Synchrotron Radiation: Properties and Production

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Advanced Photon Source

Filling in for:

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Scientific User Facilities

National School for Neutron and X-ray Scattering
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Welcome to Chicago

Hope you enjoyed your visit to the city yesterday, and the rest of the week here at Argonne.

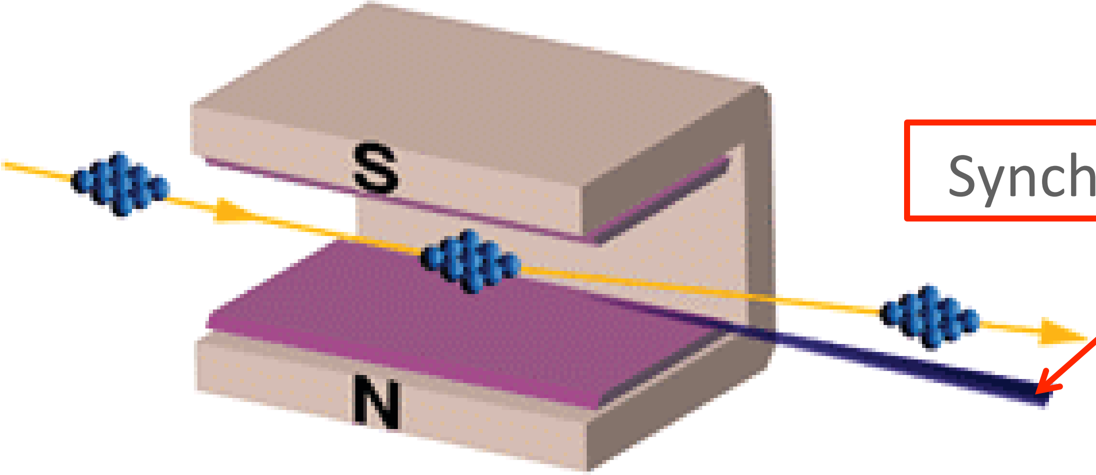
Discovery of x-rays ~100 years ago

- X-rays were discovered (accidentally) in 1895 by Wilhelm Konrad Roentgen.
- Roentgen won the first Nobel Prize in 1901 “for the discovery with which his name is linked for all time: the... so-called Roentgen rays, as he himself called them, X-rays...”

”X-ray Cream Furniture Polish - Guaranteed since 1891”
Umm… x-rays weren't discovered until 1895.

Synchrotron Radiation

Lorentz Force: $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$



Synchrotron Radiation

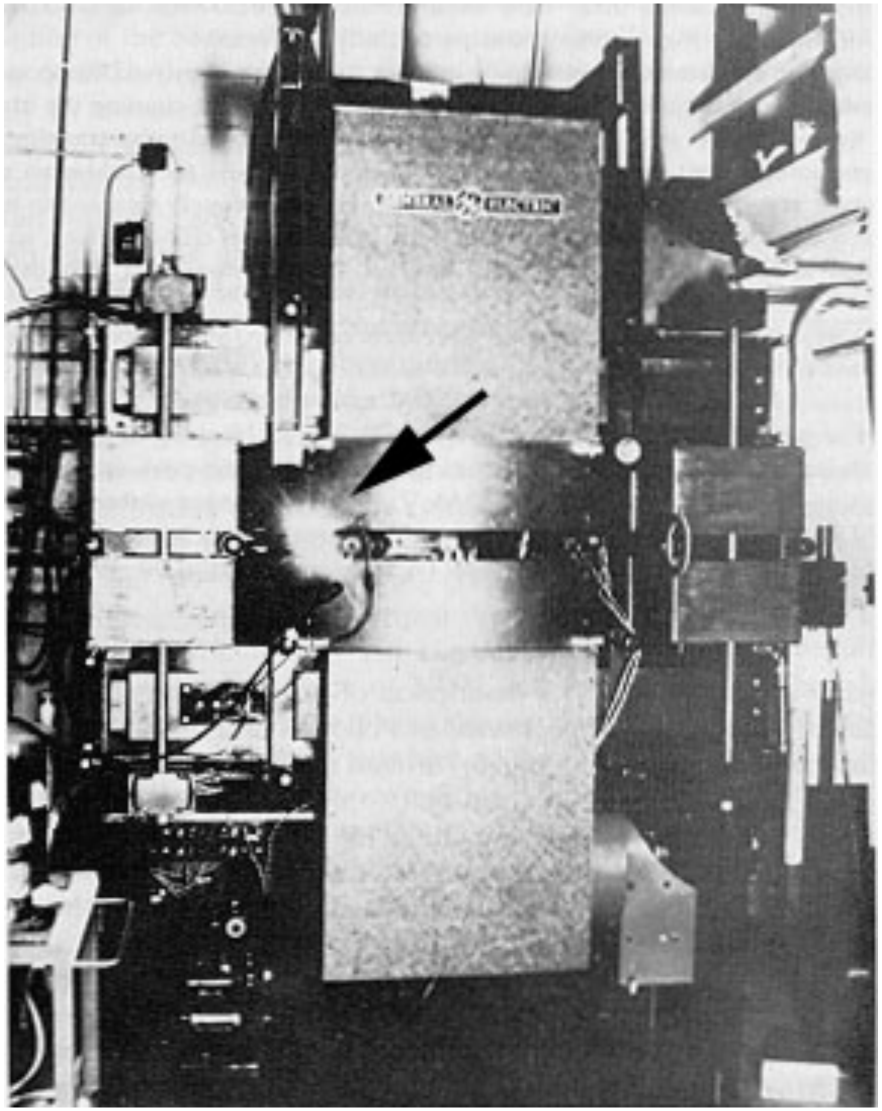
Electromagnetic radiation produced by relativistic charged particles accelerated in circular orbits

Discovery of Synchrotron Radiation - ~50 years ago

Synchrotron radiation (SR) was first observed (accidentally) from a 70 MeV synchrotron 1947.

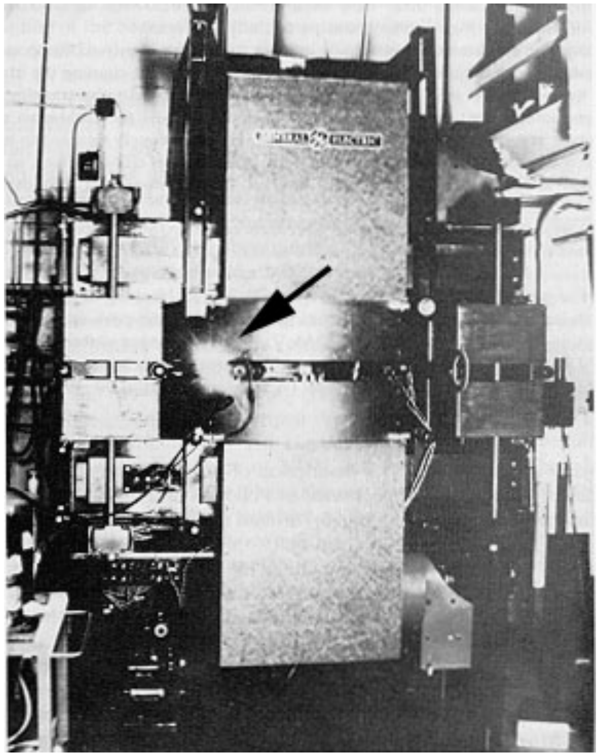
> *On April 24,[1947] Langmuir and I [Herbert Pollack] were running the machine and as usual were trying to push the electron gun and its associated pulse transformer to the limit. Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cherenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.*

Excerpted from Handbook on Synchrotron Radiation, Volume 1a, Ernst-Eckhard Koch, Ed., North Holland, 1983.



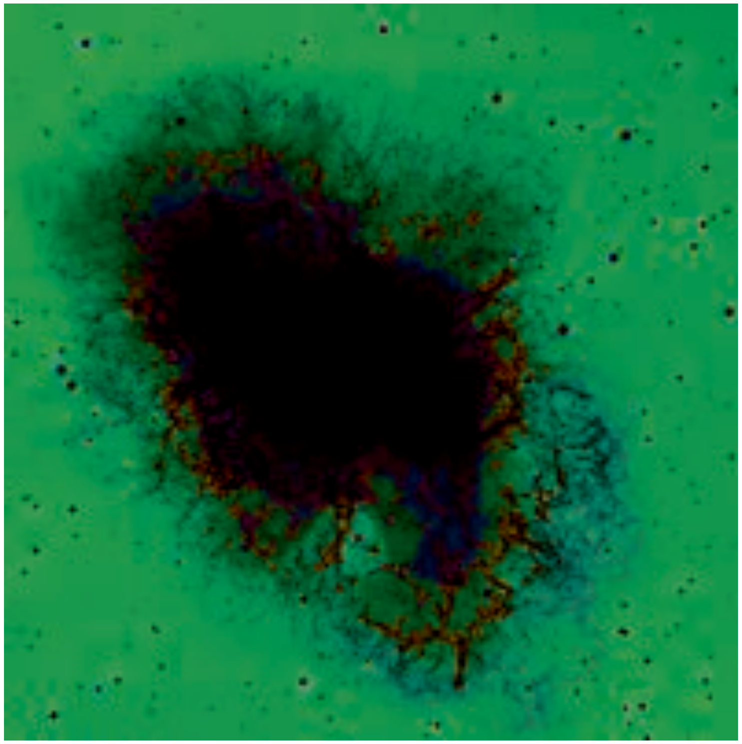
Synchrotron Radiation

GE Synchrotron
New York State



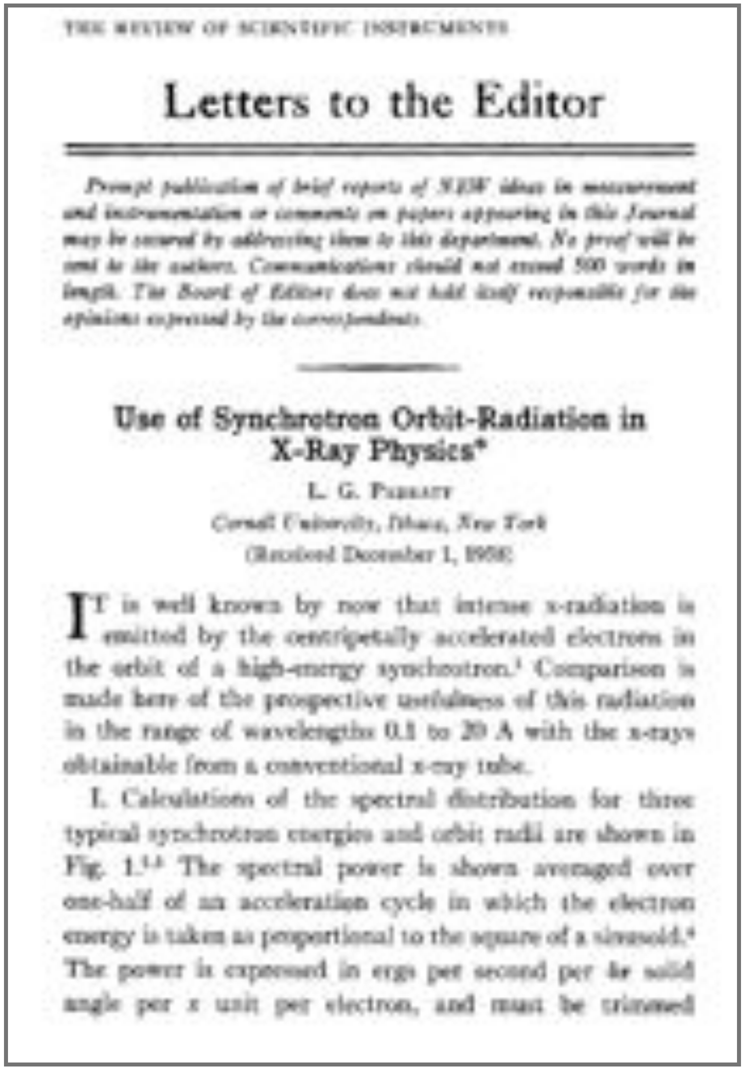
First Light observed
1947

Crab Nebula
6000 light years away



First Light observed
1054 AD (1758 AD)

From Synchrotrons to Storage Rings



THE REVIEW OF SCIENTIFIC INSTRUMENTS

Letters to the Editor

Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Use of Synchrotron Orbit-Radiation in X-Ray Physics*

L. G. PARRATT

Cornell University, Ithaca, New York

(Received December 1, 1958)

IT is well known by now that intense x-radiation is emitted by the centripetally accelerated electrons in the orbit of a high-energy synchrotron.[1] Comparison is made here of the prospective usefulness of this radiation in the range of wavelengths 0.1 to 20 A with the x-rays obtainable from a conventional x-ray tube.

I. Calculations of the spectral distribution for three typical synchrotron energies and orbit radii are shown in Fig. 1.[1-3] The spectral power is shown averaged over one-half of an acceleration cycle in which the electron energy is taken as proportional to the square of a sinusoid.[4] The power is expressed in ergs per second per 4π solid angle per x unit per electron, and must be trimmed

- Synchrotrons were first used as sources of SR. However, the particles' constantly changing energy was not attractive and the advent of storage rings provided a far more attractive source.

- We now use the name synchrotron radiation to describe radiation that is emitted from charged particles traveling at relativistic speeds, regardless of the accelerating mechanism.

- Although synchrotron radiation can cover the entire electromagnetic spectrum, we are interested (mostly) in radiation in the x-ray regime.

$$(\lambda[\text{Å}] = 12.4 / E[\text{keV}])$$

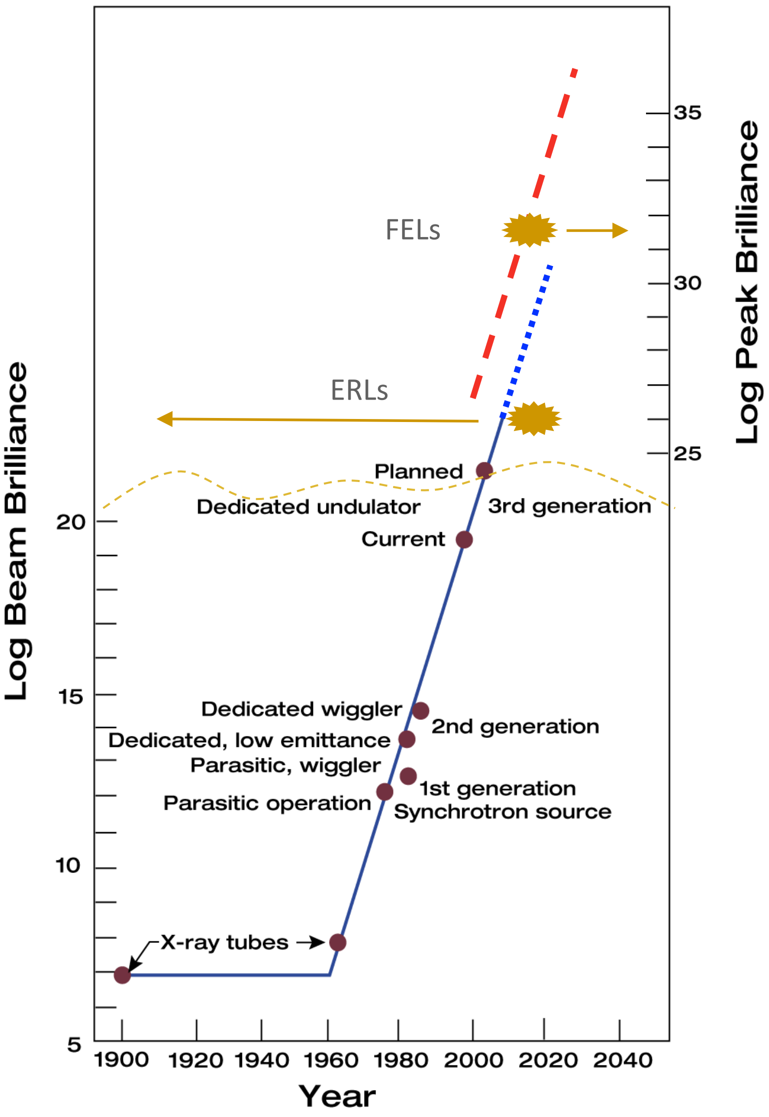
Square brackets indicate the units to be used in the calculation.

Evolution of Synchrotron Radiation Sources

Synchrotron radiation has been used a research tool for nearly 50 years. (X-ray, VUV, IR)

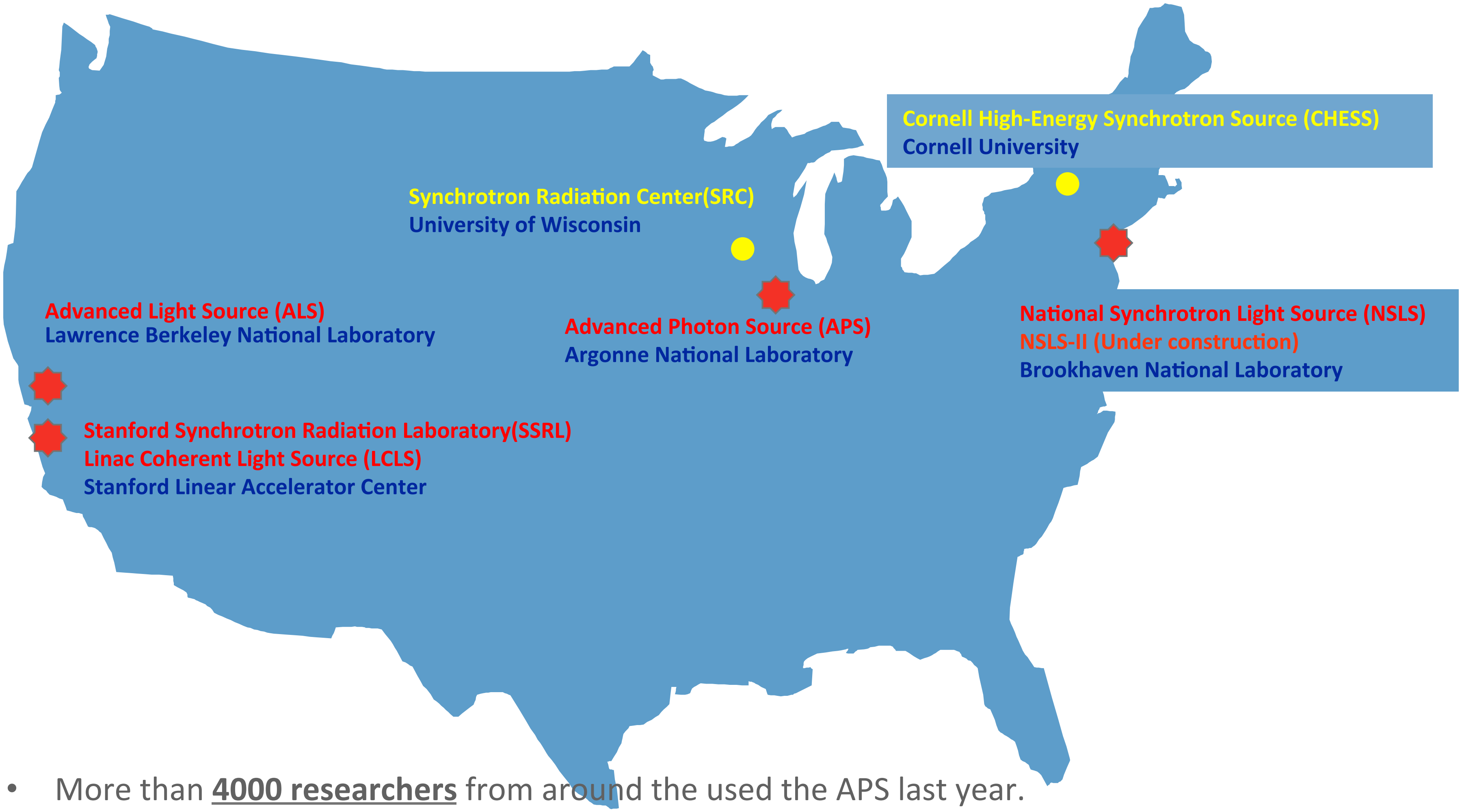
- 1st Generation Sources
  Ran parasitically on high energy physics accelerators (CHESS)
- 2nd Generation Sources
  Built to optimize synchrotron radiation from the bending magnets (NSLS)
- 3rd Generation Sources
  Built to optimize synchrotron radiation from insertion devices (APS)
- 4th Generation Sources
  fully coherent sources
  X-ray Free Electron Lasers (X-FELs)
  Energy Recovery Linacs (ERLs)
  Ultimate Storage Rings

History of (8-keV) X-Ray Sources



Synchrotron Facilities in the United States

There are currently 7 major synchrotron facilities in the US, 5 operated by the Department of Energy and 2 by the National Science Foundation



Cornell High-Energy Synchrotron Source (CHESS)
Cornell University

Synchrotron Radiation Center(SRC)
University of Wisconsin

Advanced Light Source (ALS)
Lawrence Berkeley National Laboratory

Advanced Photon Source (APS)
Argonne National Laboratory

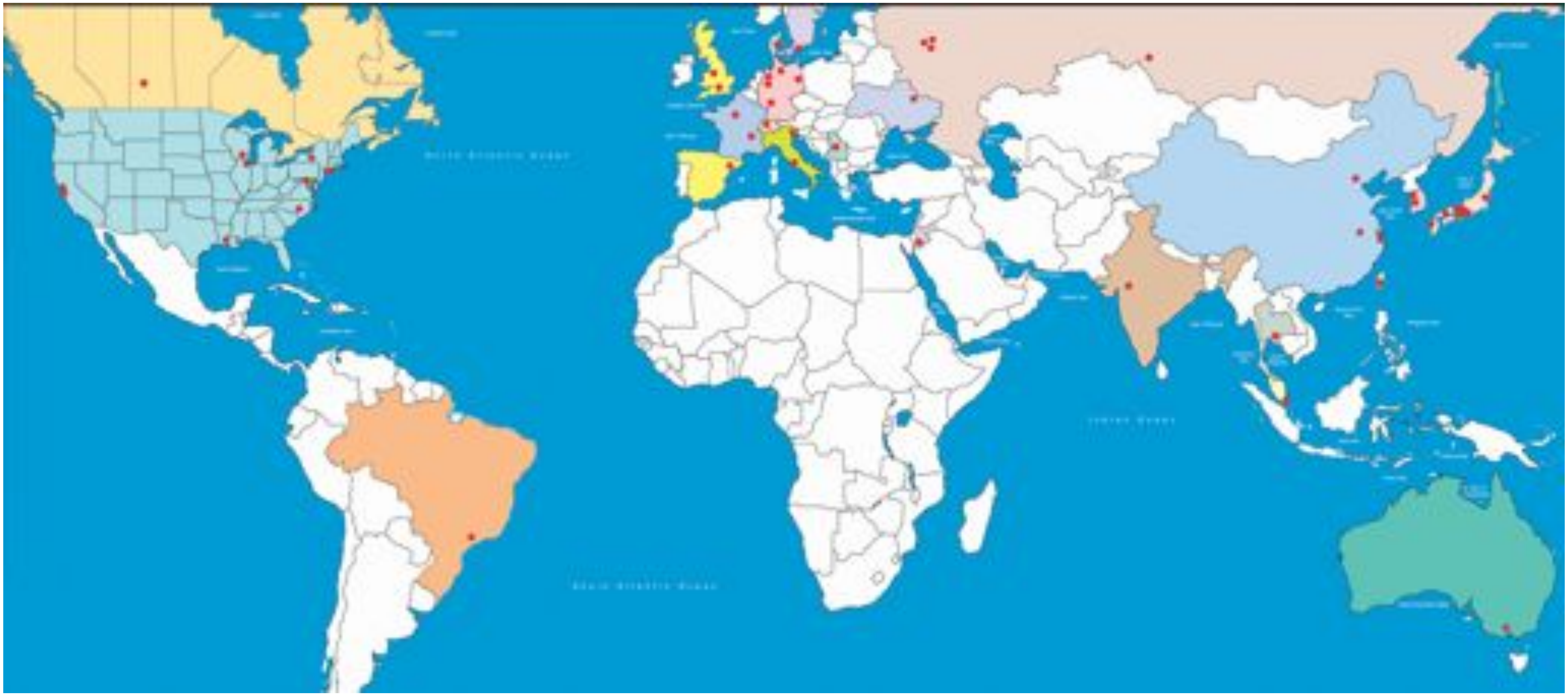
National Synchrotron Light Source (NSLS)
NSLS-II (Under construction)
Brookhaven National Laboratory

Stanford Synchrotron Radiation Laboratory(SSRL)
Linac Coherent Light Source (LCLS)
Stanford Linear Accelerator Center

- More than **<u>4000 researchers</u>** from around the used the APS last year.
- Over 8000 unique scientists use one or more of the US synchrotrons

Synchrotron Radiation Facilities Around the World

World-wide there are ~35 major synchrotron facilities

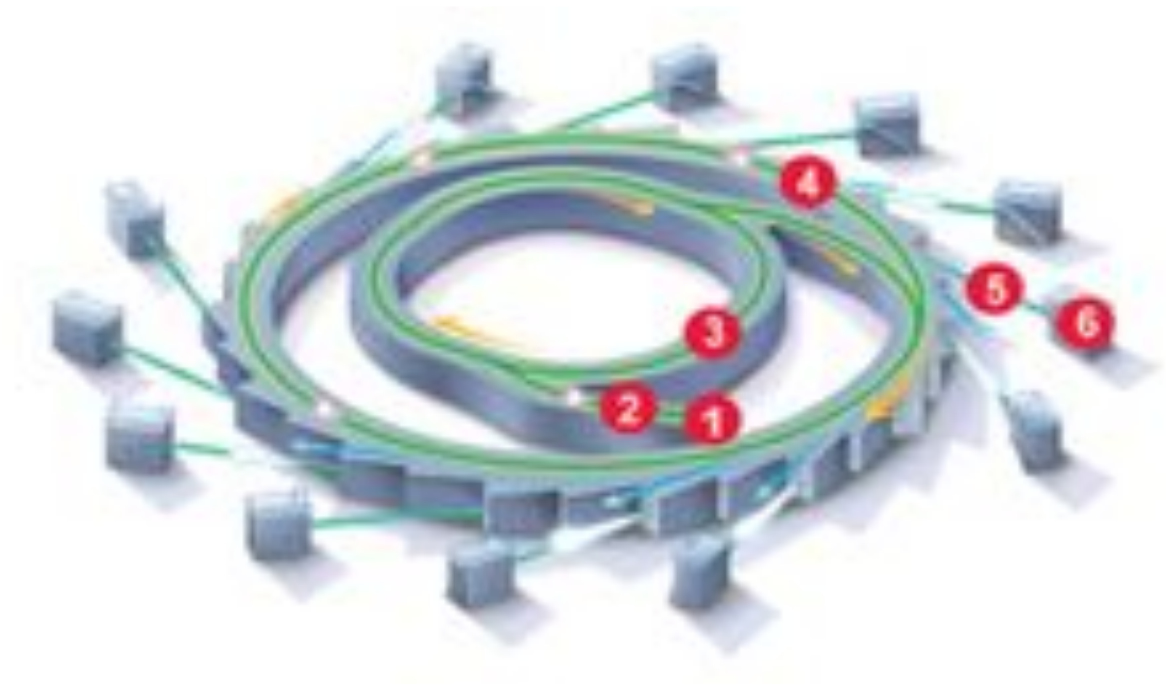


- There are approximately 40,000 users of these facilities.

3rd Generation High-Energy Sources

- One of four third-generation hard x-ray sources around the world
  - ESRF, Grenoble, France, 6 GeV, (1994)
  - APS, Argonne, IL, 7 GeV, (1996)
  - SPring-8, Harima, Japan, 8 GeV, (1997)
  - Petra-III, Hamburg, Germany, 6 GeV (2009)

Typical SR Facility Complex



1. Electron gun
2. Linear Accelerator LINAC
3. Booster Synchrotron
4. Storage Ring (SR)
5. Beamline
6. Experiment station

(Courtesy: Australian Synchrotron,
Illustrator: Michael Payne)

APS Linear Accelerator and Booster

LINAC

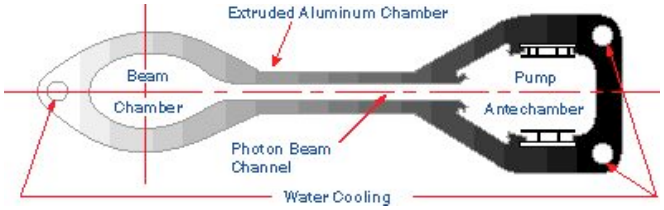
- Accelerates electrons from 0 to 450 MeV
- Operates at 48 Hz
- 50 meters in length

Booster

- Accelerates electrons from 450 Mev to 7 GeV
- Operates at 2 Hz
- 368 m circumference

APS Storage Ring

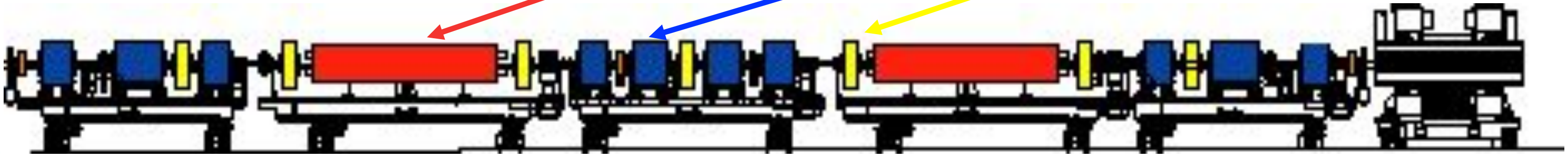
Looks round but 40 sided polygon → tetracontagon



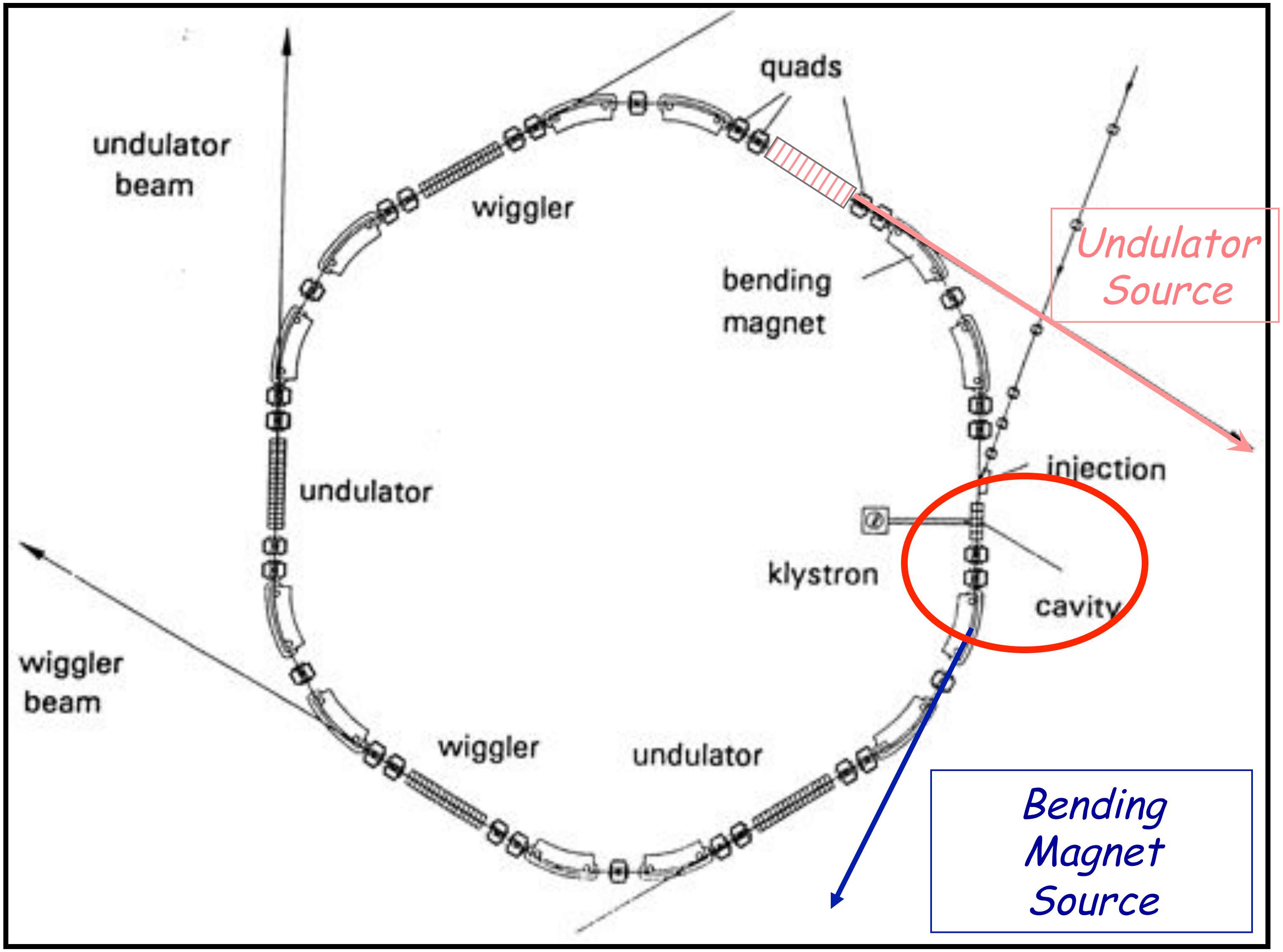
Cross-section of the Al storage ring vacuum chamber

- Present operation:
  - 7 GeV and 100 mA
  - bunch length 70 psec

- 1104 m circumference (3.68 microsecond period)
  - 80 dipoles
  - 240 quads
  - 20 skew quads
  - 280 sextupoles



Storage Rings



Undulator Source

Bending Magnet Source

Radiation from Moving Charges

Maxwell's Equations

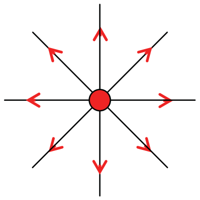
$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \mathbf{B} = 0$$

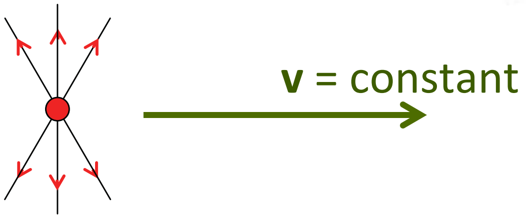
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

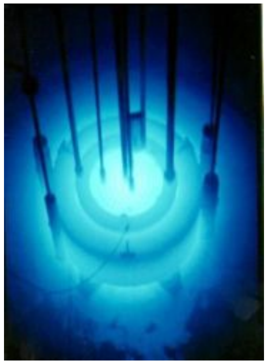
$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$

Charge at rest: Coulomb field, no radiation

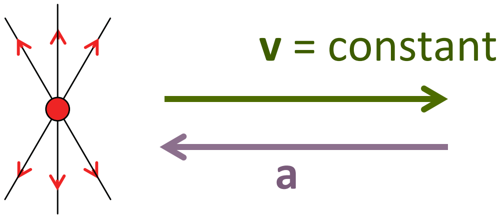


Uniformly moving charge, no radiation
(*Cherenkov Radiation)

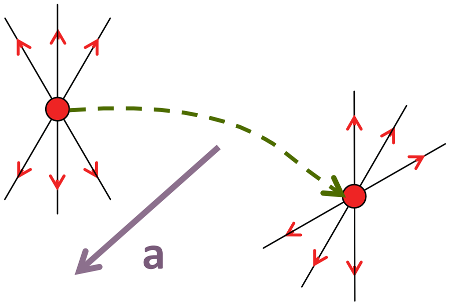




Accelerating charge



- **v** || **a** → Bremsstralung, Antennas
- **v**⊥ **a** → Synchrotron Radiation



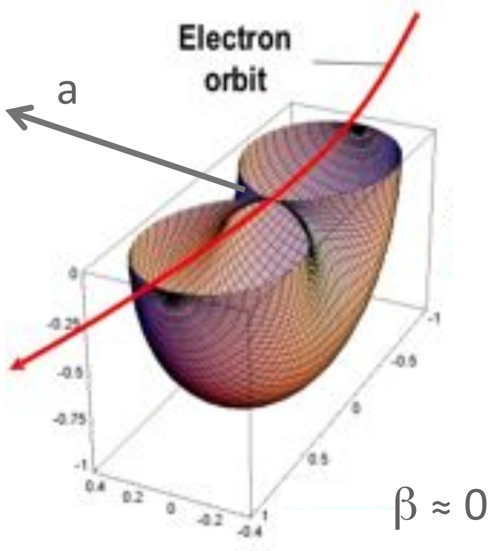
Radiation Patterns From Accelerating Charges

Definitions:

$$\gamma = 1/\sqrt{1-\beta^2} = E/m_oc^2$$

and

$$\beta = v/c$$



$\beta \approx 0$

At the APS with E = 7 GeV,

$$\gamma = E/m_oc^2 = 7\ \mathrm{GeV}/0.511\ \mathrm{MeV}$$

$$\gamma = 1.4 \times 10^4$$

$$1/\gamma = 73 \times 10^{-6}$$

$$\beta = 99.999999\%$$

When v << c, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.

And so as you crank up β, the radiation pattern begins to deform (in the lab frame).

See Appendix 0 for more details

Radiation Patterns When v Approaches c

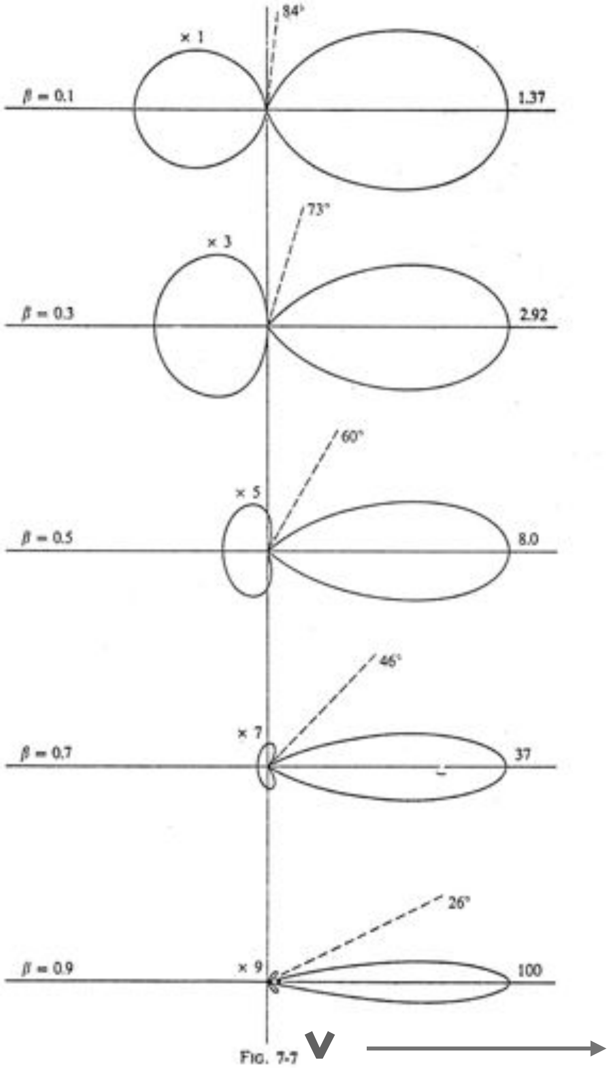


FIG. 7-7

So as β approaches 1 :

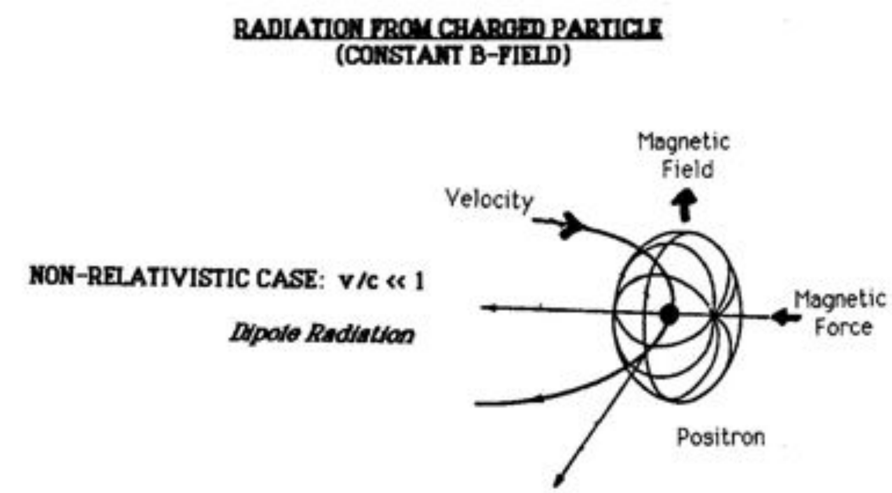
a. The shape of the radiation pattern is changing; it is more forward directed

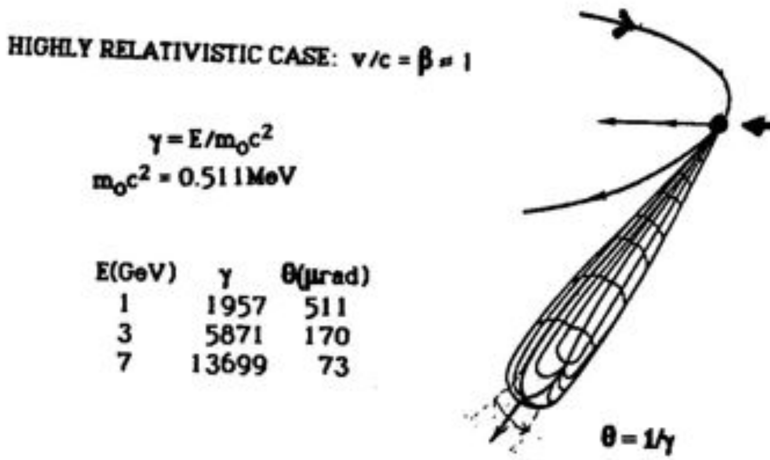
b. The size of the radiation pattern is changing; it is getting bigger

So at $\beta \approx 1$, the node at $\theta' = 90°$ (in the frame of the radiating particle) transforms to:

$$\tan\theta_{lab} = \frac{\sin\theta'}{\gamma\left(\cos\theta' + \beta\right)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$

Radiation from Highly-Relativistic Particles





E(GeV)	γ	θ(μrad)
1	1957	511
3	5871	170
7	13699	73

In fact, the opening angle in both the horizontal and vertical directions, is given approximately by:

$$\theta = 1/\gamma,$$

when $\beta \approx 1$.

Relativistic velocities are good!!
- radiation forward directed
- radiated power ∝ E^4


THE

PHYSICAL REVIEW

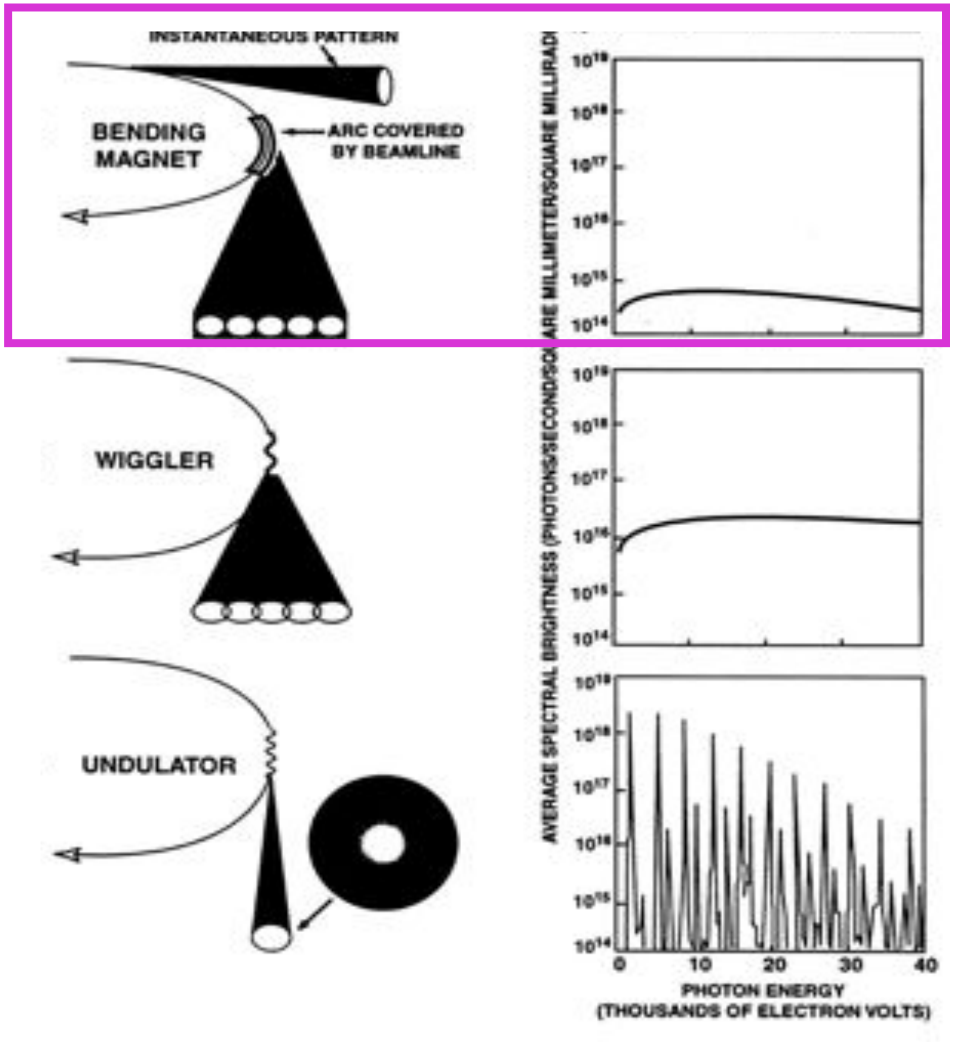
A journal of experimental and theoretical physics established by E. L. Nichols in 1893

SECOND SERIES, VOL. 102, NO. 6 JUNE 15, 1956

Spectral and Angular Distribution of Ultraviolet Radiation
from the 300-Mev Cornell Synchrotron*

D. H. TOMBOULIAN AND P. L. HARTMAN
Department of Physics, Cornell University, Ithaca, New York

Radiation Sources at 3rd Generation Facilities



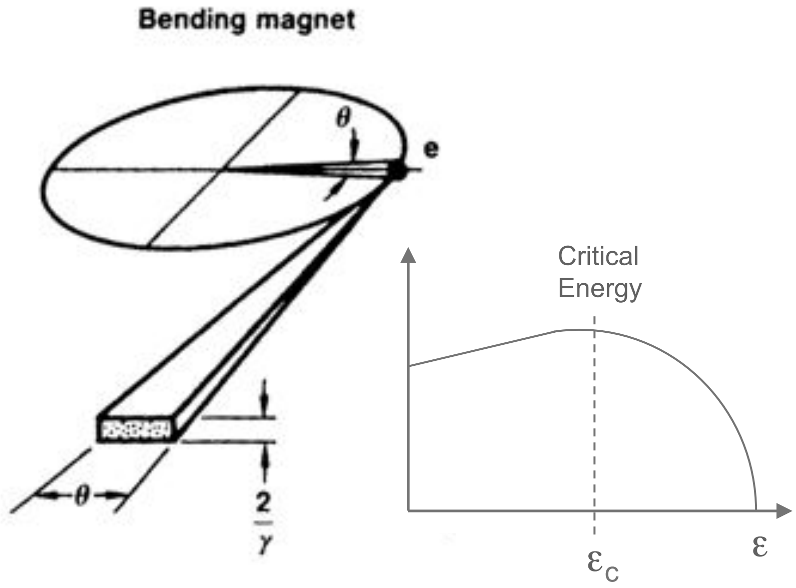
There are two different sources of radiation at 3rd generation sources:

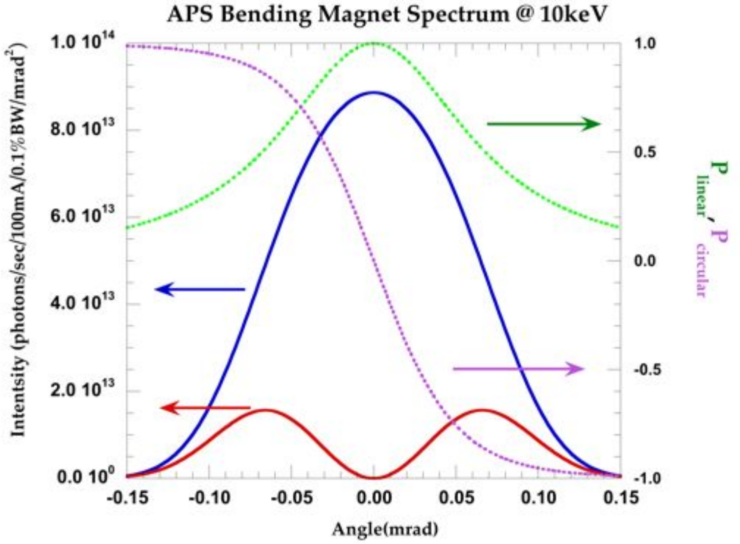
- bending magnets (BMs)
- insertion devices (IDs); periodic arrays of magnets inserted between the BMs (wigglers or undulators)

The important parameters to know about each one is:

- Spectral distribution
- Flux (number of x-rays/sec - 0.1%bw)
- Brightness (flux/source size-source divergence)
- Polarization (linear, circular)

Bending Magnet Sources

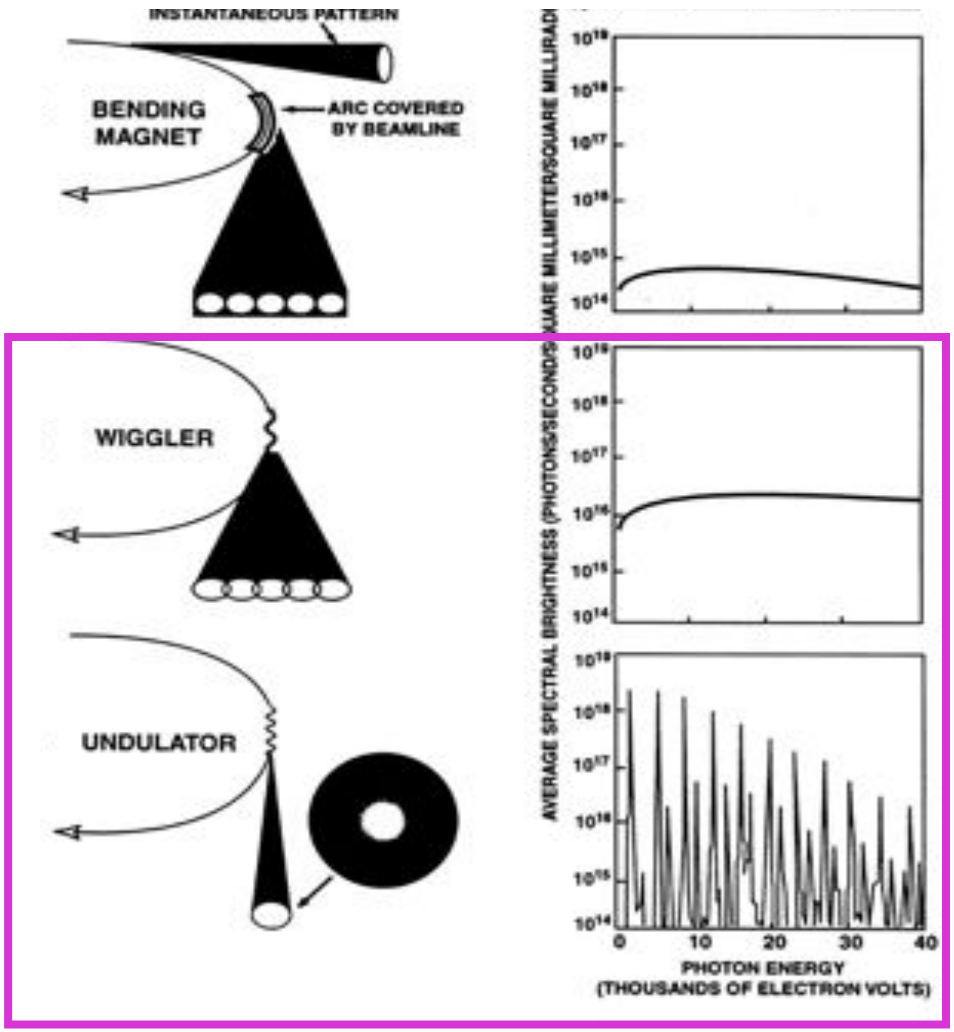




Bend Magnet Radiation (see Appendix 1)

- Spectrum characterized by the critical energy:
  $E_c = 3hc\gamma^3/4\pi r$. ~19 keV @ APS
- Flux “typically” 10^{13} photons/sec/0.1% BW /mrad from 3rd generation source
- Vertical opening angle $1/\gamma$. For the APS:
  $1/\gamma = 73 \times 10^{-6}$ radians
- Horizontal opening angle determined by apertures
- In the plane of the orbit, the polarization is linear and parallel to the orbital plane.
- Out of the plane, there is a component perpendicular to the orbit and out of phase by $\pi/2$ with respect to the parallel component and so the off-axis beam is elliptically polarized.

Radiation Sources at 3rd Generation Facilities



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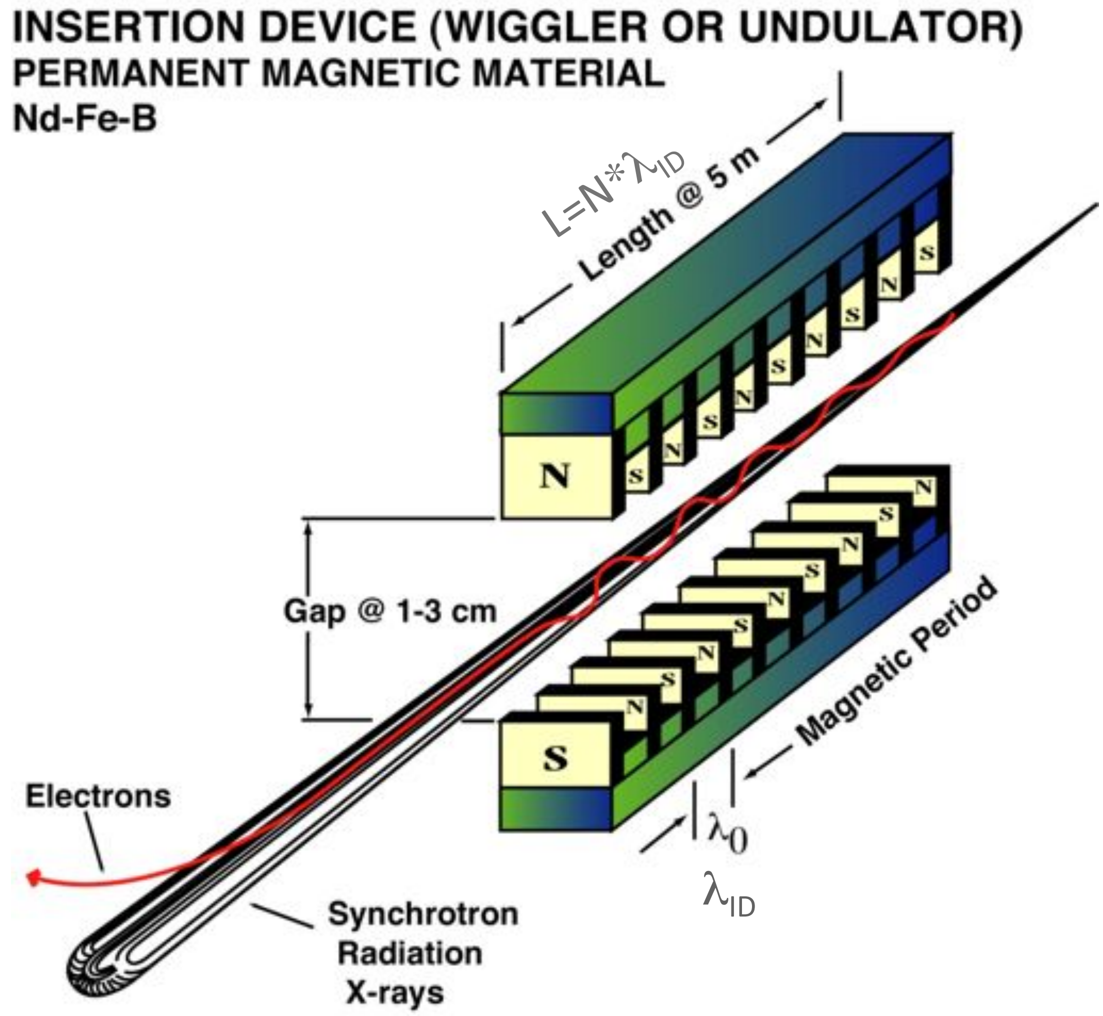
Planar Insertion Devices



Beam of electrons continues around the ring

Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

Characterizing Insertion Devices

IDs are characterized by the so-called “deflection parameter”, K **(See Appendix 2)**:

$$K = eB_o\lambda_{ID}/2\pi m_o c = 0.0934\ \lambda_{ID}[\mathrm{cm}]\ B_o[\mathrm{kG}]$$

where λ_{ID} is the period of the insertion device and B_o the peak magnetic field. (The length of the ID, L, is equal to the number of periods, N, times the length of the period, i.e., $L = N\lambda_{ID}$.)

The maximum deflection angle of the particle beam, θ_{max}, is given by:

$$\theta_{max} = \pm(K/\gamma)$$

and the amplitude of the oscillation of the particles, x_{max}, by:

$$x_{max} = (K/\ \gamma)(\lambda_{ID}/2\pi)$$

APS Undulator A has a period of 3.3 cm and operates with K ≈ 1, therefore:

$\theta_{max} \approx 1/\gamma$ **and** $x_{max} \approx 0.38$ *microns.*

Wigglers and Undulators

Wigglers (K>>1) :

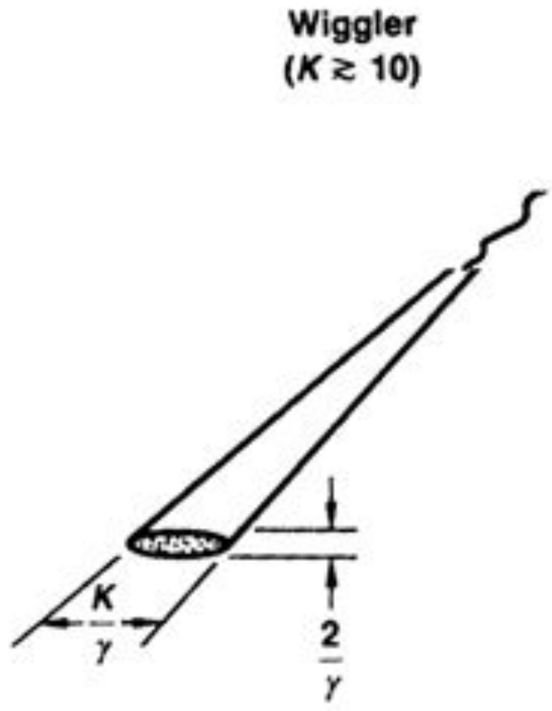
- $\theta_{max} = (K/\gamma) >> 1/\gamma$, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation ($1/\gamma$).
- radiation spectrum looks like 2N dipole sources (N = number of periods)

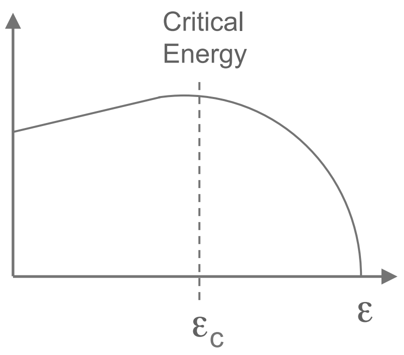
Undulators (K ≈ 1):

- θ_{max} is comparable to the natural opening angle of the radiation ($1/\gamma$) and so the radiation from each pole overlaps causing interference effects in the spectral distribution.
- radiation spectrum does not looks like 2N dipole sources

APS 2.4 m long Undulator A ($\lambda_{ID} = 3.3$ *cm*)

Wiggler Radiation Sources

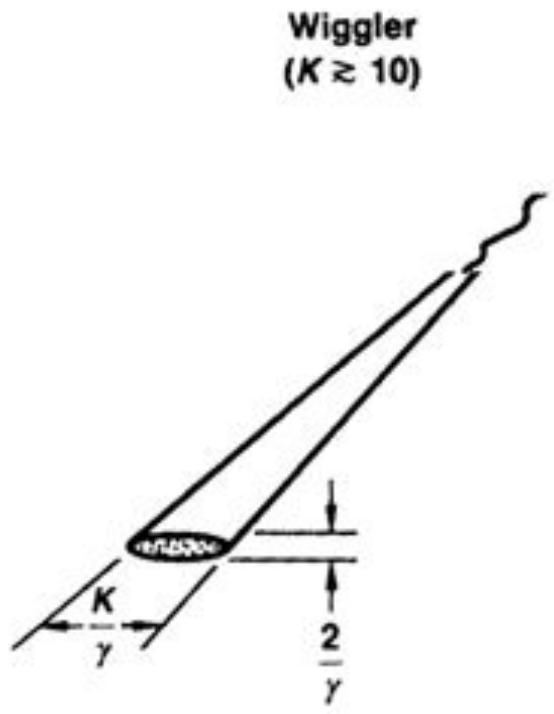




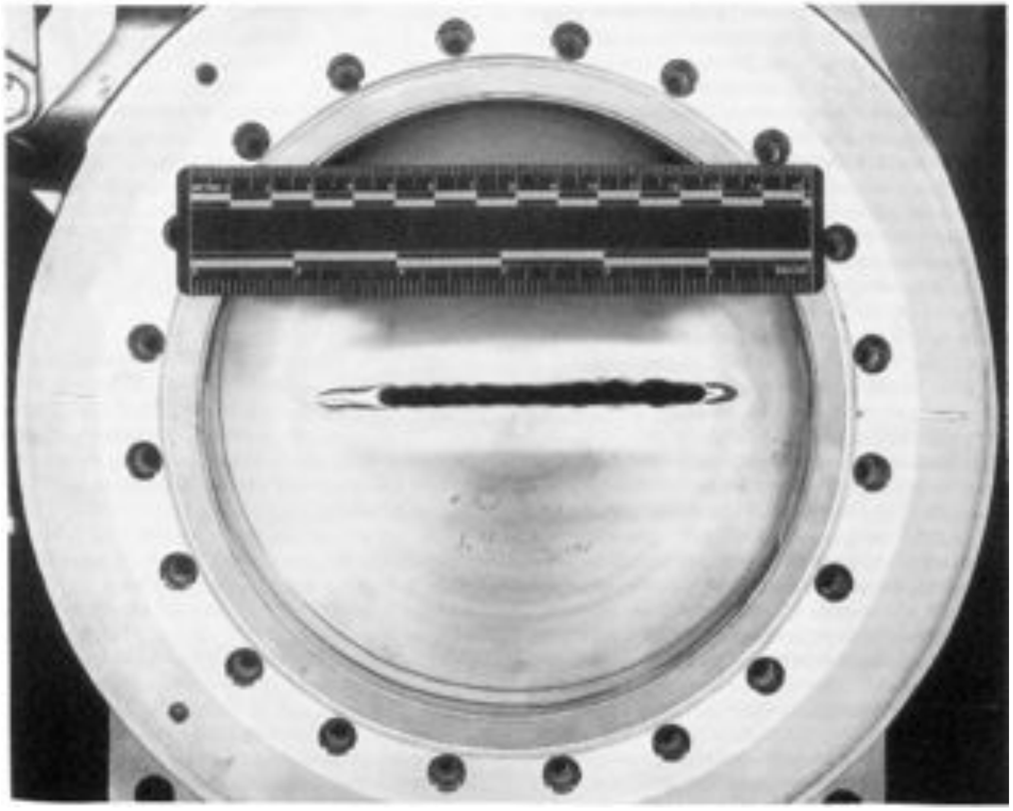
Wiggler Radiation
(see Appendix 3)

- like BM radiation where each pole is a "source"
- spectrum characterized by the critical energy (which may be different than BM critical energy)
- flux "typically" 10^{14} to 10^{15} photons/ sec/0.1% BW/ mrad (10-100x Bend magnet)
- vertical radiation opening angle $1/\gamma$ (73×10^{-6} radians for APS)
- Presently, there are NO planar wigglers installed at the APS. Wigglers with fields in both the x and y directions) produce elliptically polarized radiation. These are sometimes called elliptical multipole wigglers (EMWs).

Wiggler Radiation Sources



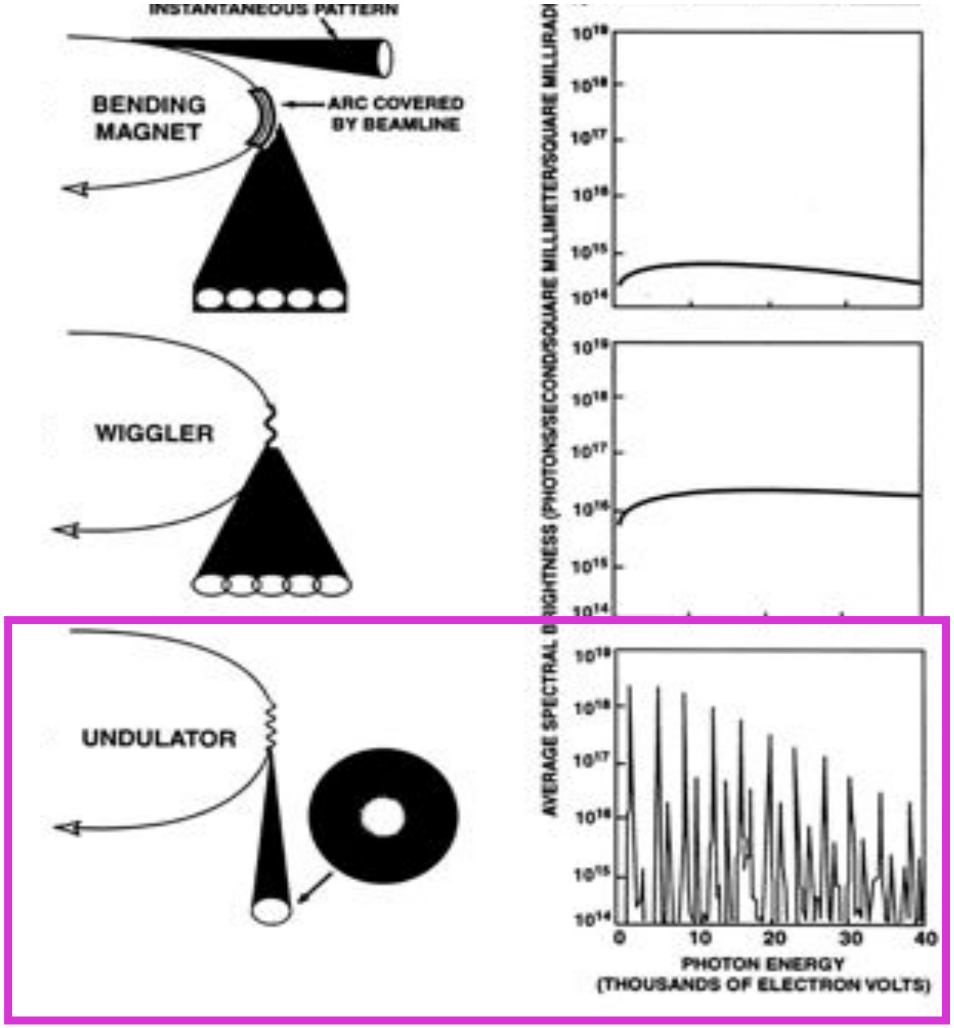
X-ray Beam has lots of power!!
1-10 kW



Makes designing optics (monochromators, mirrors) a challenge

White beam from wiggler incident on Gate Valve for ~2-10 minutes @ NSLS

Radiation Sources at 3rd Generation Facilities



There are two different sources of radiation at 3rd generation sources:

- bending magnets (BMs)
- insertion devices (IDs); periodic arrays of magnets inserted between the BMs (wigglers or undulators)

The important parameters to know about each one is:

- Spectral distribution
- Flux (number of x-rays/sec - 0.1%bw)
- Brightness (flux/source size-source divergence)
- Polarization (linear, circular)

Undulator Radiation

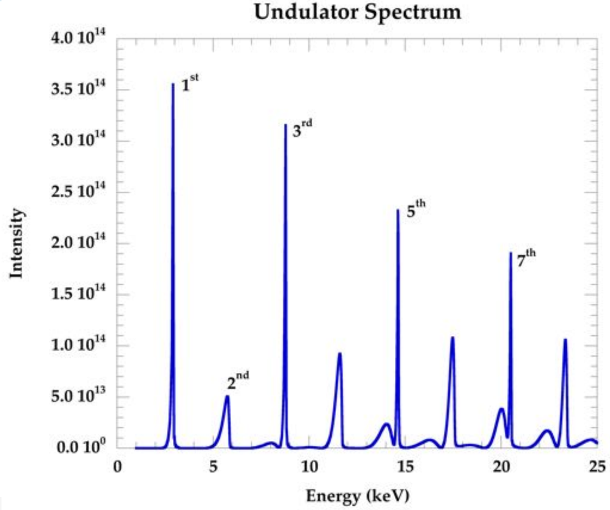
Undulator radiation is the coherent super-position of radiation from each pole of the undulator. Interference from different parts of the particle's trajectory in the undulator causes the radiation to be squeezed into discrete spectral lines and into a narrower emission angle.

Constructive interference occurs at wavelengths given by:

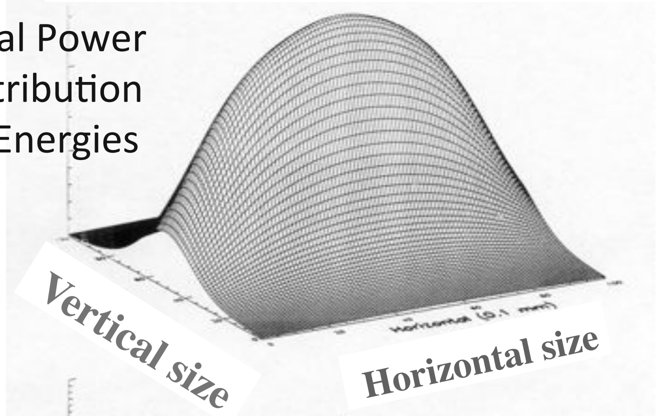
$$\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2 + \gamma^2\theta^2),$$

where n is the harmonic number. (Only odd harmonics are observed on axis due to symmetry arguments.)

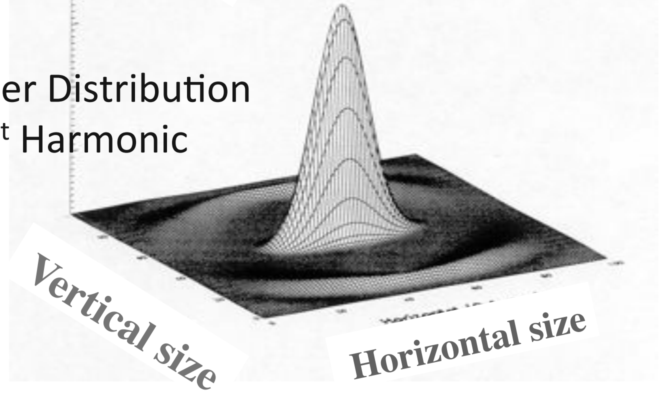
This bundle of radiation from the odd harmonics in the center of the power envelope is called the "central cone".



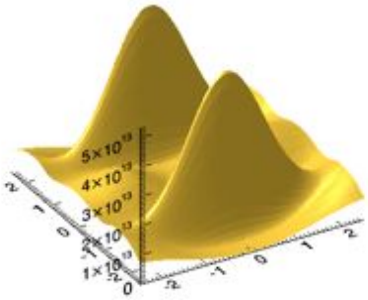
Total Power Distribution All Energies



Power Distribution in 1st Harmonic



Power Distribution at E= 2.5* 1st Harmonic



1 Å X-rays from a 3 cm Period Magnetic Field?

Where does the $1/\gamma^2$ come from in the equation: $\lambda_n^{x\text{-ray}} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$?

1) Lorentz Contraction: Consider the electron in its rest frame:

- It does not see a static magnetic field from the undulator, but rather a time-varying B-field and associated E-field (due to the relativistic transformation of the magnetic field of the device).
- The period of the E and B field are Lorentz contracted so that: $\lambda_{e\text{-frame}} = \lambda_{ID} / \gamma$ (3.3 cm to 2.3 μm) and so the electron oscillates (and hence radiates) with that same period driven by the EM fields.

2) Doppler shift: Back in the lab frame:

- Due to the fact that the electron is traveling towards us, the radiation emitted by the electron is Doppler shifted to higher frequencies (shorter wavelengths). The relativistic Doppler shift goes as $\sqrt{1-\beta} / \sqrt{1+\beta} \approx 1/2\gamma$, and so the wavelength observed in the lab is:

3) Combination: $\lambda_{lab} \approx (\lambda_{ID}/\gamma)(1/2\gamma) = (\lambda_{ID}/2\gamma^2)$

$$\lambda_{ID} = 3.3\text{cm}$$

$$\gamma \approx 14000\,(\text{APS})$$

$$\lambda_{x-ray}(\text{cm}) \cong 0.8\,\text{x}\,10^{-8}$$

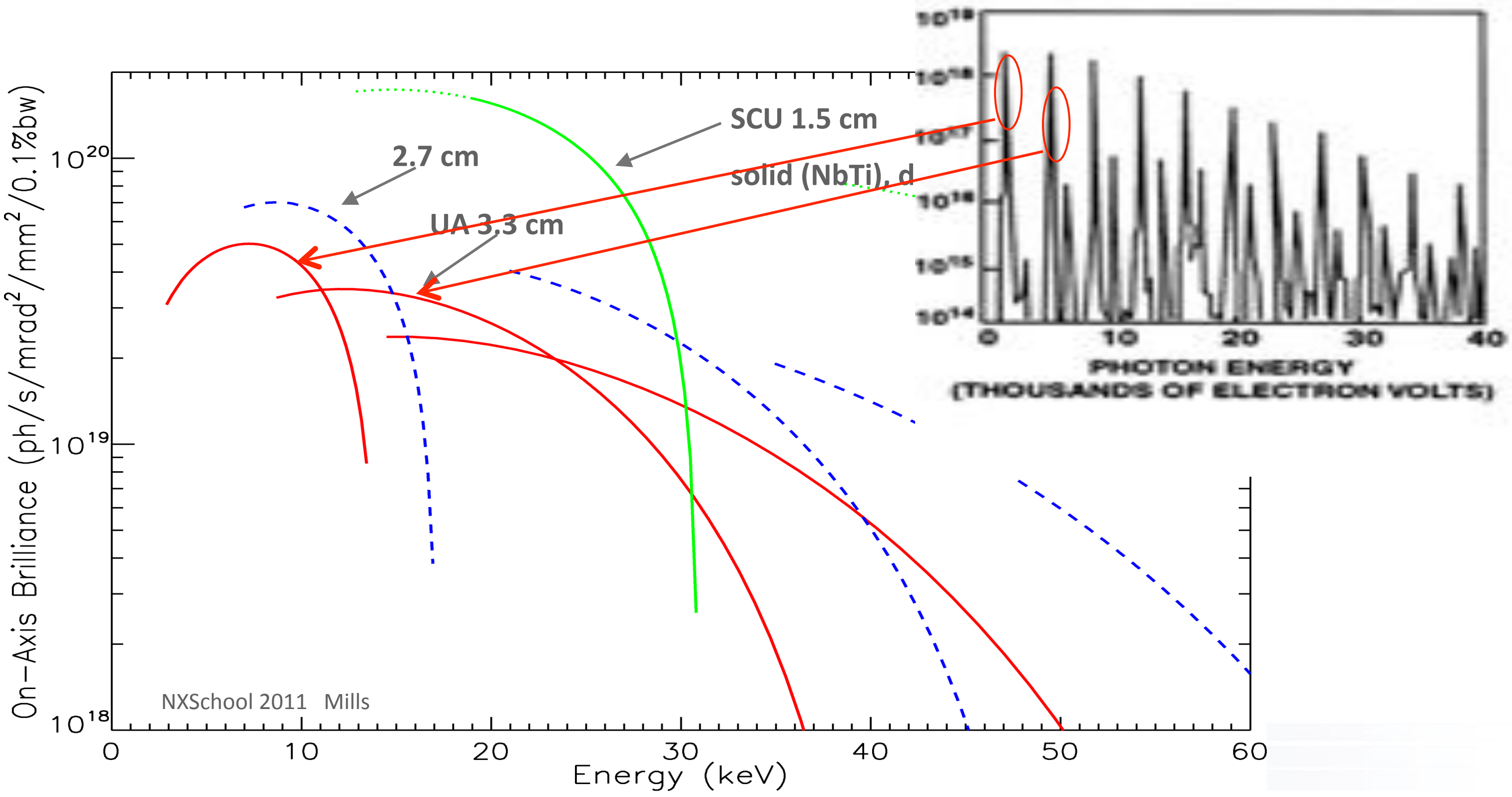
$$\cong 0.08\ nm$$

Tuning the Peaks of Undulator Radiation

On-axis (θ =0), we can write:

$$\lambda_n^{x\text{-}ray} = (\lambda_{ID}/2\gamma^2 n)(1 + K^2/2)$$

$\lambda_n^{x\text{-}ray}$, can be adjusted by B, i.e. by varying K (= 0.0934 λ_{ID}[cm] B_o[kG]).
This can be achieved by varying the current in the windings of electromagnetic devices or by varying the separation between the upper and lower poles ("the gap") in permanent magnet devices. You can't reduce the gap too much since you will cut into the particle beam and lifetime will go to pot!

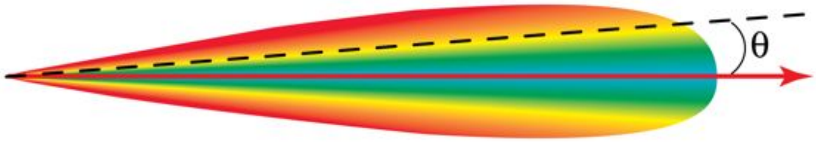


Undulator Energy Spread and Angular Distribution

The energy spread of the interference peak (central cone) is given by:

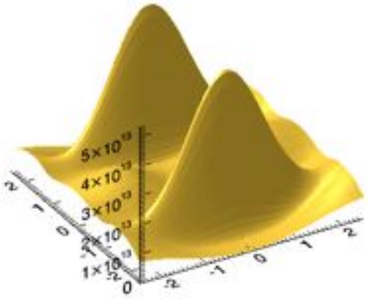
$$\Delta E/E = \Delta\lambda/\lambda \approx 1/nN$$ (like a grating!).

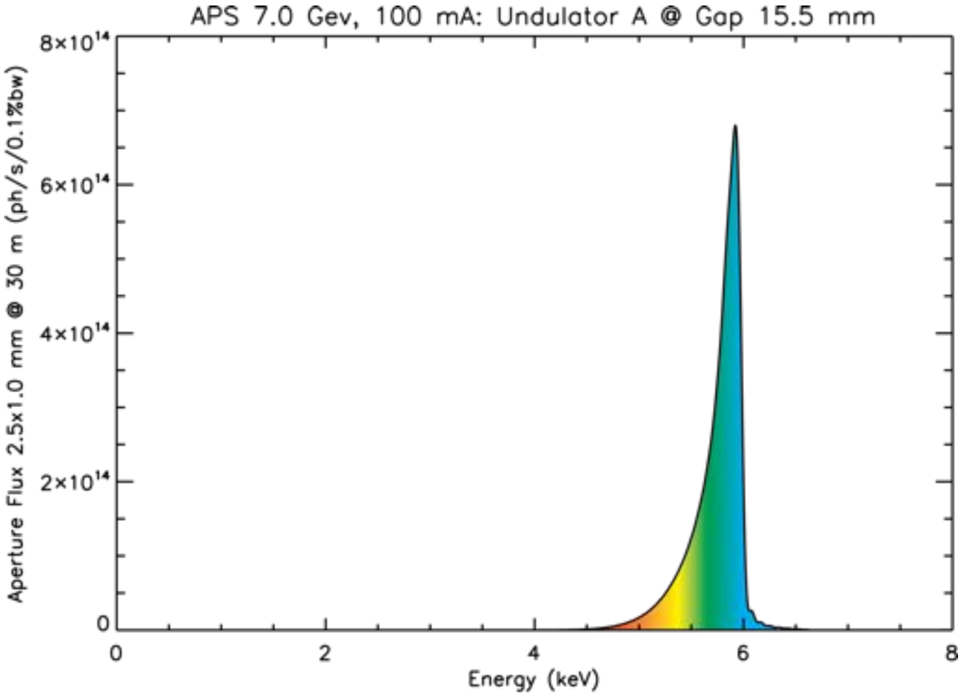
For a given K-value (gap), the wavelength at angle θ is $\lambda_1 = (\lambda_{ID}/2\gamma^2)(1 + K^2/2 + \gamma^2\theta_1^2)$



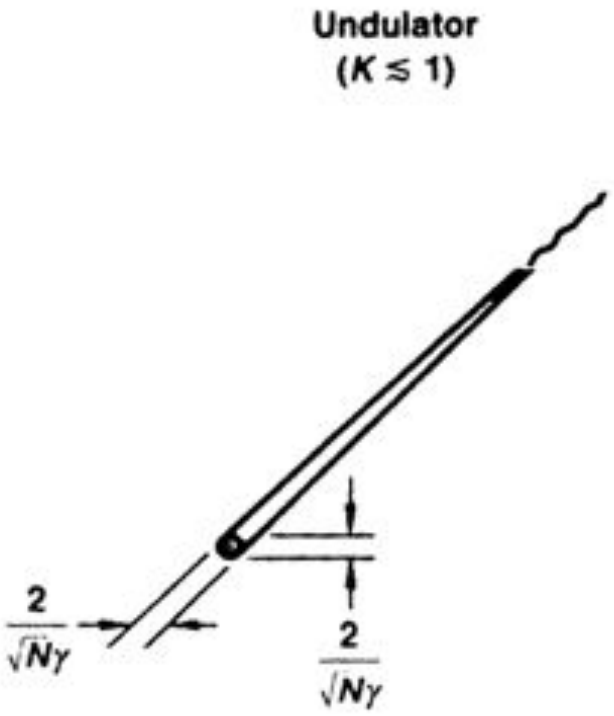
The central cone opening angle, θ_1, for the odd harmonics is given by:

$$\theta_1/ 2 = (\lambda^{x\text{-}ray}/2L)^1$$





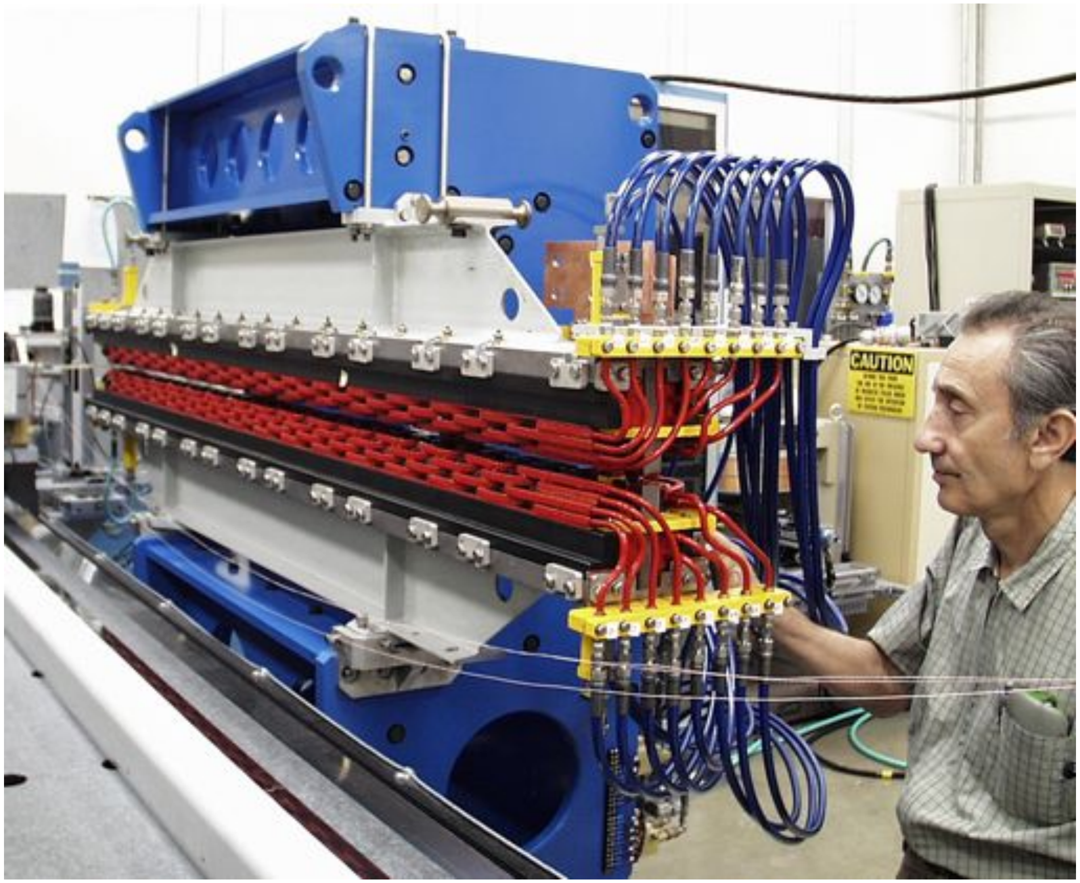
Undulator Radiation Patterns and Spectra



Undulator Radiation
(see Appendix 4)

- undulators defined as IDs with horizontal deflection angle ≈ $1/\gamma$, i.e., $K \approx 1$
- spectrum peaked at x-ray specific x-ray energies, but peaks are tunable by varying K (K = 0.94 B[T] λ_{ID}[cm])
- at the peaks (harmonics) the horizontal and vertical opening angles of the radiation is given by:
  $(\lambda_{x\text{-ray}} / 2L)^{1/2}$ [typically a few microradians]
- to get the true opening angle, need to consider the opening angle of the emitting particles
- Highly linearly polarized on harmonic. Can use specialized devices to control polarization

Electromagnetic Helical Undulator (4-ID-C)



Full polarization control from source

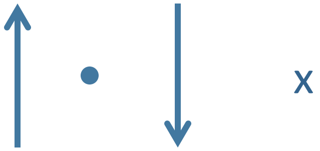
Circular switching rate

~100 ms (0.5 Hz) (~200 A)

Linear switching rate

~30 s (~1200 A)

Magnetic Field along device



Electro-magnets used to make electron go in a helical orbit. This produces circularly polarized x-rays (good for magnetism experiments).

Time Structure of the Radiation

- Particles are grouped together by the action of the RF cavities into bunches (350 MHz).

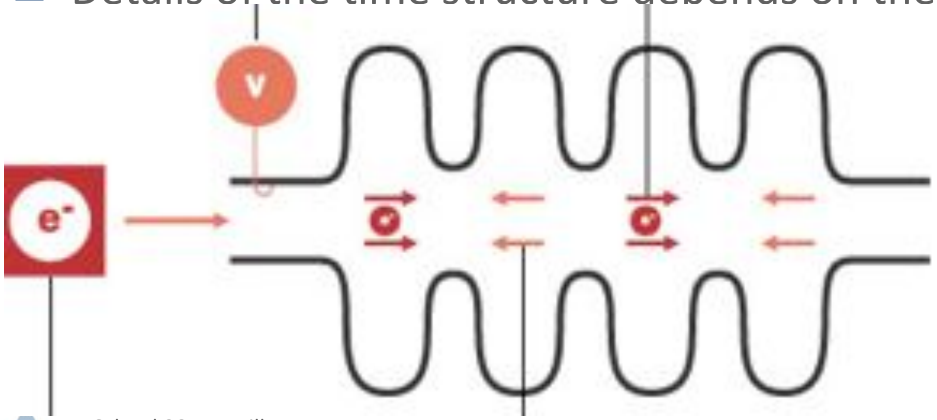
  At the APS:

  – natural bunch length (i.e. in the limit of zero current) 35 psec FWHM

  – typically about 100 psec FWHM

- 1104 m circumference (3.68 microsecond period; ~270 kHz)
  - – harmonic number 1296 i.e. 1296 evenly spaced "RF buckets" around the ring
  - – minimum spacing is 2.8 nsec

- Details of the time structure depends on the fill



Typical APS Filling Patterns

- 324 equally spaced bunches

  - approximately 11 nsec between bunches

  -approximates a continuous source

- 24 equally spaced bunches

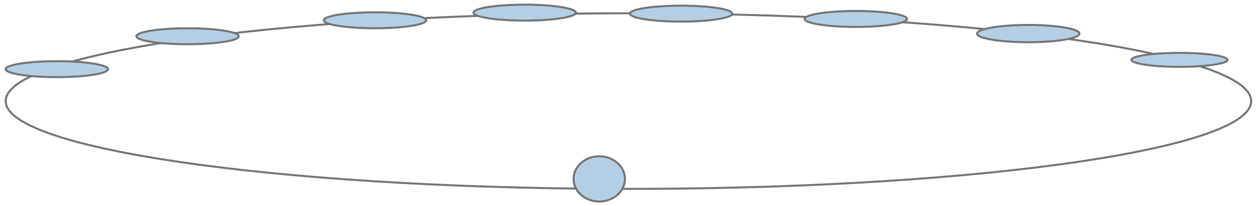
  - approximately 154 nsec between bunches

  -compromise between continuous source and pulsed source

- 1 + 7x8 (hybrid mode)

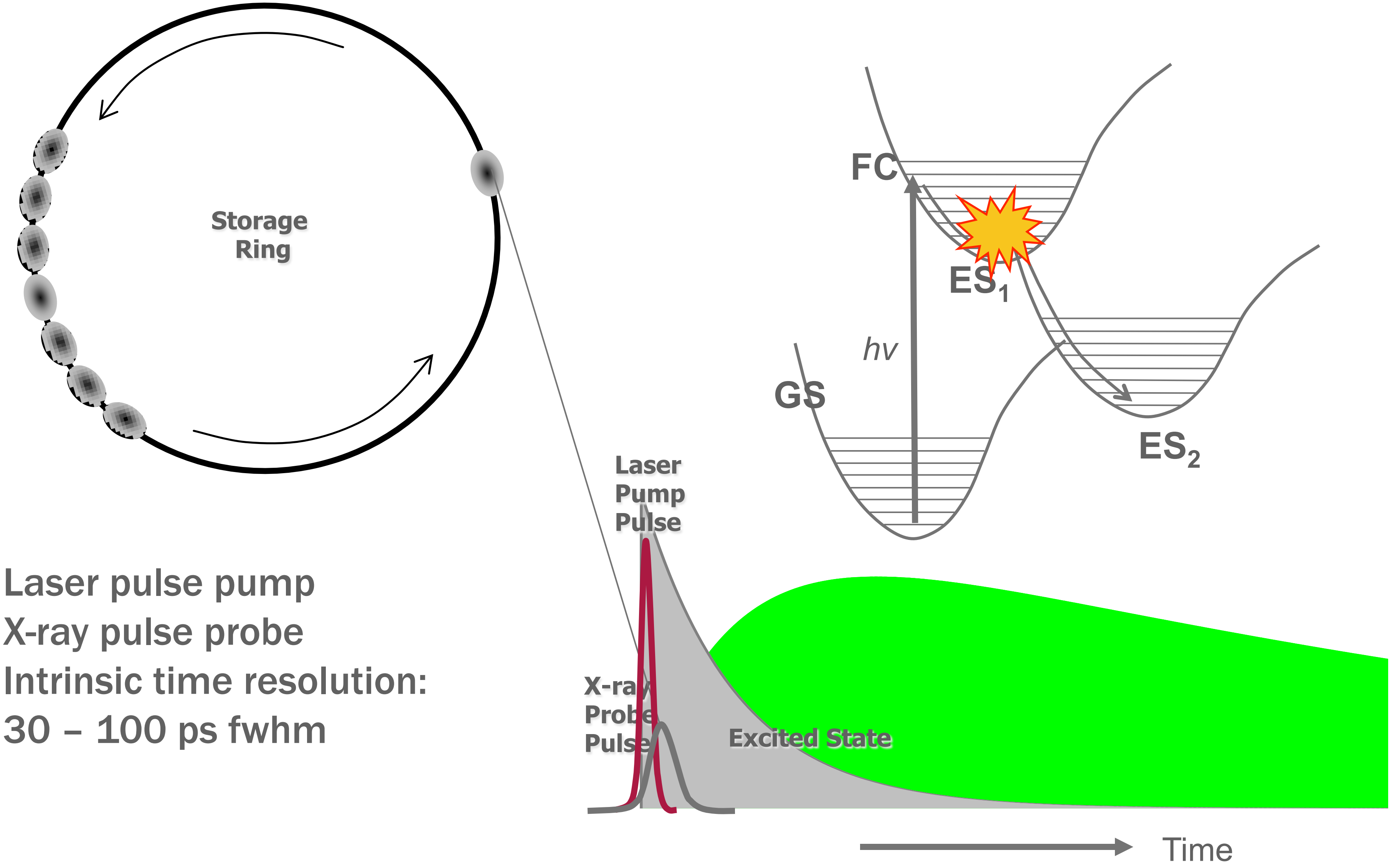
  - a single bunch followed by 8 groups of 7 bunches

  - timing experiments



Schematic of APS hybrid mode

Pump-probe Chemistry



Laser pulse pump
X-ray pulse probe
Intrinsic time resolution:
30 – 100 ps fwhm

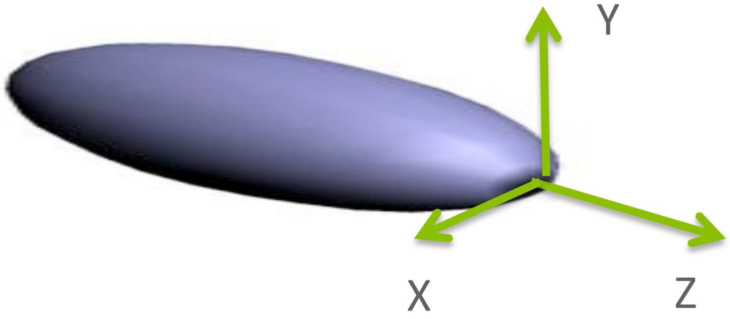
Transverse Properties of Particle Beams

- Up until now, we have calculated the radiation properties from a single electron, however in a storage ring, the radiation is emitted from an ensemble of electrons with some finite size and divergence distribution.

- Both the transverse and longitudinal properties of the particle beam in a storage ring are the equilibrium properties of the particle beam, but here we are interested in the ***transverse*** properties.

- The product of the particle beam size and divergence is proportional to a parameter of the beam called the emittance (units are length x angle). (See Appendix 6)

- The emittance is a constant of the storage ring, although one can trade off beam size for divergence as long as the (something proportional to the product) remains constant.



Transverse Properties and Photon Beam Brightness

Why do we need to know about the transverse particle beam properties?

Although the flux from BM and ID sources can be determined without detailed knowledge of the source size and divergence, one very important characteristic of the beam, namely **brightness,** requires a more information on the particle beam's size and divergence.

Brightness has units of:

photons/sec/0.1% BW/source area/source solid angle

$$\boxed{\text{Flux}/4\pi^2\,\Sigma_h\,\Sigma_v\,\Sigma_h'\,\Sigma_v'}$$

where Σ_i (Σ_i') is the effective one sigma value of the source size (divergence) in the i^th^ direction. The effective source size and divergence has contributions from both the particle beam and the radiation itself.

Stuff a Storage Ring Source can't do

- **What parameters would users like to see "enhanced"?**
  - Increased brightness (i.e., larger coherent fraction = $F_{coherent} / F_{total}$)
  - Shorter pulses (from 100 ps to 100 fsec)

- **We are about at the limits of what storage rings can do:**
  - Particle beam emittance (source size x divergence):
    - Horizontal emittance: 3×10^{-9} m-rads
    - Average brightness: 10^{19}-10^{20} x-rays/sec-0.1% bw-mm^2-mrad^2
  - Particle beam longitudinal properties:
    - Bunch length: 20 mm (70 psec)

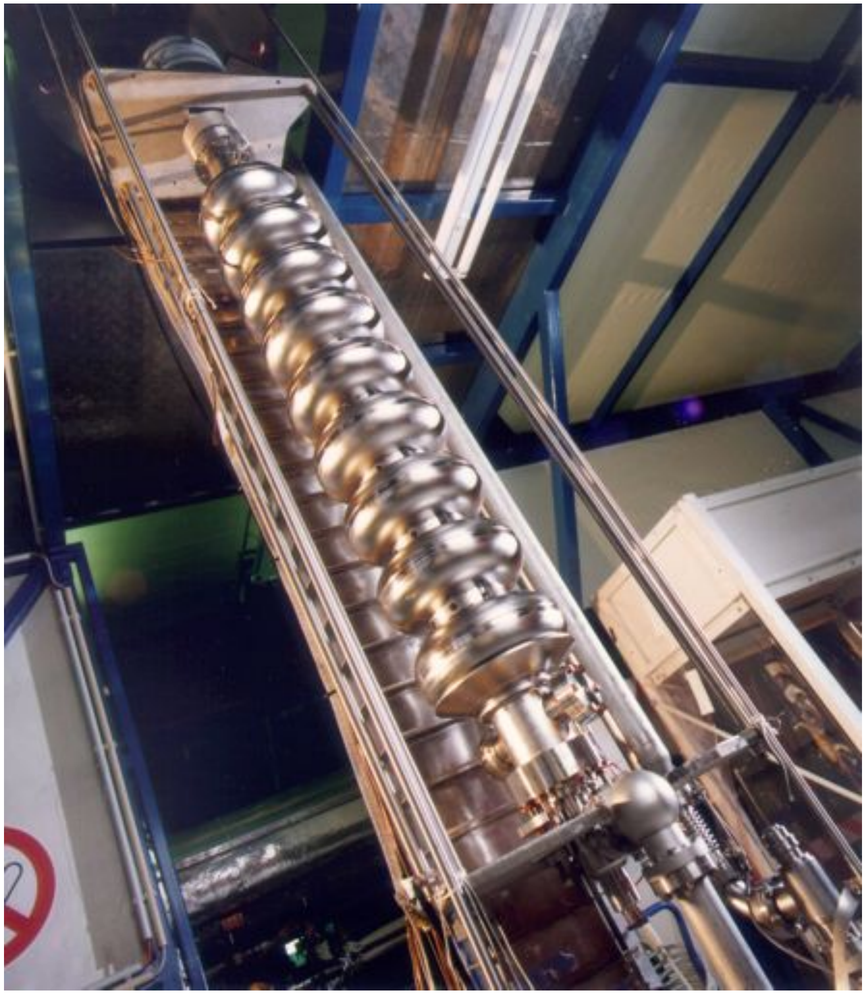
- **"Ultimate Storage Rings and Energy Recovery Linacs have the potential to deliver (near) fully spatially coherent beams but if we want femtosecond pulses and full coherence, use the low emittance and short pulses that can be generated by linear accelerators.**

X-ray Free Electron Lasers (X-FELs)

- Linac-based x-ray free electron lasers (XFELs) should provide many of the desired improvements.
  - Full coherence!
  - Femtosecond pulse lengths!

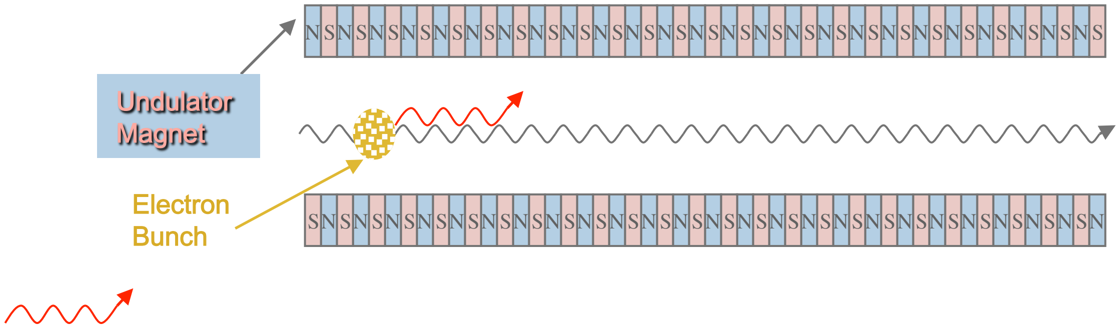
- An **x-ray FEL** uses the high brightness of an **electron gun** coupled to an emittance-preserving linac.

- The gain in the laser is obtained through a process called **Self-Amplified Spontaneous Emission** *or* **SASE.**



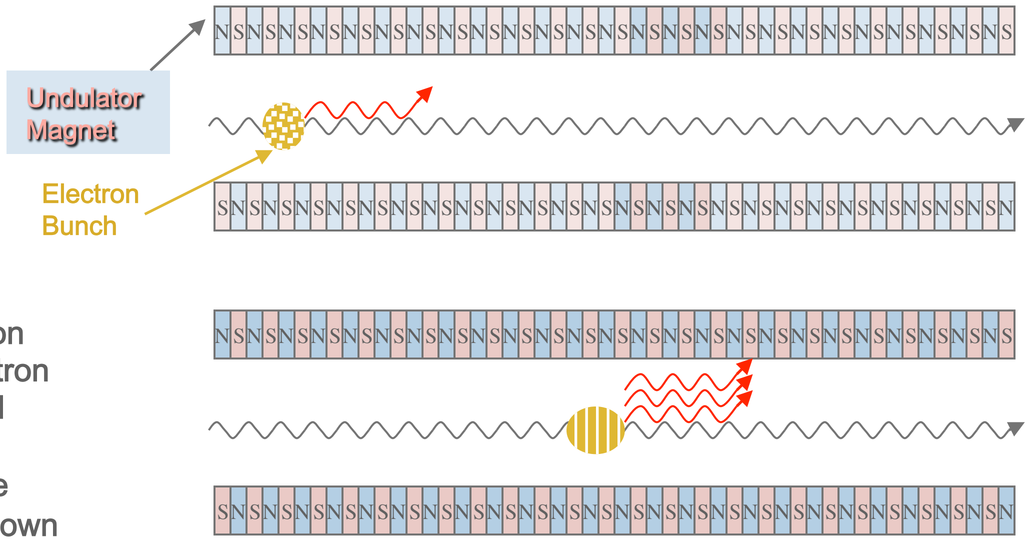
Self-Amplified Spontaneous Emission

The LCLS produces extraordinarily bright pulses of synchrotron radiation in a process called “self-amplified spontaneous emission” (SASE). In this process, an intense and highly collimated electron beam travels through an undulator magnet. The alternating north and south poles of the magnet force the electron beam to travel on an approximately sinusoidal trajectory, emitting synchrotron radiation as it goes.



John Galayda, LCLS

Self-Amplified Spontaneous Emission

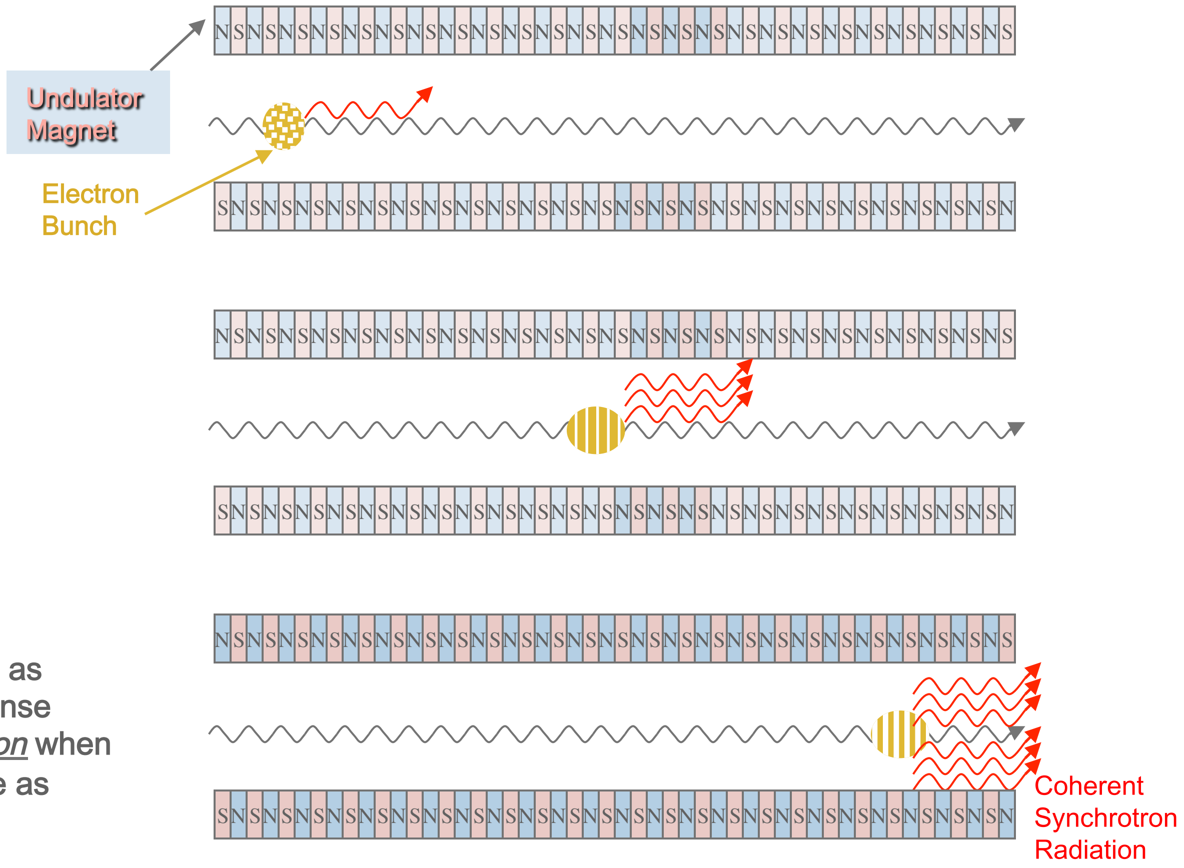


The electron beam and its synchrotron radiation are so intense that the electron motion is modified by the electric and magnetic fields of its own emitted synchrotron light. Under the influence of both the undulator magnet and its own synchrotron radiation, the electron beam is forced to form micro-bunches, 

separated by a distance equal to the wavelength of the emitted radiation.

John Galayda, LCLS

Self-Amplified Spontaneous Emission



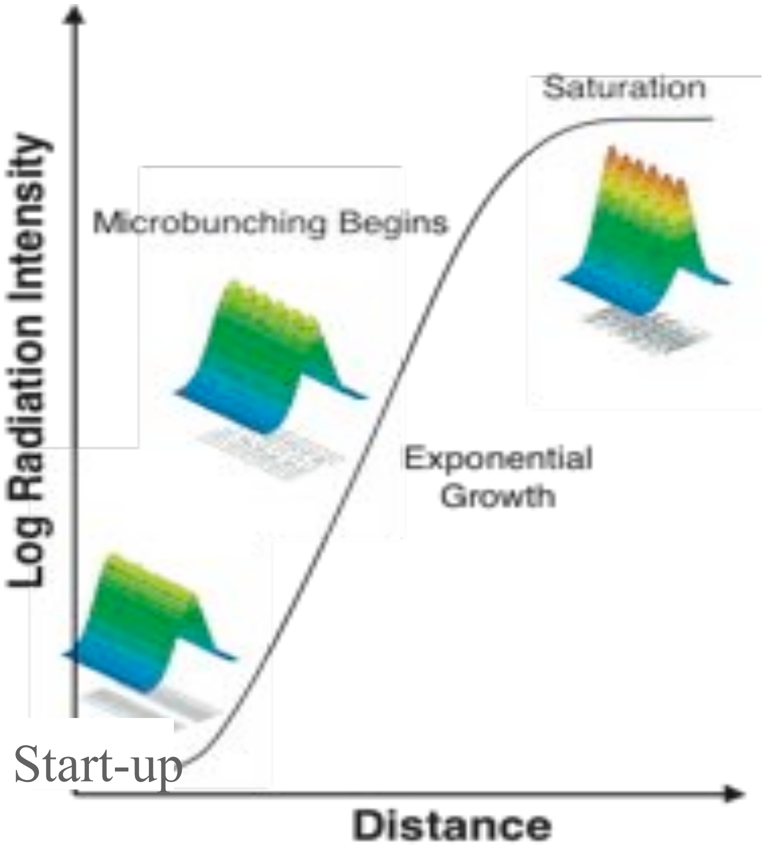
These micro-bunches begin to radiate as if they were single particles with immense charge. The process reaches *saturation* when The micro-bunching process has gone as far as it can go.

John Galayda, LCLS

Free Electron Lasers (FELs)

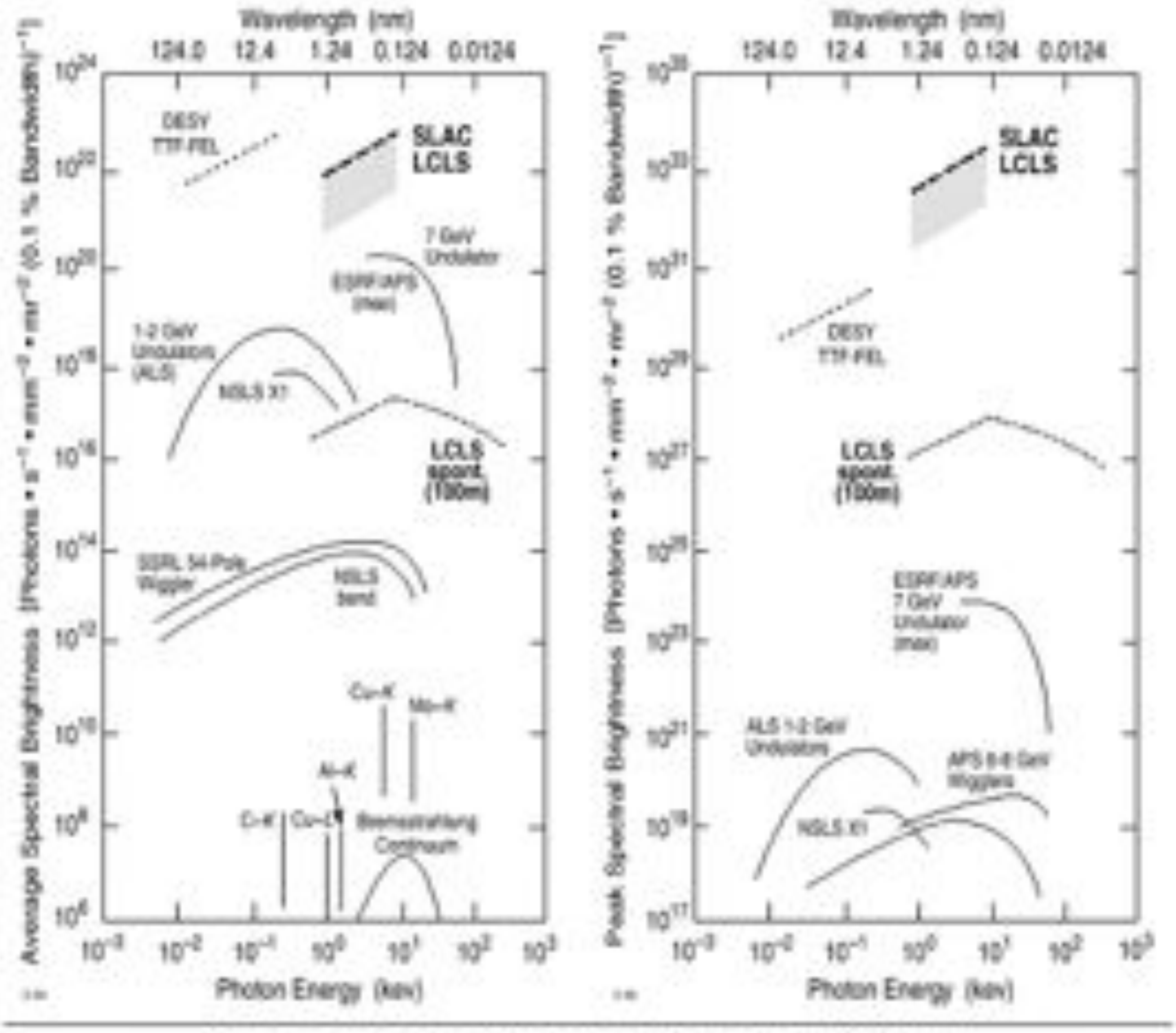
Start-up stage
External signal or spontaneous radiation interacts with the e-beam resonantly at undulator λ

Energy modulation ➔ density modulation (microbunching) ➔ coherent radiation at λ ➔ exponential growth (L_G)



Undulator Hall at the Linac Coherent Light Source (LCLS) at SLAC

Spectral Properties for X-ray Free Electron Lasers

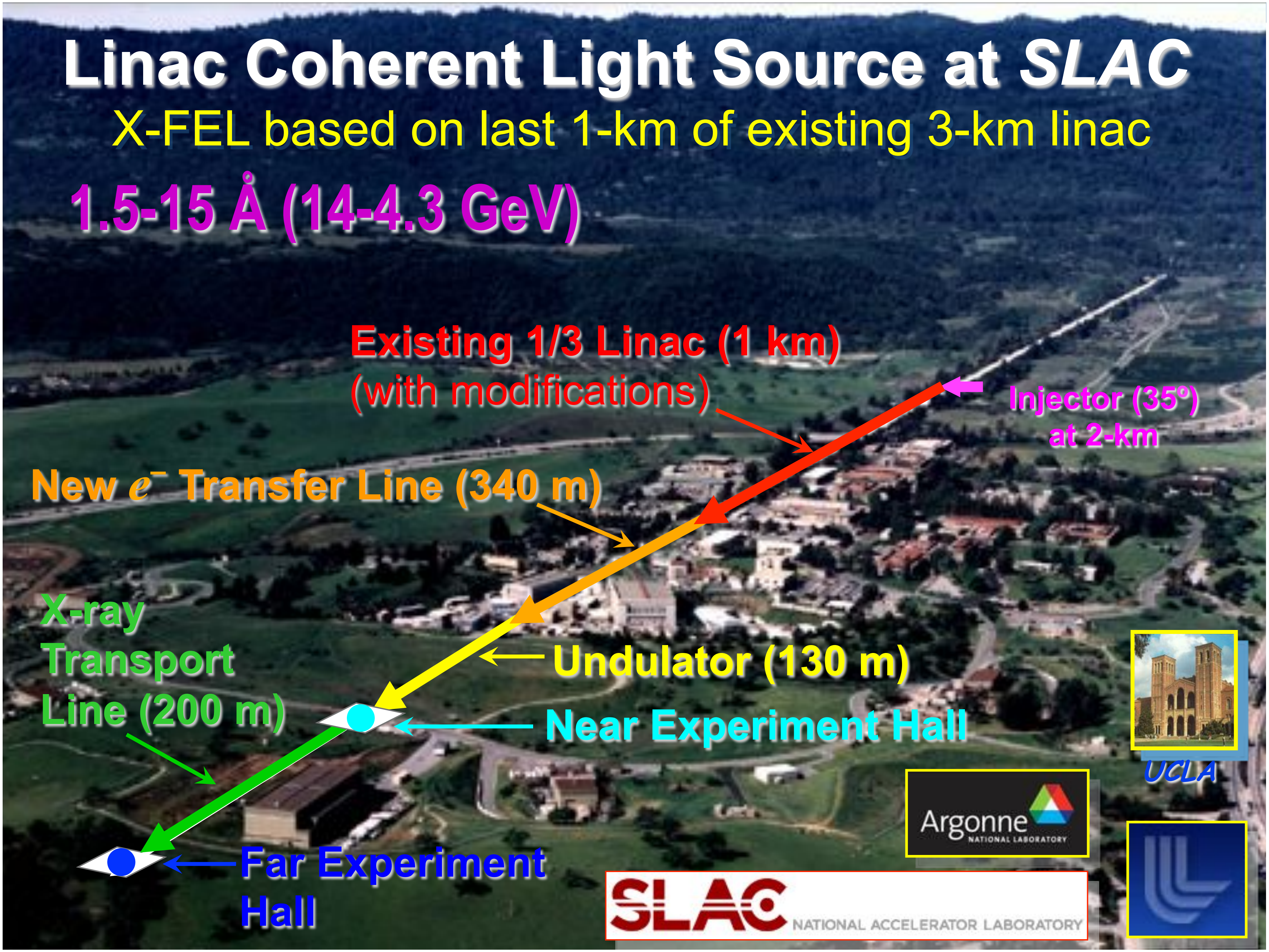


Average and peak brightness calculated for the LCLS and for other facilities operating or under construction.

Linac Coherent Light Source at *SLAC*

X-FEL based on last 1-km of existing 3-km linac

1.5-15 Å (14-4.3 GeV)



Existing 1/3 Linac (1 km)
(with modifications)

Injector (35°) at 2-km

New e^- Transfer Line (340 m)

Undulator (130 m)

X-ray Transport Line (200 m)

Near Experiment Hall

Far Experiment Hall

UCLA

Argonne NATIONAL LABORATORY

SLAC NATIONAL ACCELERATOR LABORATORY

Advanced Photon Source Upgrade

- APS currently has a proposal to do a major upgrade of the facility over the next 5-7 years.
- Estimated cost is ~$350M, and will bring the facility to the state-of the-art for a 3rd generation storage ring source.
- Been approved by DOE. Currently in the design phase
- First upgrades will start in the Fall of 2013

  - New and upgraded beamlines
  - Increase current to 150 mA.
  - Deliver 100 times better temporal resolution, down to 1 picosecond.

APS-U Project Baseline Scope

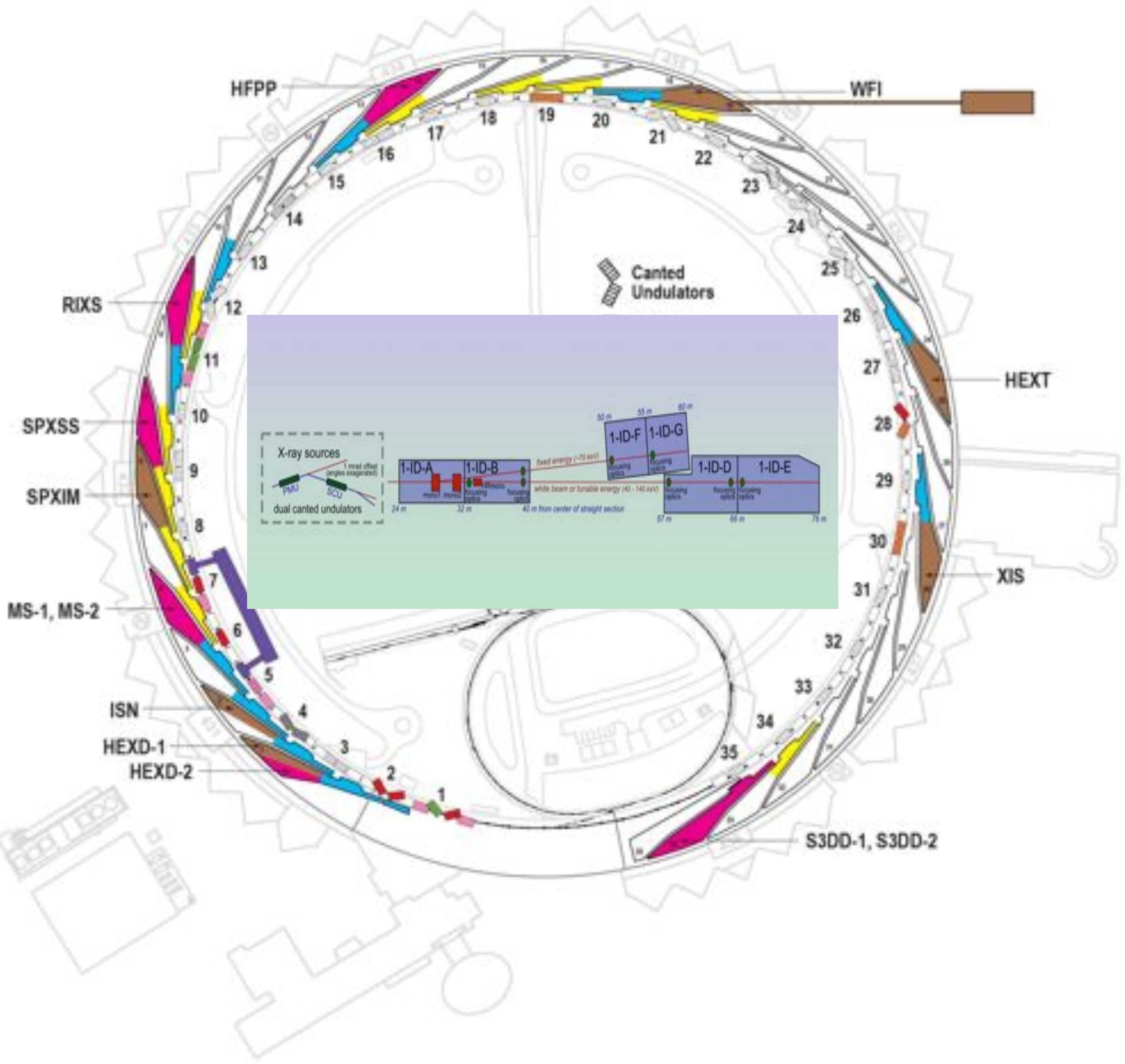
- 🟨 10 Renovated Front Ends
- 🟦 11 New Front Ends

- 🟪 4 Long Straight Sections

- 🟧 5 Planar Undulators
- ⬛ 2 Polarized Undulators
- 🟥 6 Revolver Undulators
- 🟩 3 Superconducting Undulators

- 🟪 SPX SRF Cavities and Cryostats

- 🟫 6 New Beamlines
- 🟥 8 Beamline Upgrades



Appendix 0: Radiated Power from Charges at Relativistic Velocities

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and α the acceleration. For a circular orbit of radius r, in the non-relativistic case, α is just the centripetal acceleration, v^2/r. In the relativistic case:

$$a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r}$$

Where $\tau = t/\gamma$ = proper time, $\boxed{\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2 \text{ and } \beta = v/c}$

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

Boxes with lines like this indicate an important equation.

Appendix 0: Dependence on Mass and Energy of Radiated Power

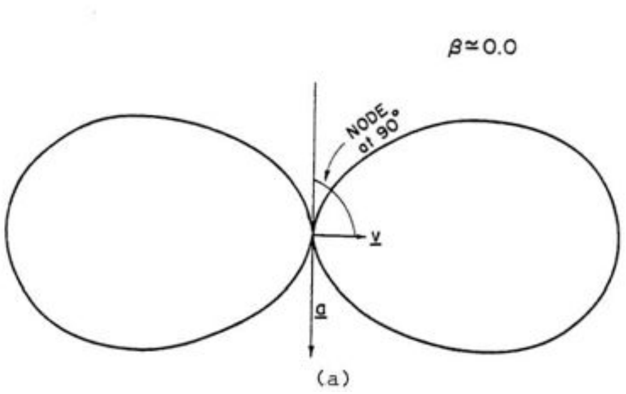
$$\boxed{P = \frac{2e^2}{3c^3}\frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2}\frac{E^4}{m_o^4 c^8}}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e^- with the same total energy, E.)

2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

Appendix 0: Radiation Patterns When V << C



When v << c, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.

Recall that special relativity says that angles transform as:

$$\tan\theta_{lab} = \frac{\sin\theta'}{\gamma\left(\cos\theta' + \beta\right)}$$

And so as you crank up β, the radiation pattern begins to deform (in the lab frame).

At the APS with E = 7 GeV,

$\gamma = E/m_o c^2 = 7\ \text{GeV}/0.511\ \text{MeV}$

$\gamma = 1.4 \times 10^4$

$1/\gamma = 73 \times 10^{-6}$

Appendix 1: BM Spectral Distribution

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c.

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and ρ ισ the radius of curvature of the trajectory. Note that the critical energy scales as γ^3. In practical units, the critical energy can be written as:

$$\boxed{E_c[\text{keV}] = 2.218\ E^3[\text{GeV}]/\ \rho[\text{m}] = 0.06651\ B[\text{kG}]\ E^2[\text{GeV}]}$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at E = 7 GeV . The critical energy of the radiation emitted from the BM is:

$$E_c[keV] = 0.06651\ B[kG]\ E^2[GeV]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5\ keV}\ \text{or}\ \underline{0.64\ Å.}$$

Appendix 1: BM Angular Distribution and Flux

- The opening angle of the entire radiation field (i.e., the power) is approximately:

  $$\theta_x = \theta_y \approx 1/\gamma,$$

  where θ_x is the horizontal angle and θ_y is the vertical angle.

- The collimation in the horizontal direction is lost and what is observed is just the vertical opening angle.

- Flux from a bending magnet is usually quoted as flux per unit horizontal angle (integrated over the vertical angle). There are no "simple" closed-form solutions for the photon flux, F, as a function of wavelength from a bending magnet however there are numerous series approximations that can easily be evaluated with a calculator/computer.

From a bending magnet (B = 5.99 kG) at the APS operating at E = 7GeV and I = 100 mA (at the critical energy, integrated over all vertical angles) we get:

$$dF/d\theta_x \approx 10^{13} \text{ photons/sec - 0.1\% BW - mrad } \theta_x$$

Appendix 2: Where did "K" come from?

Equation of motion for a relativistic charged particle in a magnetic field

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v}\times\vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{1D}}\right)$$

$$\dot{v}_x = \frac{ecB_0}{\gamma m_0}\sin\left(\frac{2\pi z}{\lambda_{1D}}\right) \qquad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0}\frac{\lambda_{1D}}{2\pi c}\cos\left(\frac{2\pi ct}{\lambda_{1D}}\right) = -\frac{eB_0}{\gamma m_0}\frac{\lambda_{1D}}{2\pi}\cos\left(\frac{2\pi ct}{\lambda_{1D}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c}\left[\frac{\lambda_{1D}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{1D}}\right) = \left[\frac{eB_0}{m_0}\frac{\lambda_{1D}}{2\pi c}\right]\frac{1}{\gamma}\left[\frac{\lambda_{1D}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{1D}}\right) = K\frac{1}{\gamma}\left[\frac{\lambda_{1D}}{2\pi}\right]\sin\left(\frac{2\pi z}{\lambda_{1D}}\right)$$

$$x_{\max} = K\frac{1}{\gamma}\left[\frac{\lambda_{1D}}{2\pi}\right] \qquad and \qquad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \qquad where \qquad K = \left[\frac{eB_0}{2\pi}\frac{\lambda_{1D}}{m_0 c}\right]$$

Appendix 3: Wiggler Radiation Spectral Distribution

Spectral Distribution:

Wiggler radiation is the incoherent superposition of radiation from each pole of the wiggler. As with the bending magnet, the spectral distribution of the emitted radiation from a wiggler is smoothly varying as a function of photon energy and is characterized using the critical energy as a parameter.

The APS has built a wiggler with a magnetic field, B_o, of 10 kilogauss, a period of 8.5 cm, and a length of 2.4 meters (N=28).

$K = 7.9$, $E_c = 32.6$ keV, $\theta_{max} = 577$ microradians, $x_{max} = 8$ microns

Wiggler Flux:

The flux from a wiggler can calculated by multiplying the bending magnet equations by 2N where N is the number periods (and by using the appropriate critical energy!).

The APS Wiggler (B_o =10 kG, λ_{ID} = 8.5 cm, L = 2.4 meters; N=28, has a flux (at the critical energy integrated over all vertical angles) of:

$dF/d\theta_x = 4.8 \times 10^{14}$ photons/sec - 0.1 % BW - mrad θ_x

Appendix 4: Undulator Flux

To determine the flux from an undulator, we *integrate over both the vertical and horizontal angular distributions of the central cone* and so the flux will have units of:

x-rays/sec-0.1% bw.

As with bending magnets, there is not a closed form expression for the flux, but it can be approximated by:

$$F_n = 0.72 \times 10^{11}\ N\ Q_n\ I[\mathrm{mA}]\ \ \mathrm{ph/sec\text{-}0.1\%\ bw}$$

where:

$$Q_n(K) \approx 1 \text{ for } K > 1.$$

For APS Undulator A (N = 72) with the storage ring running at 7 GeV and 100 milliamps, typical flux values for the first harmonic (n=1) are:

$F = 3\text{-}5 \times 10^{14}$ ph/sec-0.1% bw.

Appendix 5: APS Electron Beam Parameters

APS runs with $\varepsilon_H = 3 \times 10^{-9}$ *m-rad and a coupling (ratio of vertical emittance to horizontal emittance) of 0.9% , therefore*

$$\varepsilon_V = 0.025 \times 10^{-9} \text{ m-rad.}$$

The particle beam source size and divergence at the straight sections are:

$$\sigma_H = 270 \text{ microns}$$
$$\sigma_H' = 11 \text{ microradians}$$
$$\sigma_V = 9 \text{ microns}$$
$$\sigma_V' = 3 \text{ microradians}$$

Appendix 5: Comparison of Radiation & Electron Beam Properties

- If Gaussian distributions are assumed for both the particle beam and the radiation itself, resultant source size and divergence is the quadrature sum of the two components, namely:

$$\Sigma_i = \sqrt{\,}[\sigma_r^2 + \sigma_i^2] \quad \text{and} \quad \Sigma_I' = \sqrt{\,}[\sigma_r'^2 + \sigma_i'^2].$$

- We can now calculate the beam brightness since

$$\mathcal{B} = \text{Flux}/4\pi^2\, \Sigma_H\, \Sigma_V\, \Sigma_H'\, \Sigma_V'$$

APS Undulator A has a length of 2.4 meters. For 1Å radiation the natural opening angle is:

$\sigma_r' = \sqrt{[\lambda/2L]} =$ **4.5 microradians.**

The corresponding source size of the radiation is:

$\sigma_r = \sqrt{[\lambda L/8\pi^2]} = 1.7$ **microns.**

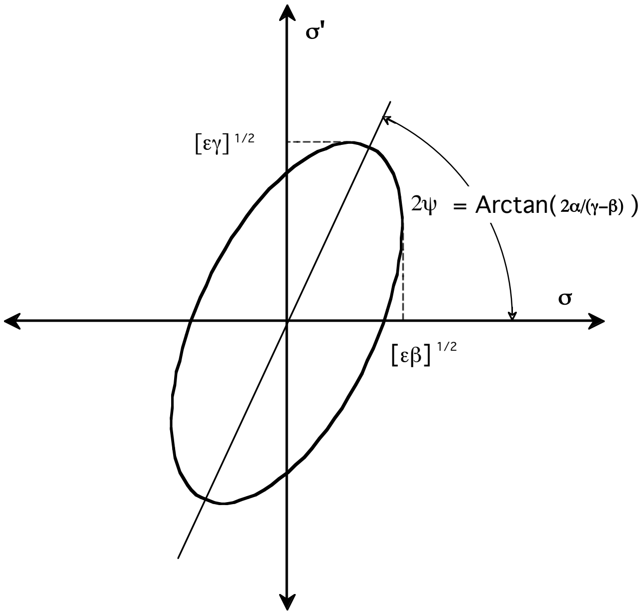
Compare this with the vertical size and divergence of the APS beam:

$\sigma_V' = 3$ microradians

$\sigma_V = 9$ microns

Appendix 6: Phase Space of the Charged Particles

- A mono-energetic, charged particle, under the action of a linear force such as is found in a storage ring, traces a contour in phase space that is an ellipse.



In the above diagram, ε is the emittance and α, β, and γ describe the shape of the ellipse at any point in the storage ring. α, β, and γ are sometimes called the Twiss parameters.

- $\sigma_{H,V}(s)$ and $\sigma_{H,V}'(s)$ are the one-sigma values of the transverse position and divergence, respectively, at some position, s, around the storage ring.

- The area of the ellipse is proportional to a parameter of the beam called the emittance (units are length x angle).

- Liouville's Theorem states that for a system such as described here, the phase space volume should remain a constant. In other words the area of the phase space ellipse (emittance) is constant even if the shape of the ellipse changes (periodically) as one goes around the particle's trajectory.

Diffraction Limited Source Size and Divergence

- The effective phase space of the radiation source (Σ_i and $\Sigma_{i'}$) has contributions from size and divergence of the particle beam generating the radiation and the intrinsic source size and divergence of the radiation itself. **Is there are limit to how small the effective phase space area (i.e., emittance) can be?** Yes, you are still bound by the Heisenberg Uncertainty Principle. Recall:

$$\Delta x \Delta p_x \geq \hbar/2$$

$$\frac{p_x}{p_z} = \Theta_x \text{ or } \frac{\Delta p_x}{p_z} = \Delta\Theta_x \text{ and } p_z = \hbar k = \hbar(2\pi/\lambda)$$

$$so: \Delta x \Delta p_x = \Delta x \Delta\Theta_x p_z = \Delta x \Delta\Theta_x[\hbar(2\pi/\lambda)] \geq \hbar/2$$

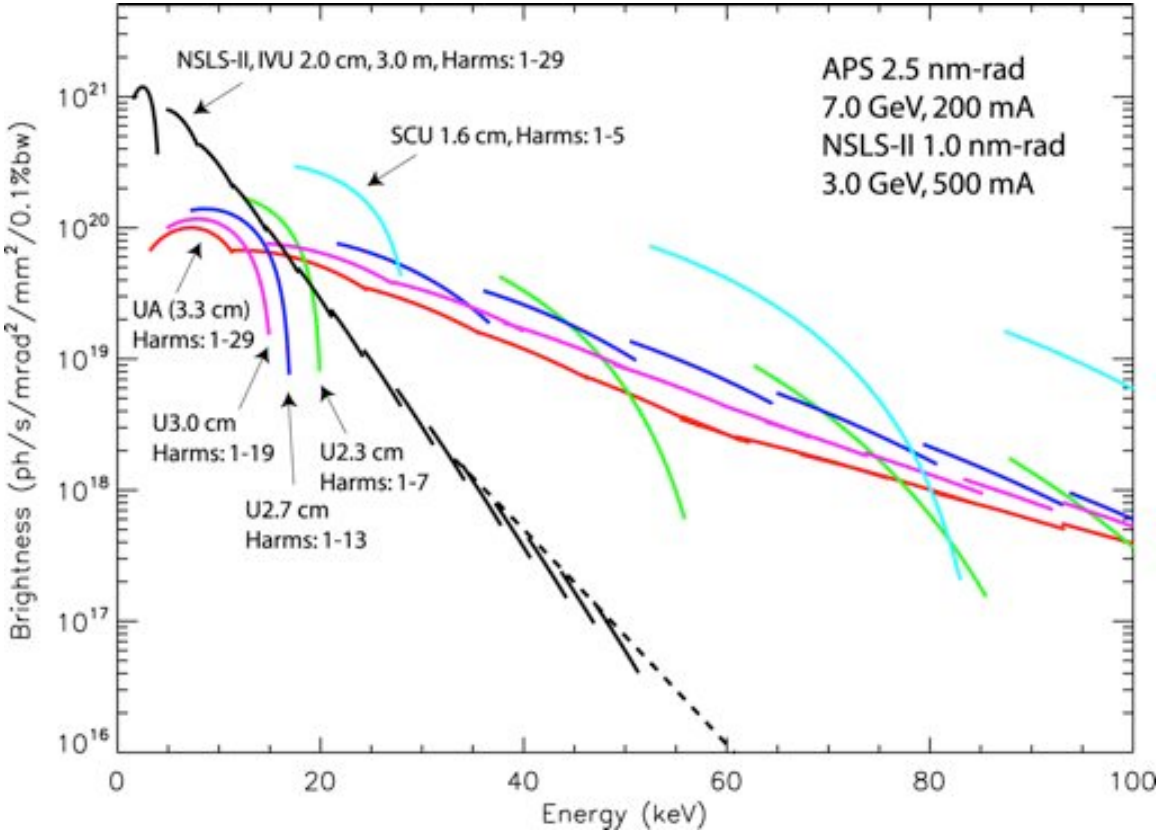
$$\Delta x \Delta\Theta_x \geq \lambda/4\pi$$

- This is the so-called diffraction limit. For central cone of the undulator:

σ_r' or $(\Delta\Theta) = \sqrt{[\lambda/2L]}$ and so

σ_r or $(\Delta x) = \sqrt{[\lambda L/8\pi^2]}$

Brightness vs. Photon Energy for Various SR Facilities



- The high brightness beams at 3rd generation sources (ALS and APS, for example) results in an x-ray beam with partial transverse (or spatial) coherence.
- This beam property can be an important parameter in some experiments such as photon correlation spectroscopy, x-ray holography, imaging, etc.

The Linac Coherent Light Source (LCLS) at SLAC

The LCLS uses the last kilometer of an existing 3 km LINAC at the Stanford Linear Accelerator Center (at Stanford University, CA)

The X-ray free electron laser (XFEL) relies on a high energy linear accelerator (LINAC) and not on a synchrotron/storage ring.