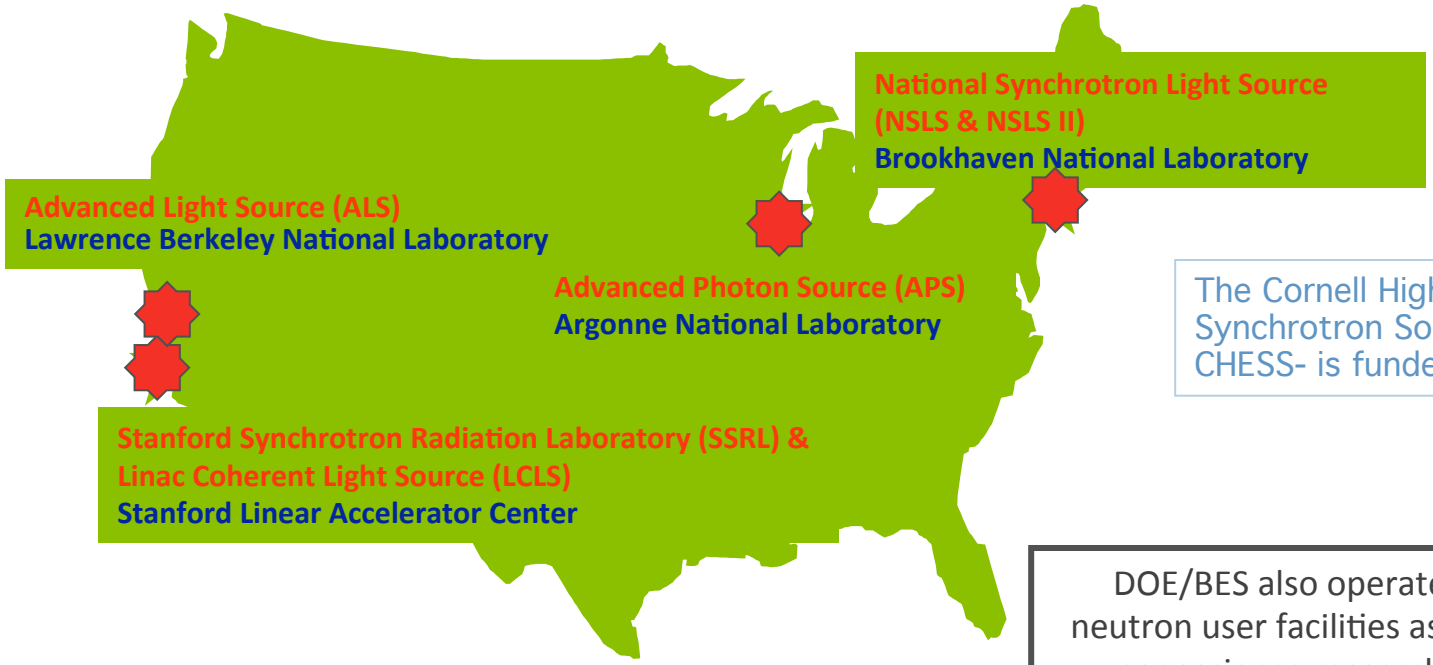
- 1st Generation Sources
  - *Ran parasitically on accelerations for high energy physics (CHESS and SSRL originally)*
- 2nd Generation Sources
  - *Built to optimize synchrotron radiation from the bending magnets (NSLS)*
- 3rd Generation Sources
  - *Built to optimize synchrotron radiation from insertion devices (ALS, APS, NSLS II [const.])*
- 4th Generation Sources
  - *fully coherent sources*
    - *X-ray Free Electron Lasers (X-FELs)*
    - *FEL-Oscillator (FEL-O)*
    - *Energy Recovery Linacs (ERLs)*
    - *Diffraction Limited Storage Rings (DLSRs)*



A Vital National Resource for Science and Technology

The **Advanced Photon Source (APS)** is a fully optimized, insertion-device-based, third generation x-ray source for the production of high intensity (brightness) hard x-ray beams.

ALS, APS. NSLS & SSRL are funded by the Department of Energy, Office of Science, Basic Energy Science (DOE/BES).



The Cornell High Energy Synchrotron Source – CHESS- is funded by NSF.

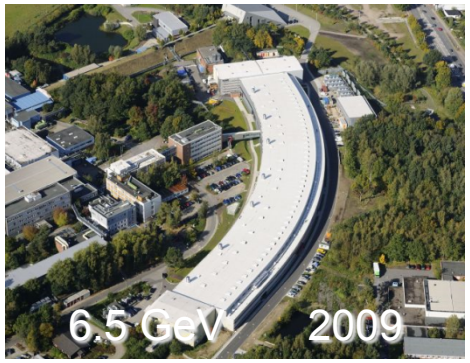
DOE/BES also operates several neutron user facilities as well as the nanoscience research centers.

Over **5000 unique researchers** from around the world came to Argonne last year to perform experiments at the APS.

Synchrotrons Around the World

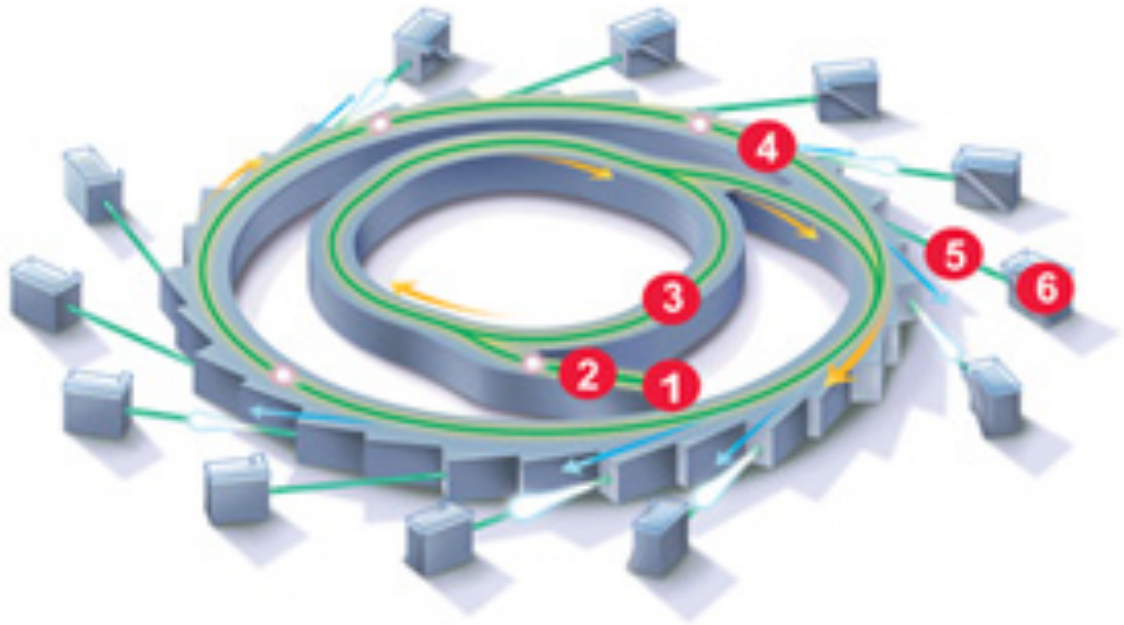
ESRF, France

PETRA-III, Germany: new



SPring-8, Japan

Typical SR Facility Complex

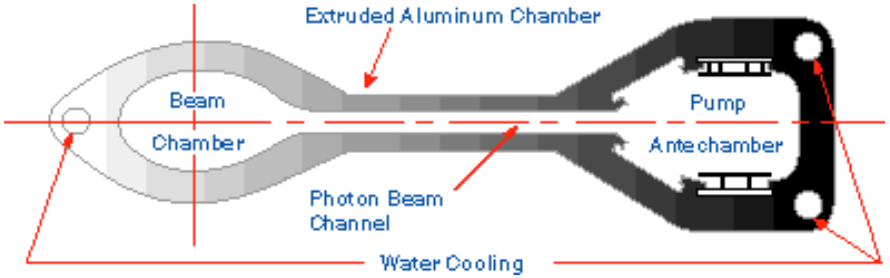


1. Electron gun
2. Linear Accelerator LINAC
3. Booster Synchrotron
4. Storage Ring (SR)
5. Beamline
6. Experiment station

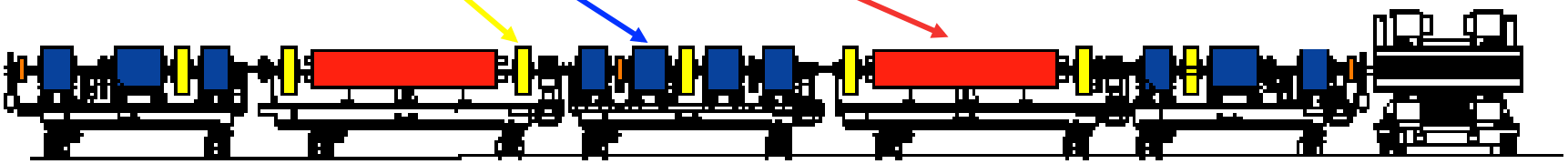
(Courtesy: Australian Synchrotron,
Illustrator: Michael Payne)

The APS Storage Ring

- Present operation:
  - 7 GeV and 100 mA
  - lifetimes > 12 hrs
  - filling time 1.1 minutes
- 40 straight sections each 5 meters long
  - 5 for accelerator purposes
  - 35 available as radiation sources
- 1104 m circumference
  - 80 dipoles
  - 240 quads
  - 20 skew quads
  - 280 sextupoles



Cross-section of the Al storage ring vacuum chamber



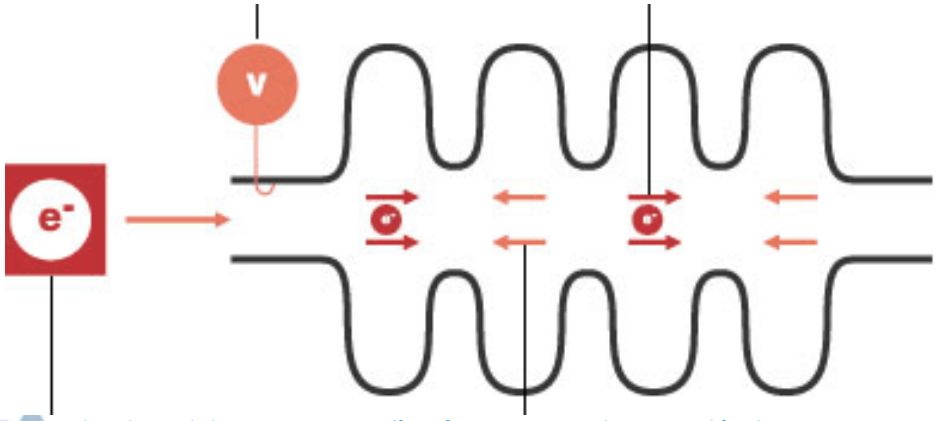
Time Structure of the Radiation

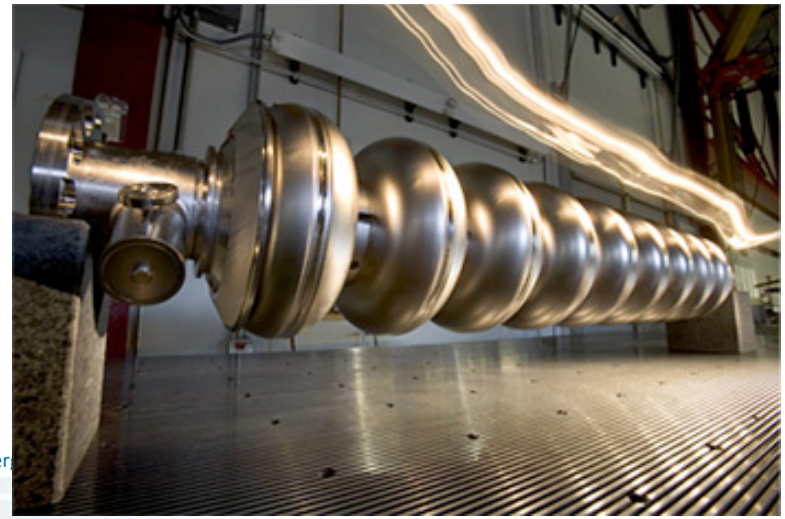
- Because the electrons are radiating x-rays, they are constantly losing energy.

- To restore the energy loss on each revolution, radio frequency (RF) cavities are installed in the storage ring to replenish the radiative energy losses.

- Particles are grouped together by the action of the radio frequency (RF) cavities into bunches. At the APS:
  - typically about 100 psec FWHM (about 3 cm in length)
  - 1104 m circumference (3.68 microsecond period)
  - there are 1296 evenly spaced “RF buckets” (stable orbit positions” around the ring)
  - minimum spacing is 2.8 nsec between bunches (determined by the RF frequency- 352MHz)

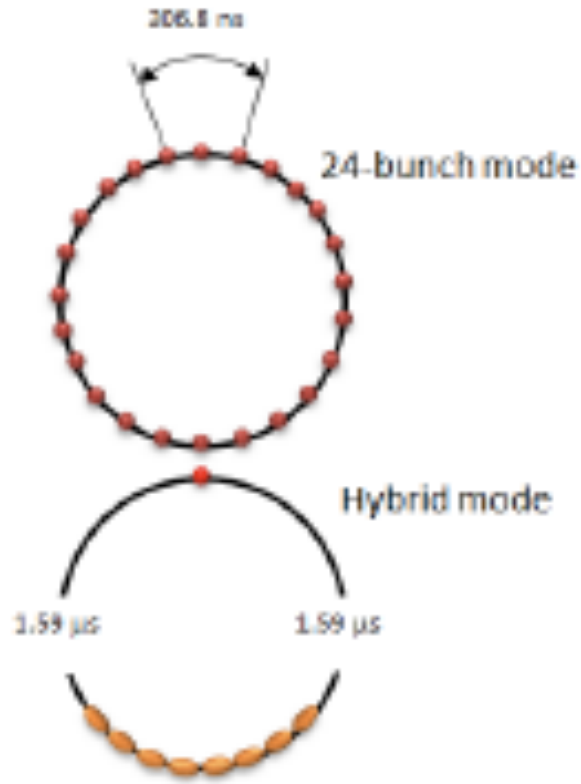
- Details of the time structure depends on the fill pattern, i.e. which RF buckets have electrons in them.



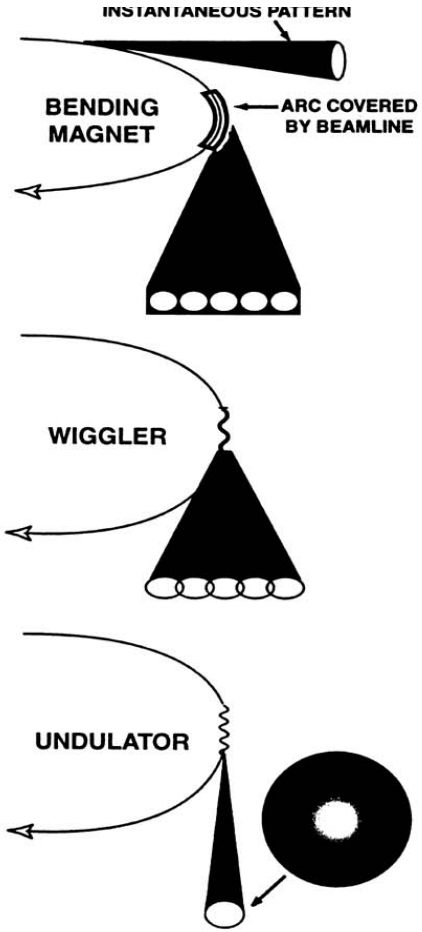


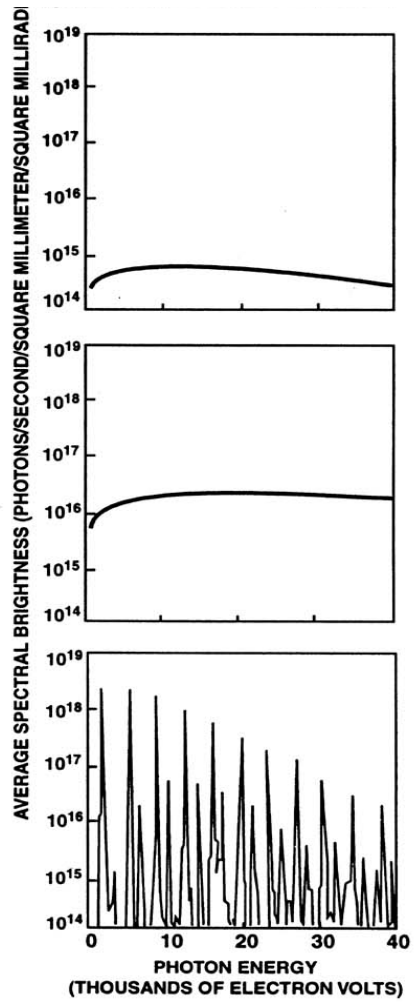
Typical APS Filling Patterns

- 324 equally spaced bunches
  - approximately 11 nsec between bunches
  - approximates a quasi-continuous source
- 24 equally spaced bunches
  - approximately 154 nsec between bunches
  - compromise between quasi-continuous source and pulsed source
- 1 + 7x8 (hybrid mode)
  - a single bunch followed by 8 groups of 7 bunches
  - Used for timing experiments (pump/probe)



Radiation Sources at 3rd Generation Facilities





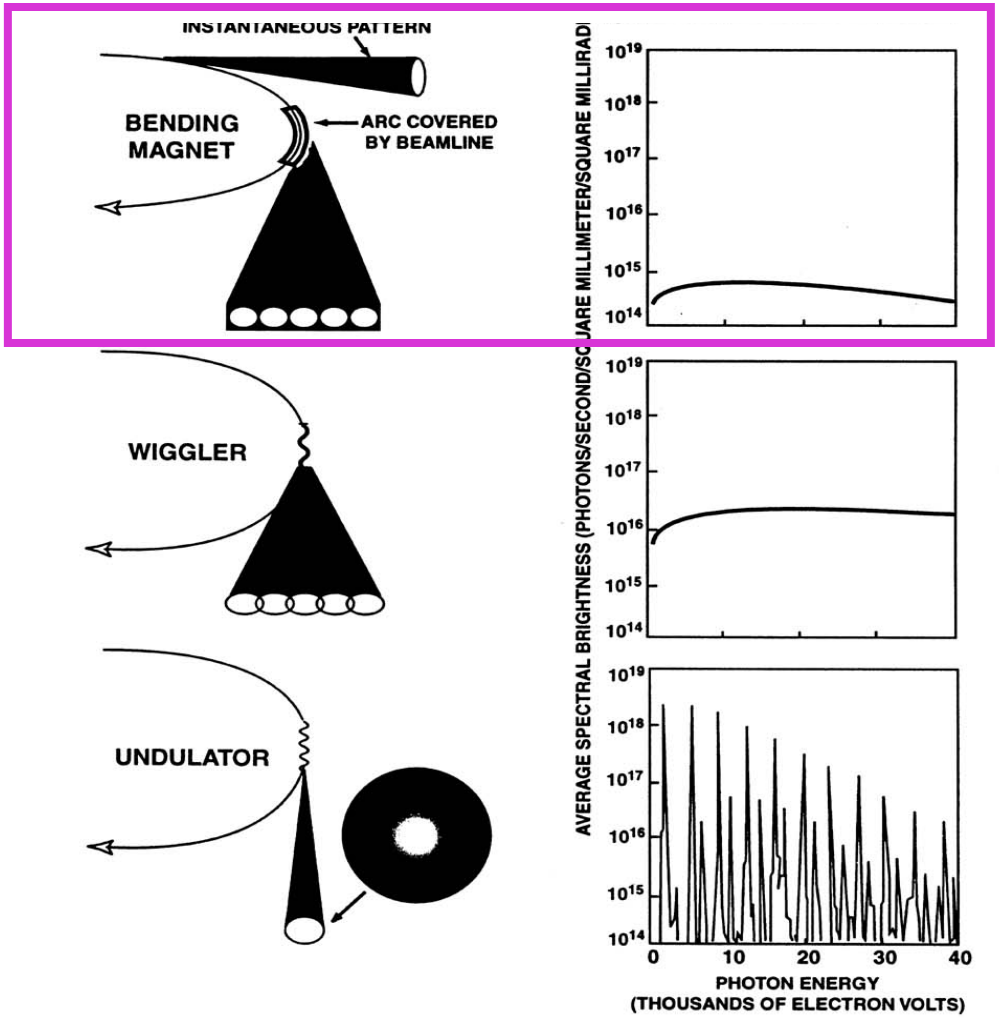
There are two different sources of radiation at 3rd generation sources:

- bending magnets (BMs)
- insertion devices (IDs); periodic arrays of magnets inserted between the BMs (wigglers or undulators)

The important parameters to know about each one is:

- Spectral distribution
- Flux (number of x-rays/sec - 0.1%bw)
- Brightness (flux/source size-source divergence)
- Polarization (linear, circular)

BM Spectral Properties



Bend Magnet Radiation

- Spectrum characterized by the critical energy:

$$E_c = 3hc\gamma^3/4\pi r.$$

- Vertical opening angle ($\Delta\phi$) is $1/\gamma$. For the APS:

$$\Delta\phi_{vert} = 1/\gamma = 73 \times 10^{-6} \text{ radians}$$

- Horizontal opening angle determined by apertures
- In the plane of the orbit, the polarization is linear and parallel to the orbital plane.

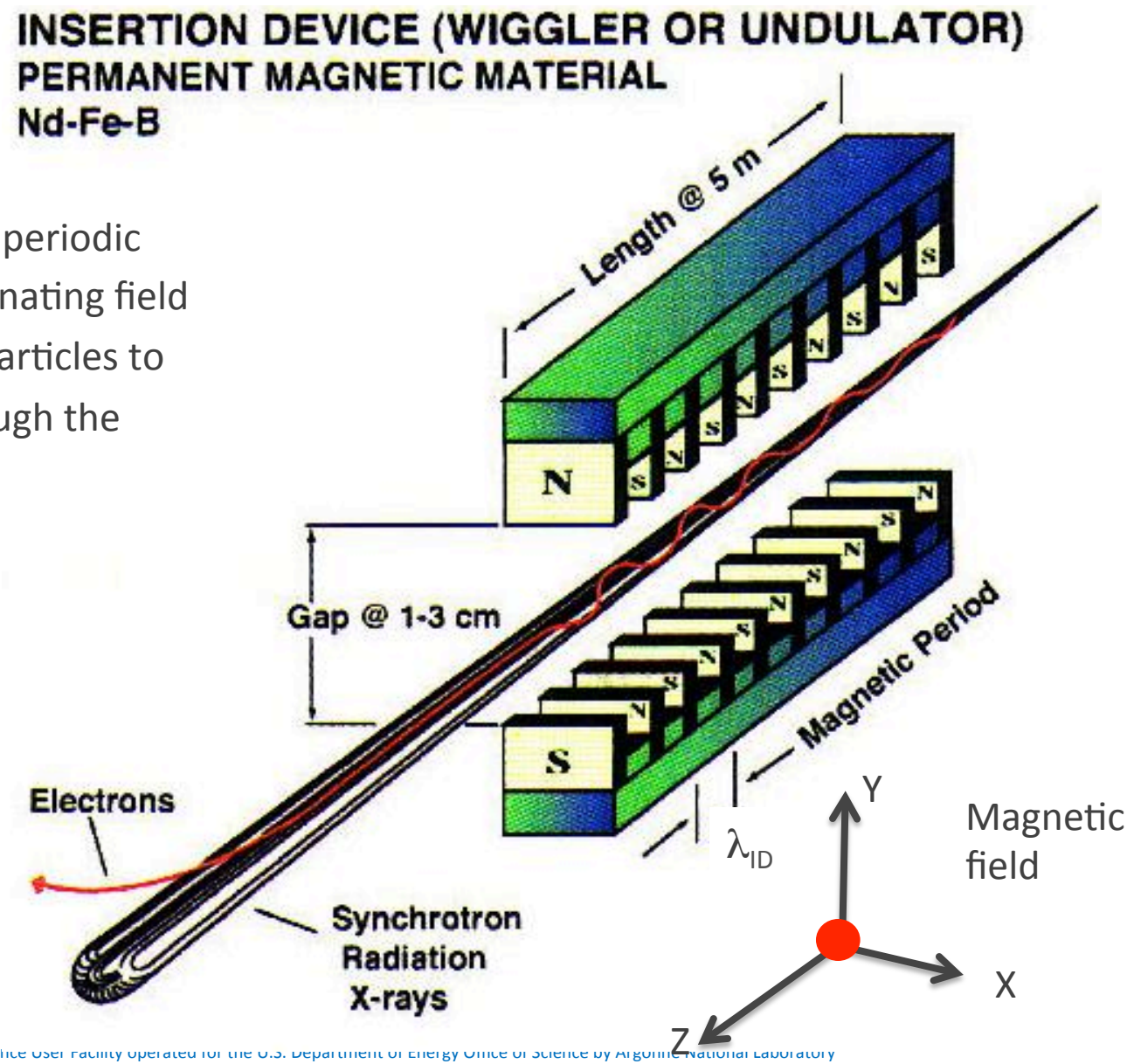
See Appendix 2 for more details

Planar Insertion Devices

Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

$L_{ID} = N\lambda_{ID}$ where:

L_{ID} = length
N = number of periods
λ_{ID} = magnetic period



Characterizing Insertion Devices

IDs are characterized by the so-called field index or deflection parameter, K **(See Appendix 2)**:

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[\text{cm}]\ B_o[\text{kG}]$$

where λ_{ID} is the period of the insertion device and B_o the peak magnetic field. (The length of the ID, L, is equal to the number of periods, N, times the length of the period, i.e., $L = N\lambda_{ID}$.)

The maximum deflection angle of the particle beam, θ_{max}, is given by:

$$\theta_{max} = \pm(K/\gamma)$$

See Appendix 3 for more details

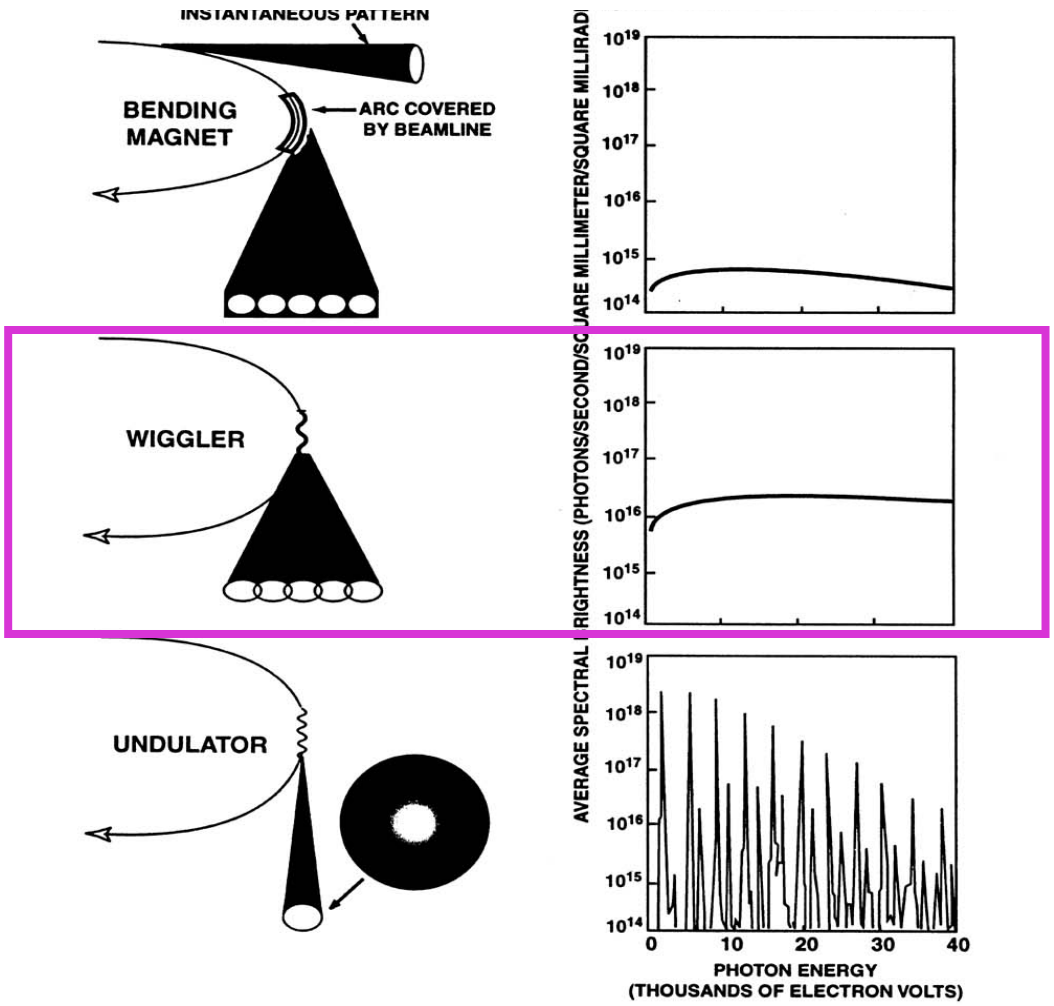
and the amplitude of the oscillation of the particles, x_{max}, by:

$$x_{max} = (K/\gamma)(\lambda_{ID}/2\pi)$$

APS Undulator A has a period of 3.3 cm and operates with K ≈ 1, therefore:

$\theta_{max} \approx 1/\gamma$ **and** $x_{max} \approx 0.38$ *microns.*

Wiggler Radiation Pattern and Spectrum



Wiggler Radiation (K>>1)

- $\theta_{max} = (K/\gamma) >> 1/\gamma$, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation ($1/\gamma$).
- radiation spectrum looks like 2N dipole sources (N = number of periods)
- spectrum characterized by the critical energy (which may be different than BM critical energy)
- Presently, there are NO planar wigglers installed at the APS. Wigglers with fields in both the horizontal and vertical directions produce elliptically polarized radiation. These are sometimes called elliptical multipole wigglers (EMWs).

Undulators

Undulators:

- $K \approx 1$
- θ_{max} is comparable $(1/\gamma)$ and so the radiation from each pole overlaps causing interference effects in the spectral distribution.
- Constructive interference occurs when:

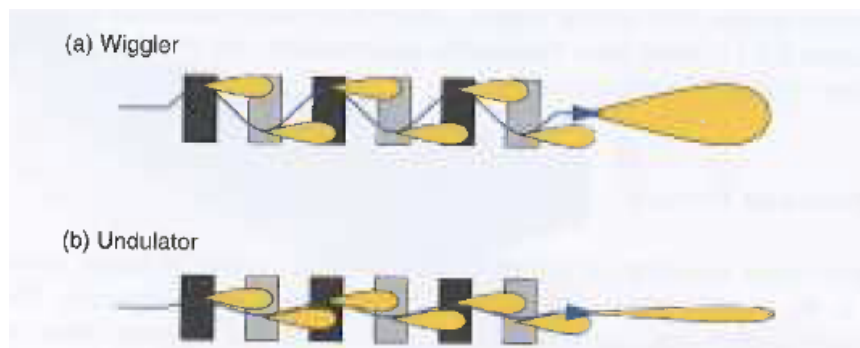
$$\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

- The wavelength where constructive interference occurs, $\lambda_n^{x\text{-}ray}$, can be adjusted by varying the magnet field, B, produced by the undulator.

$$K = 0.0934\ \lambda_{ID}[\text{cm}]\ B_o[\text{kG}]$$

- Most undulators are made of permanent magnets, so to vary the B-field that the electrons see, the gap between the upper and lower magnet arrays is varied.

APS 2.4 m long Undulator A (λ_{ID} = 3.3 cm)



Tuning Curves for Undulators

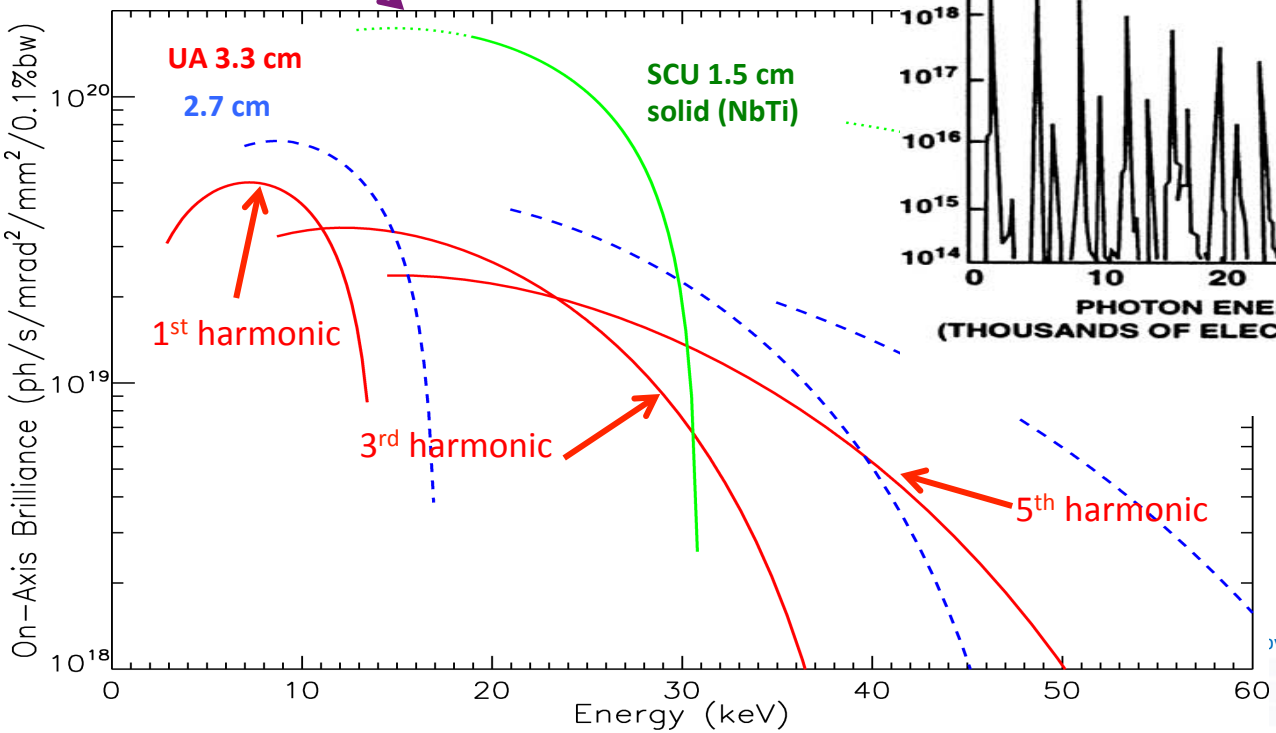
Recall:

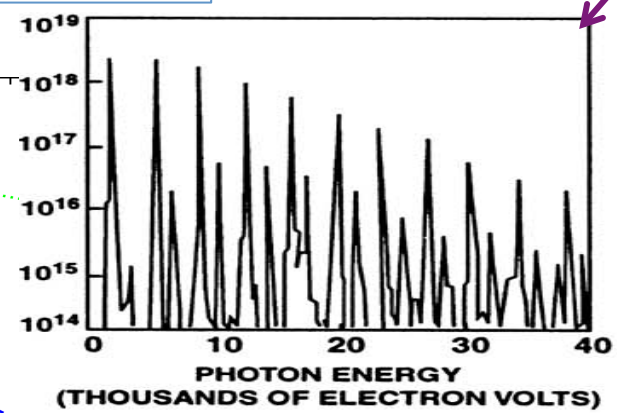
$$\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

Note that you do not get just the fundamental (n=1), but you get all the odd harmonics (n=3,5,7…)

Here is the spectrum over the tuning range of the undulator. The low energy is fixed by the minimum magnet gap (determined by the vacuum chamber). As the gap is opened, the x-ray energy increases, but as you open it further, the magnetic field decreases and so does the intensity of the x-rays.

Here is the spectrum for a particular gap (i.e., a particular K value).





The range of energy that can be covered can be adjusted varying the period, λ_{ID}, of the undulator.

Undulator Radiation Spectra

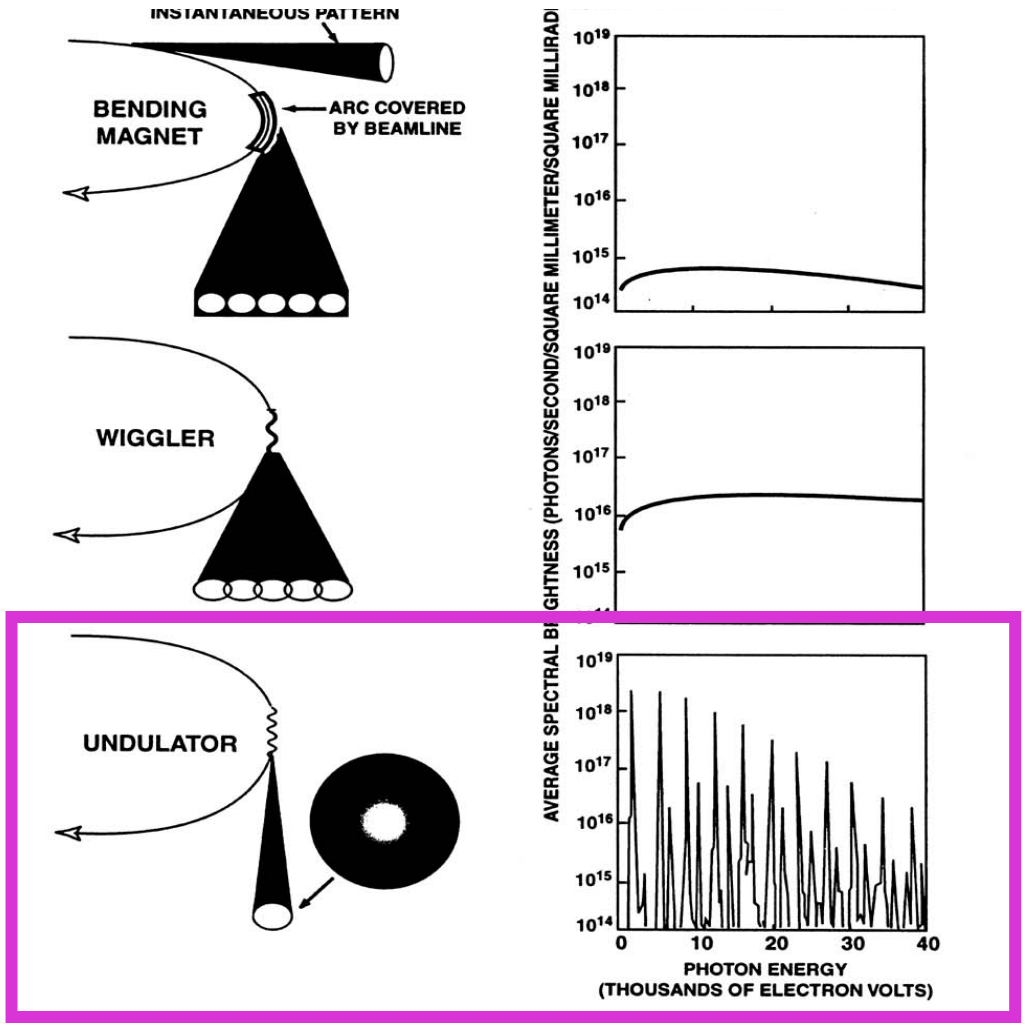
Undulator Radiation

- undulators defined as IDs with horizontal deflection angle ≈ $1/\gamma$, i.e., $K \approx 1$
- spectrum peaked but peaks are tunable by varying K ($K = 0.94\ B[T]\ \lambda_{ID}[cm]$)
- at the peaks (harmonics) the horizontal and vertical opening angles of the radiation is given by:

$$\Delta\phi = 1/\gamma\ (1/N)^{1/2}$$

where N = number of periods [typically 100]

- to get the true opening angle, need to consider the opening angle of the emitting particles (- more later on!)



See Appendix 4 for more details

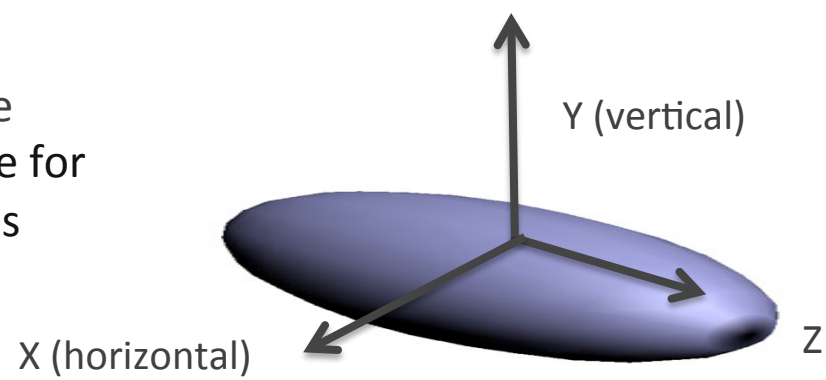
Transverse Properties of Particle Beams

- Up until now, we have calculated the radiation properties from a single electron, however in a storage ring, the radiation is emitted from an ensemble of electrons with some finite size and divergence distribution.

- Both the transverse and longitudinal properties of the particle beam in a storage ring are the equilibrium properties of the particle beam, but here we are interested in the ***transverse*** properties.

- The product of the particle beam size and divergence in the horizontal (X) direction is (proportional) to a parameter of the beam called the horizonal emittance or ε_H (units are length x angle). Similarly for the vertical (Y).

- The horizontal and vertical emittance is a constant around the length of the storage ring, although one can trade off beam size for divergence as long as the product remains constant.



X, Y are transverse directions
Z is the longitudinal direction

Transverse Properties and Photon Beam Brightness

Why do we need to know about the transverse particle beam properties?

Although the flux from BM and ID sources can be determined without detailed knowledge of the source size and divergence, one very important characteristic of the beam, namely **brightness,** requires a more detailed knowledge of the particle beam's size and divergence.

Brightness has units of:

photons/sec/0.1% BW/source area/source solid angle

$$\text{Flux}/4\pi^2\, \Sigma_h\, \Sigma_v\, \Sigma_h'\, \Sigma_v'$$

where Σ_i (Σ_i') is the effective one sigma value of the source size (divergence) in the ith direction. The effective source size and divergence has contributions from both the particle beam and the radiation itself.

For source size $\Sigma = [\sigma_{x\text{-}ray}^2 + \sigma_{electron}^2]^{1/2}$ and for source divergence $\Sigma' = [\sigma'^2_{x\text{-}ray} + \sigma'^2_{electron}]^{1/2}$

When the electron emittance, ε ($\sigma_{electron}\sigma'_{electron}$) ↓ then brightness ↑

See Appendix 5 for more details

Diffraction Limited Sources and Coherence

- The Heisenberg Uncertainty Principle sets a lower limit for the emittance of radiation. Recall:

$$\Delta x \Delta p_x \geq \hbar / 2$$

$$\frac{p_x}{p_z} = \phi_x \text{ or } \frac{\Delta p_x}{p_z} = \Delta\phi_x \text{ and } p_z = \hbar k = \hbar(2\pi / \lambda)$$

$$so: \Delta x \Delta p_x = \Delta x \Delta\phi_x p_z = \Delta x \Delta\phi_x [\hbar(2\pi / \lambda)] \geq \hbar / 2$$

$$\Delta x \Delta\phi_x \geq \lambda / 4\pi$$

- So once the particle beam emittance for a given x-ray wavelength λ is less than $\lambda/4\pi$, the source is said to be at the diffraction limit in that direction. If both the horizontal and vertical emittance of the beam is less than $\lambda/4\pi$, then the radiation emitted is fully coherent. For 1Å (12 keV) x-rays, the particle beam emittance would have to be less than:

1Å /4π = 10^{-10} meters /4π ≈ 8 x 10^{-12} m or 8 picometers – radian (radians are dimensionless)

- Coherence can be an important parameter in some experiments such as photon correlation spectroscopy, x-ray holography, imaging, etc.

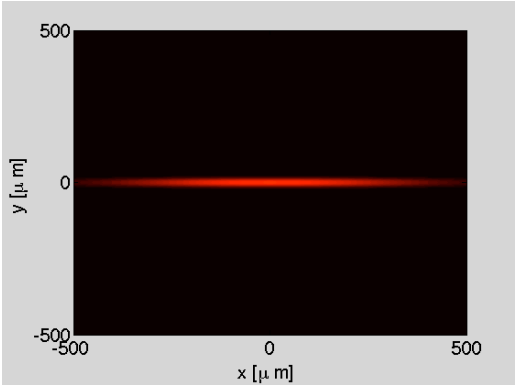
Partial Coherent Sources

For 1Å (12 keV) x-rays → 8 picometers – radian for fully coherent beam .

- APS operates with:

  $\varepsilon_H = 3 \times 10^{-9}$ m-rad or 3000 picometer-radian

  $\varepsilon_V = 0.025 \times 10^{-9}$ m-rad or 25 picometer-radian



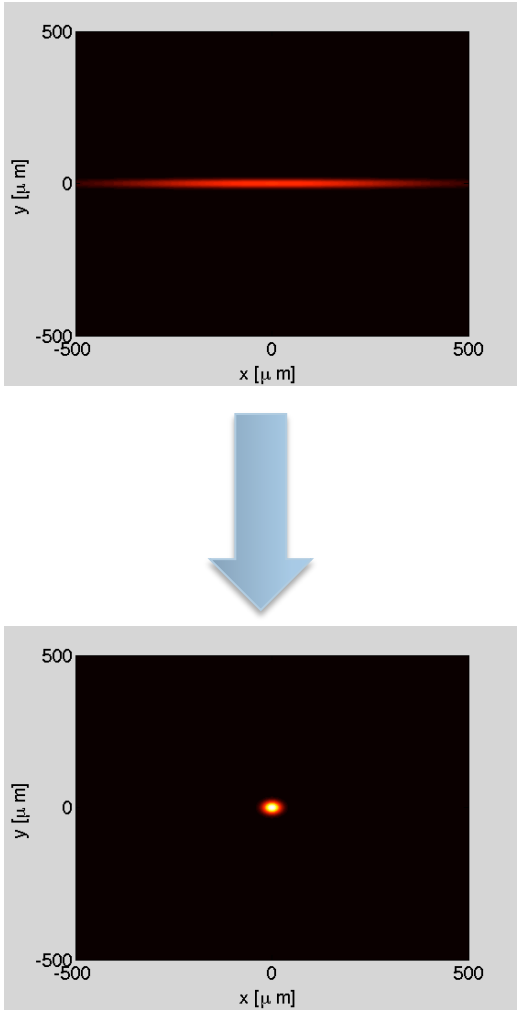
- Hence the APS is a partially coherent source at 1 Å.
- Partially coherent sources are sometimes characterized by the coherent fraction.

  Coherent fraction = ratio of diffraction-limited emittance to total emittance, or the the fraction of the x-ray flux that is coherent.

- For the APS at 1Å, the coherent fraction is ≈ 10^{-3}.
- So there is a general trend to try to reduce the particle beam emittance to increase coherence.

Beyond the Capabilities of Today's Storage Rings

- What parameters would users like to see "enhanced"?
  - Increased brightness (i.e., larger coherent fraction = $F_{coherent} / F_{total}$)
  - Shorter pulses (from 100 ps to 100 fsec)



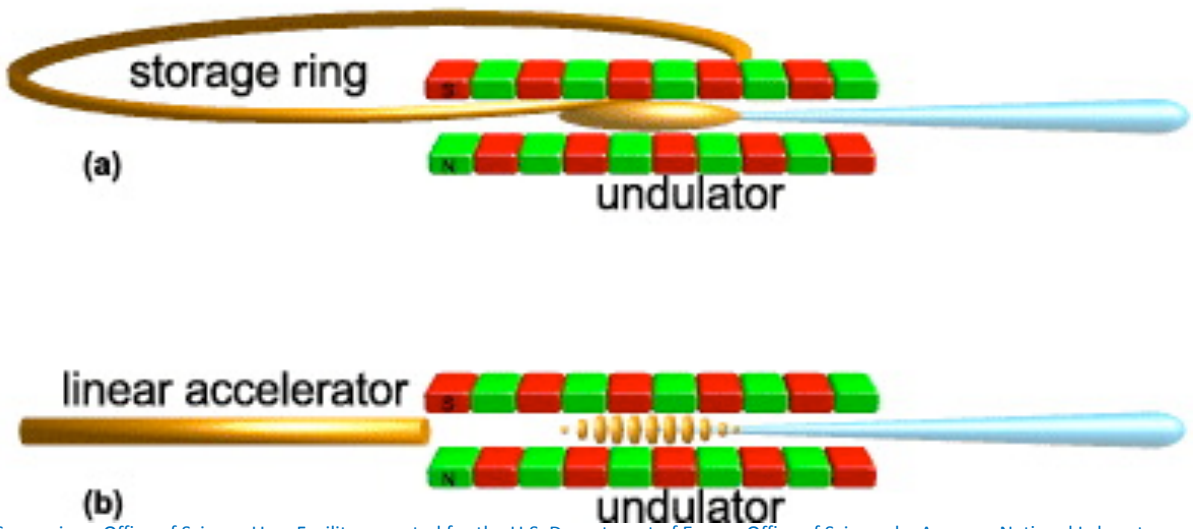
- What are the options?
  - Diffraction Limited Storage Rings (DLSRs)
    - Very large storage rings to further drive down the particle beam emittance and hence increase the brightness
    - May lose timing options as there needs to be many bunches with fewer particles per bunch
  - Energy Recovery Linacs
    - Have the potential to deliver spatially coherent beams but the technology is not there yet (active R&D programs going in at CHESS at Cornell Unversity and Photon Factory in Japan)
  - X-ray Free Electron Lasers (X-FELs)

X-ray Free Electron Lasers (X-FELs)

- Linac-based x-ray free electron lasers (XFELs) can provide many of the desired improvements.
  - Full transverse (spatial)coherence!
  - Femtosecond pulse lengths!

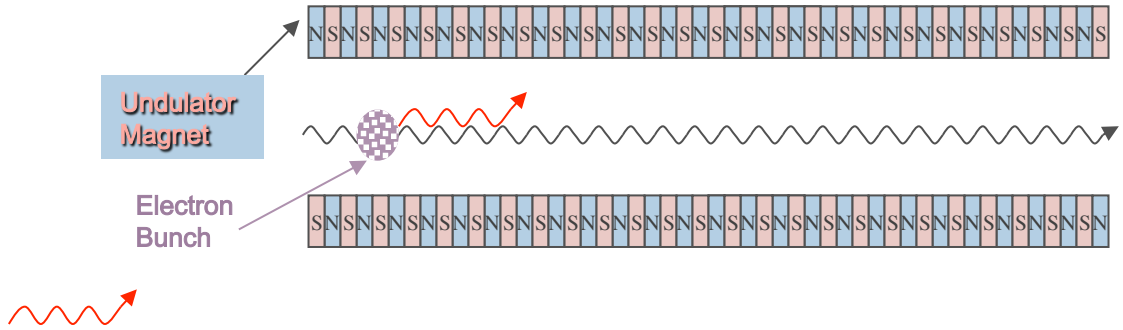
- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an emittance-preserving linac.

- The gain in the laser is obtained through a process called **Self-Amplified Spontaneous Emission** *or* **SASE.**



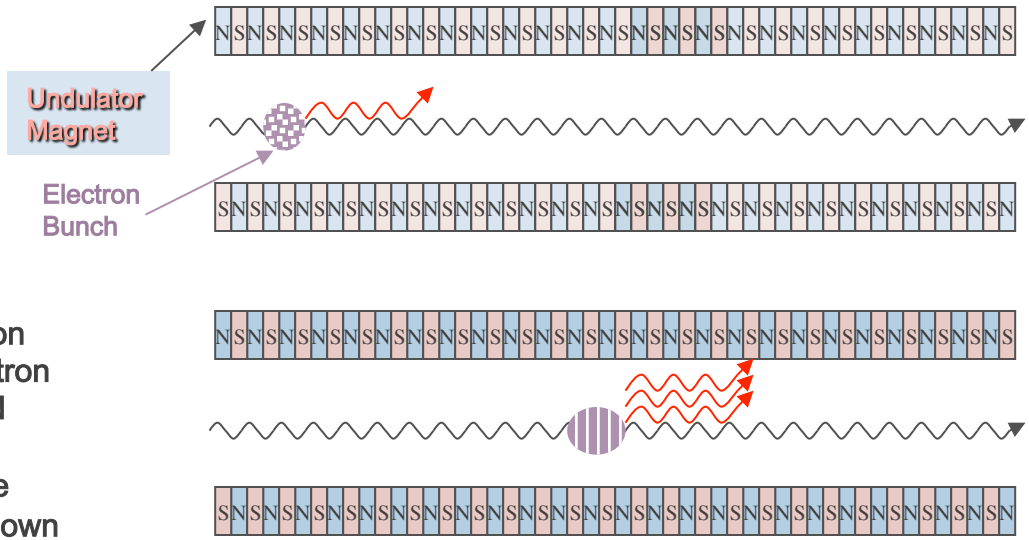
Self-Amplified Spontaneous Emission

The LCLS produces extraordinarily bright pulses of synchrotron radiation in a process called “self-amplified spontaneous emission” (SASE). In this process, an intense and highly collimated electron beam travels through an undulator magnet. The alternating north and south poles of the magnet force the electron beam to travel on an approximately sinusoidal trajectory, emitting synchrotron radiation as it goes.



The Advanced Photon Source is an Office of Science User Facility operated for the U.S. Department of Energy Office of Science by Argonne National Laboratory

Self-Amplified Spontaneous Emission

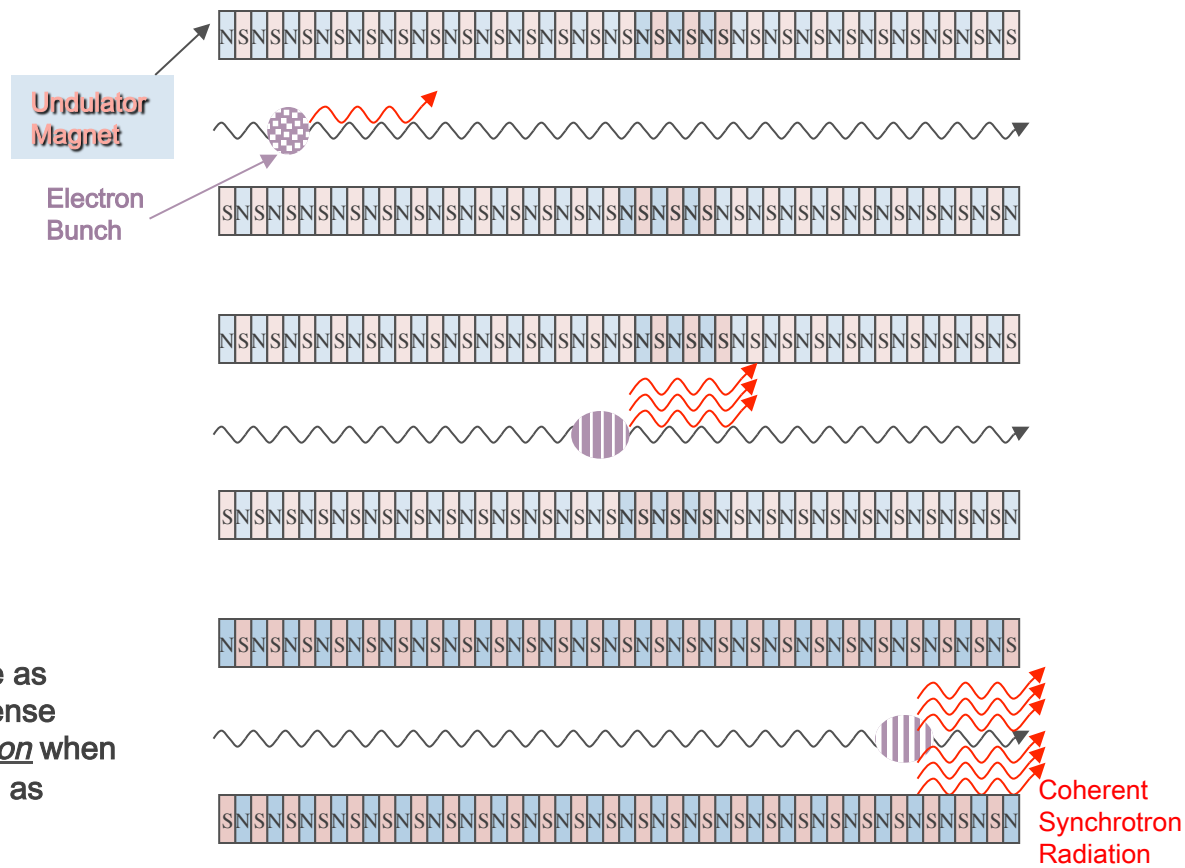


The electron beam and its synchrotron radiation are so intense that the electron motion is modified by the electric and magnetic fields of its own emitted synchrotron light. Under the influence of both the undulator magnet and its own synchrotron radiation, the electron beam is forced to form micro-bunches,

separated by a distance equal to the wavelength of the emitted radiation.

John Galayda, LCLS

Self-Amplified Spontaneous Emission



These micro-bunches begin to radiate as if they were single particles with immense charge. The process reaches *saturation* when the micro-bunching process has gone as far as it can go.

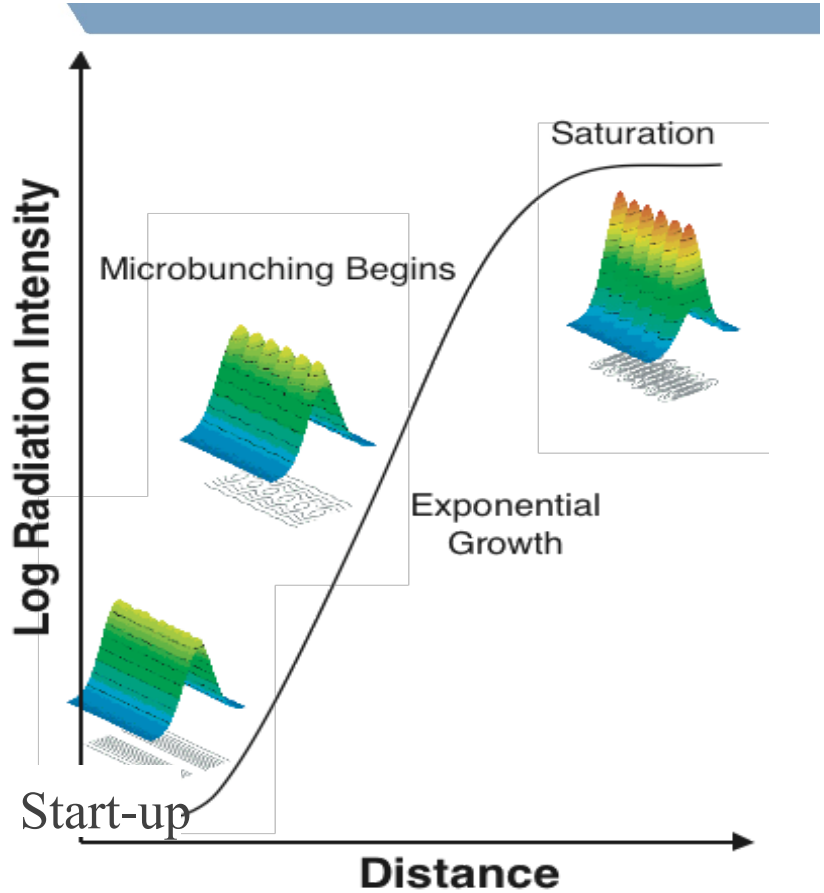
John Galayda, LCLS

Free Electron Lasers (FELs)

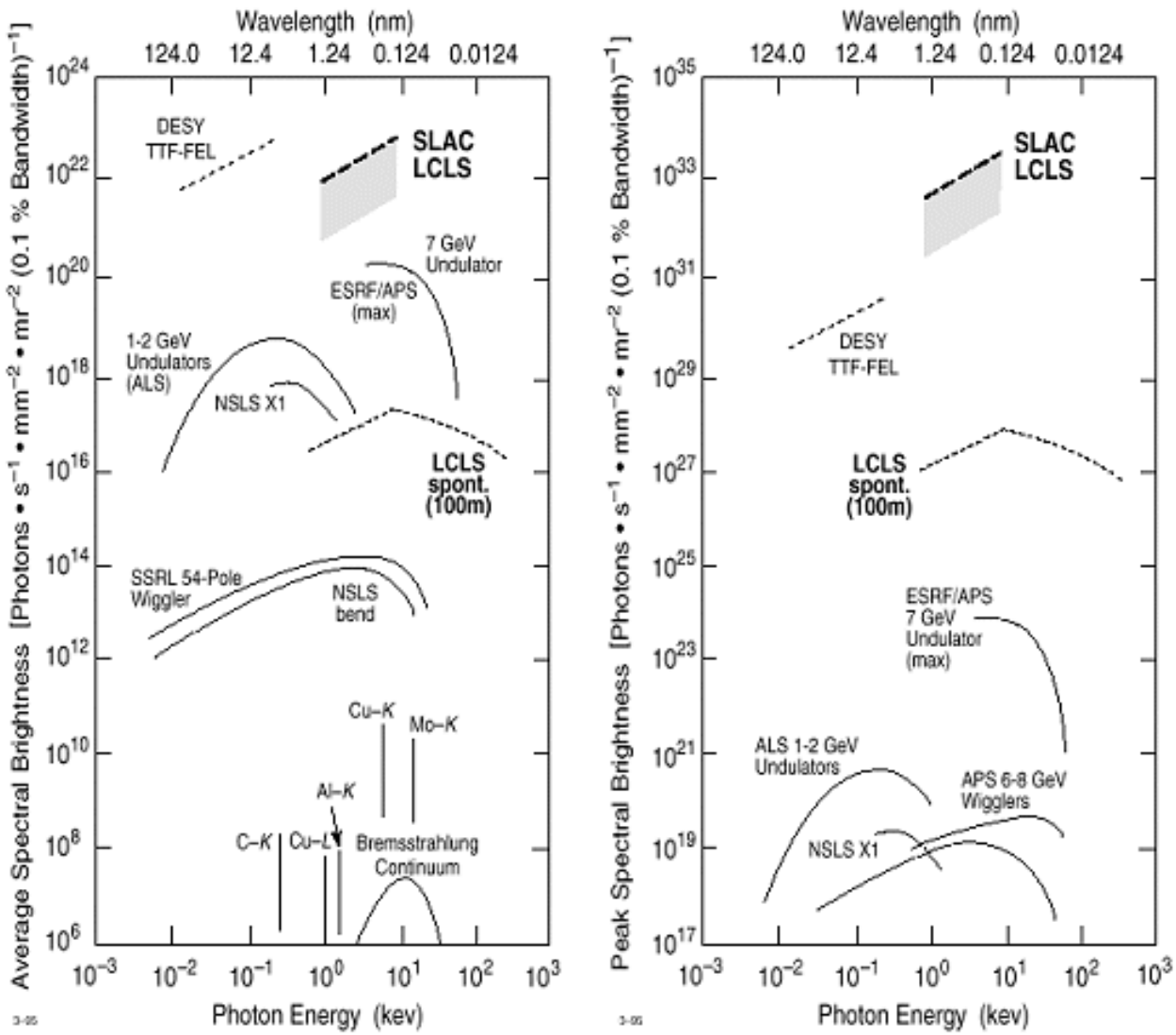
Start-up stage

External signal or spontaneous radiation interacts with the e-beam resonantly at undulator λ

Energy modulation ➔ density modulation (microbunching) ➔ coherent radiation at λ ➔ exponential growth (L_G)



Spectral Properties for X-ray Free Electron Lasers

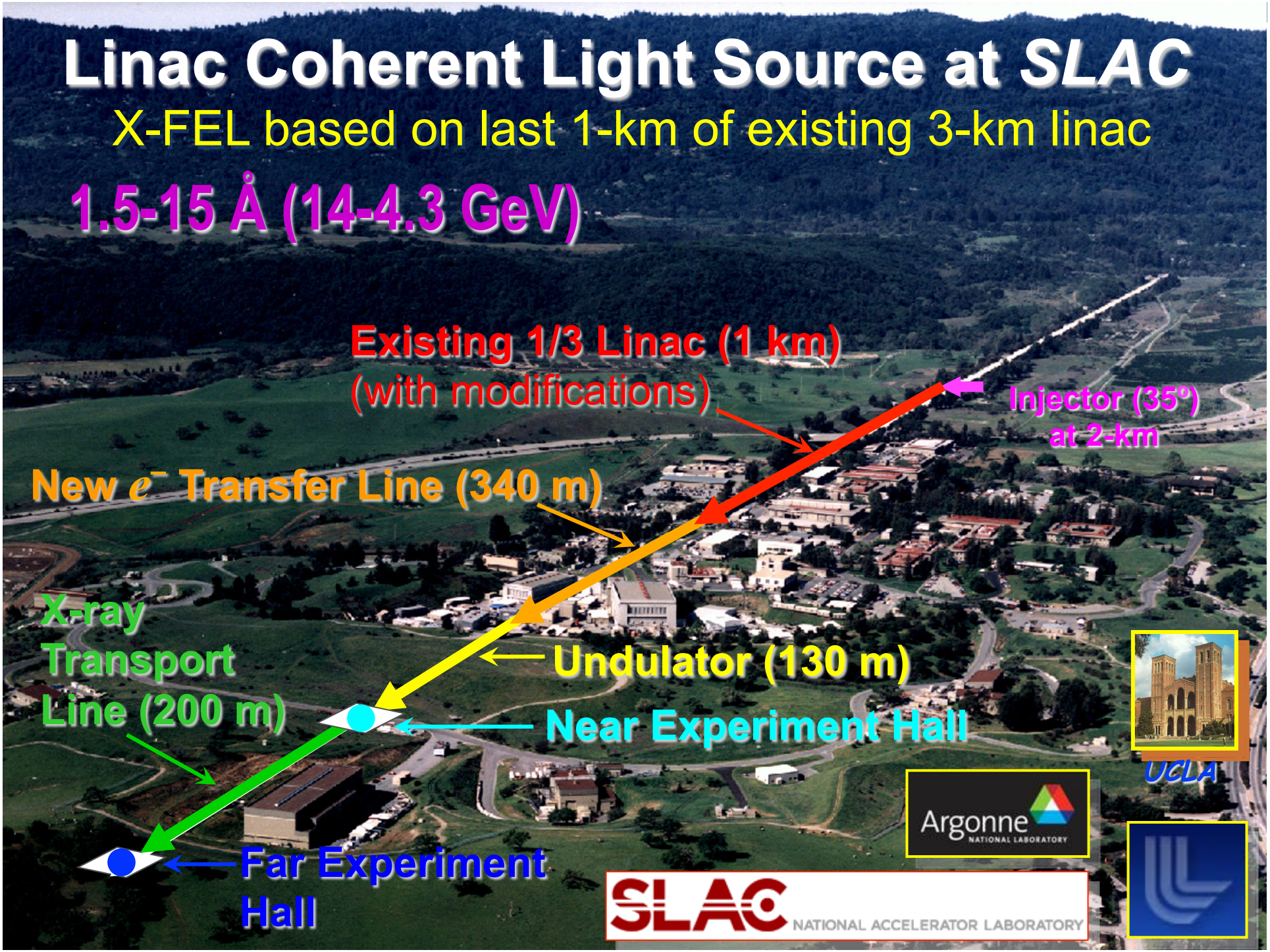


Average and peak brightness calculated for the LCLS and for other facilities operating or under construction.

Linac Coherent Light Source at *SLAC*

X-FEL based on last 1-km of existing 3-km linac

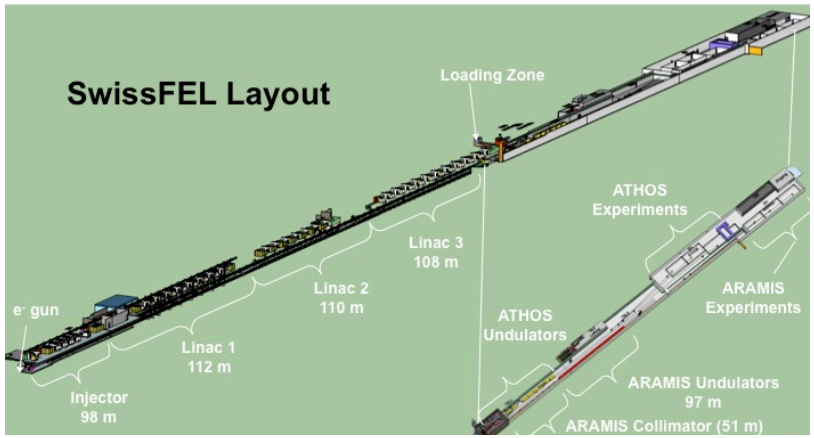
1.5-15 Å (14-4.3 GeV)



X-ray Free Electron Lasers

SACLA XFEL at SPring-8 Facility (Japan) - Operational

European XFEL at DESY (Germany) – Construction



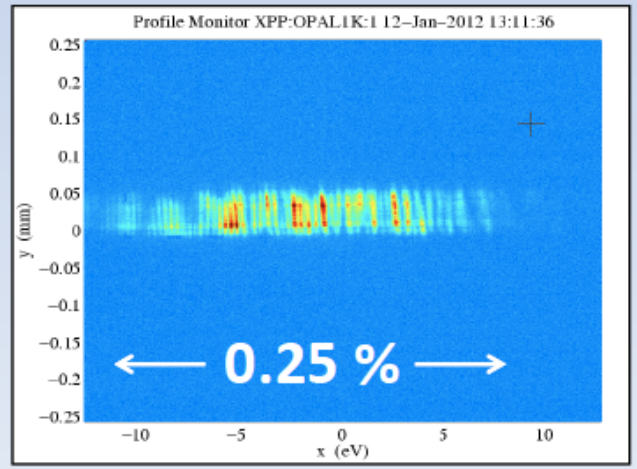
SwissFEL at PSI (Switzerland) Construction

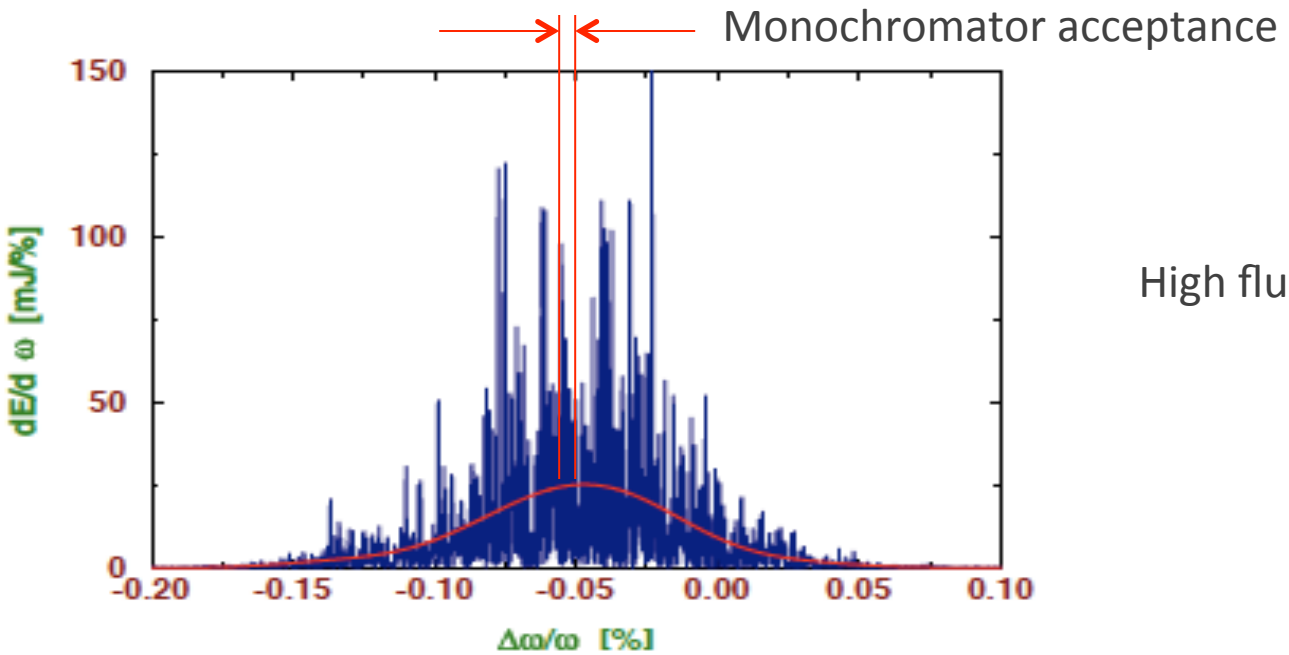
PAL XFEL at Pohang (Korea) - Construction

The Self-Amplified Spontaneous Emission (SASE) Process

- The lasing starts up from the random microbunching (i.e., shot noise) on the electron beam instead of being coherently produced by an input "seed" source.

Measured SASE spectrum (LCLS)



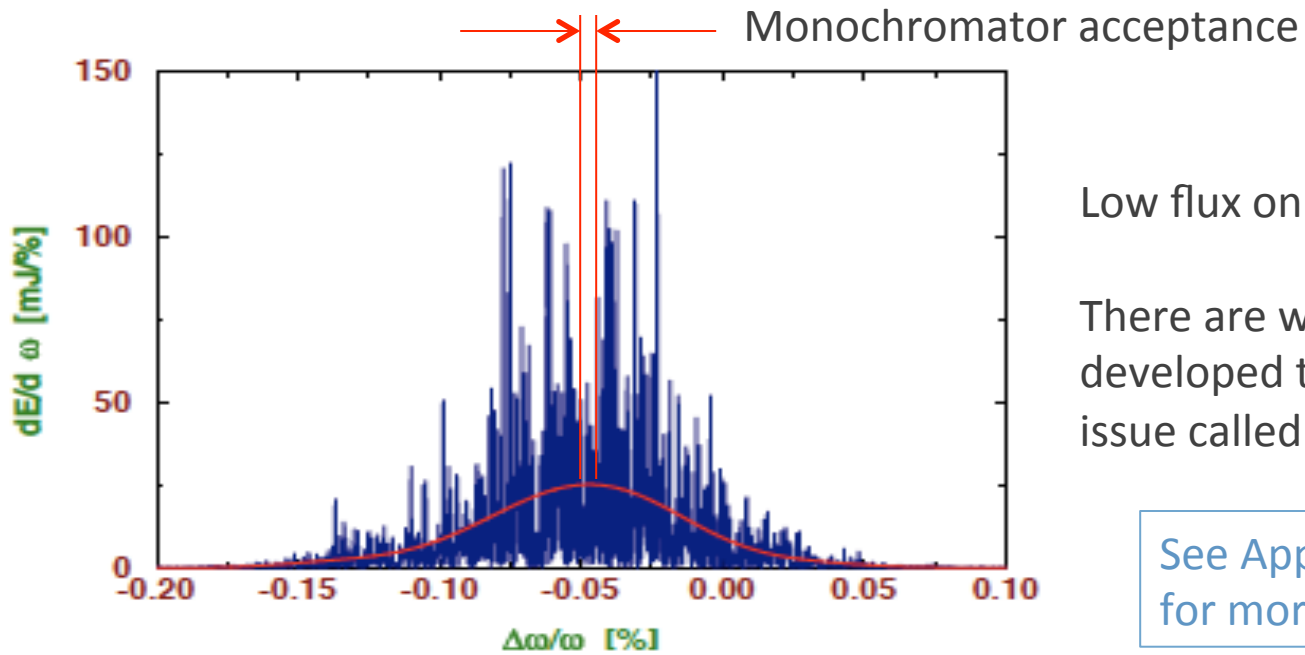


High flux on sample

The Self-Amplified Spontaneous Emission (SASE) Process

- The lasing starts up from the random microbunching (i.e., shot noise) on the electron beam instead of being coherently produced by an input "seed" source.

The shot-to-shot intensity of a monochromated beam varies with each pulse!



Low flux on sample

There are ways being developed to mitigate this issue called "seeding".

See Appendix 6 for more details

What's Next?

NSLS II – Under construction, will be operational in FY 15

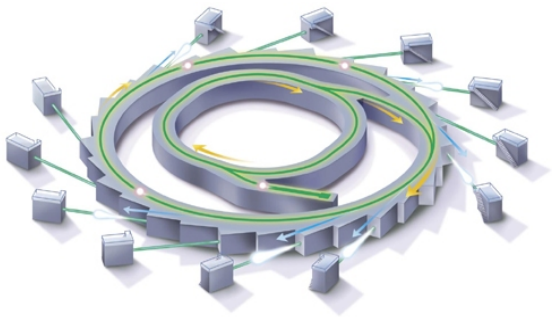
APS-Upgrade – started, with completion scheduled in FY20

LCLS II – started, with completion scheduled in FY20

Next Generation Light Source-NGLS (LBNL) – a high repetition rate soft x-ray FEL is under design

Stay tuned…….

Grand Challenge Science on Diffraction-Limited Storage Rings



A consensus report on future opportunities from scientists at

ALS, LBNL

APS, ANL

NSLS-II, BNL

SSRL, SLAC

together with a broad community of scientists at laboratories and universities.

BESAC Subcommittee on Future Light Sources

July 10-12, 2013

Appendix 1: Radiated Power from Charges at Relativistic Velocities

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o}\frac{dp}{d\tau} = \frac{1}{m_o}\gamma\frac{d\gamma m_o v}{dt} = \gamma^2\frac{dv}{dt} = \gamma^2\frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_oc^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

Appendix 1: Dependence on Mass and Energy of Radiated Power

$$P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

Appendix 2: BM Spectral Distribution

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c.

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and ρ ισ the radius of curvature of the trajectory. Note that the critical energy scales as γ^3. In practical units, the critical energy can be written as:

$$E_c[\text{keV}] = 2.218\ E^3[\text{GeV}]/\ \rho[\text{m}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at E = 7 GeV . The critical energy of the radiation emitted from the BM is:

$$E_c[\text{keV}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]$$

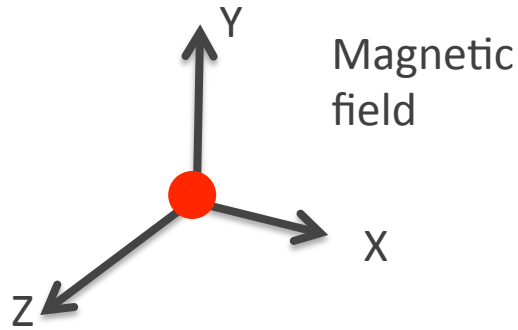
or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5\ \text{keV}}\ \text{or}\ \underline{0.64\ \text{Å}}.$$

Appendix 3: Where did "K" come from?

Equation of motion for a relativistic charged particle in a magnetic field

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$



$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{ID}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi c}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right) = -\frac{eB_0}{\gamma m_0}\frac{\lambda_{ID}}{2\pi}\cos\left(\frac{2\pi ct}{\lambda_{ID}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{ID}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{ID}}\right) = \left[\frac{eB_0}{m_0}\frac{\lambda_{ID}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{ID}}\right)$$

$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{ID}}{2\pi}\right] \qquad \textit{and} \qquad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \qquad \textit{where} \qquad K = \left[\frac{eB_0}{2\pi}\frac{\lambda_{ID}}{m_0 c}\right]$$

Appendix 4: How Do You Get 1 Å X-rays from a 3 cm Period Magnetic Field?

Where does the $1/\gamma^2$ come from in the equation: $\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$?

1) <u>Consider the electron in its rest frame:</u>
   - It does not see a static magnetic field from the undulator, but rather a time-varying B-field and associated E-field (due to the relativistic transformation of the magnetic field of the device).
   - The period of the E and B field are Lorentz contracted so that: $\lambda_{e\text{-}frame} = \lambda_{ID} / \gamma$ and so the electron oscillates (and hence radiates) with that same period driven by the EM fields.

2) <u>Back in the lab frame:</u>
   - Due to the fact that the electron is traveling towards us, the radiation emitted by the electron is Doppler shifted to higher frequencies (shorter wavelengths). The relativistic Doppler shift goes as $\sqrt{1-\beta} / \sqrt{1+\beta} \approx 1/2\gamma$, and so the wavelength observed in the lab is:

$$\lambda_{lab} \approx (\lambda_{ID}/\gamma)(1/2\gamma) = (\lambda_{ID}/2\gamma^2)$$

Appendix 5: Calculating X-ray Beam Brightness

APS Electron Beam Parameters

APS operates with $\varepsilon_H = 3 \times 10^{-9}$ m-rad and a coupling (ratio of vertical emittance to horizontal emittance) of 0.9% , therefore

$$\varepsilon_V = 0.025 \times 10^{-9} \text{ m-rad.}$$

The particle beam source size and divergence at the locations of the IDs are:

$$\sigma_H = 270 \text{ microns}$$
$$\sigma'_H = 11 \text{ microradians}$$
$$\sigma_V = 9 \text{ microns}$$
$$\sigma'_V = 3 \text{ microradians}$$

APS Undulator Beam Parameters

APS Undulator A has a length of 2.4 meters. For 1Å radiation the natural opening angle is:

$$\sigma'_{x\text{-ray}} = 1/\gamma\,(1/N)^{1/2}$$
$$= \sqrt{[\lambda/2L]} = 4.5\times10^{-6} \text{ rad}$$

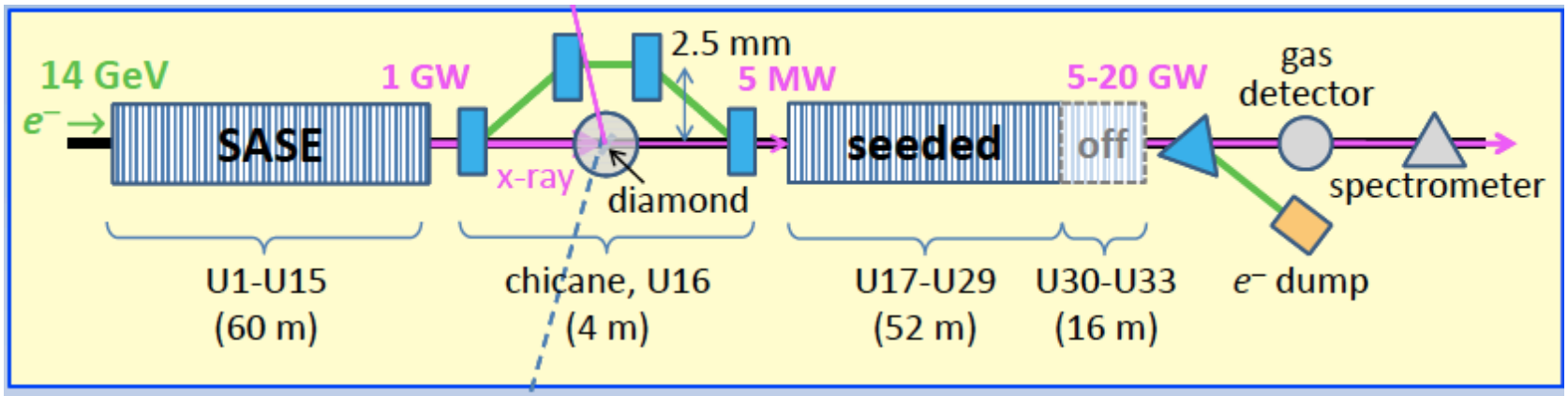
The corresponding source size of the radiation is:

$$\sigma_{x\text{-ray}} = \sqrt{[\lambda L/8\pi^2]} = 1.7 \text{ microns.}$$

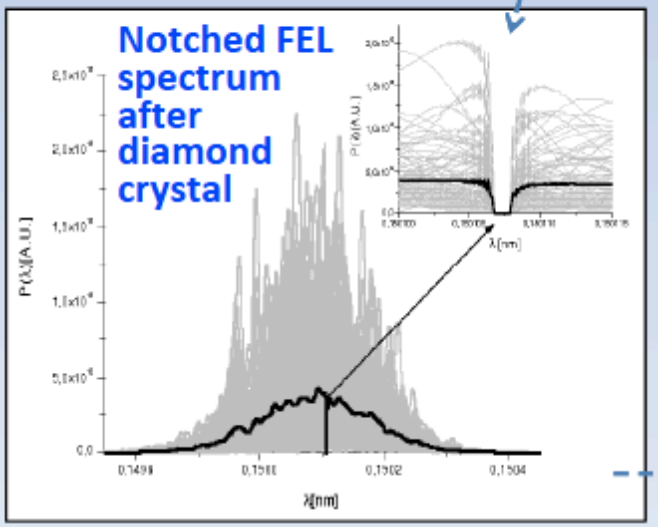
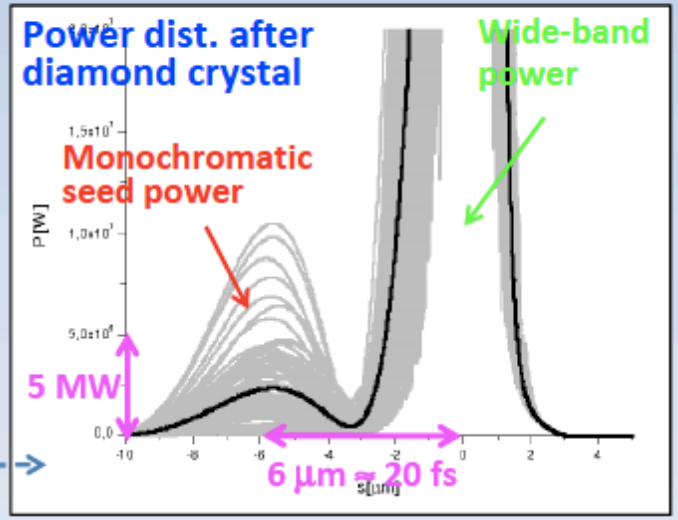
For source size $\Sigma = [\sigma_{x\text{-ray}}^2 + \sigma_{electron}^2]^{1/2}$ and source divergence $\Sigma' = [\sigma'^2_{x\text{-ray}} + \sigma'^2_{electron}]^{1/2}$

$$\mathcal{B} = \text{Flux}/4\pi^2\, \Sigma_H\, \Sigma_V\, \Sigma_H'\, \Sigma_V'$$

Appendix 6: Self-Seeding at the LCLS



Geloni, Kocharyan, Saldin (*DESY 10-133*)



Use short, low-charge bunch to self-seed at 1.5 Å (20-40 pC)