### Introduction to Inelastic Neutron Scattering

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Neutrons: Properties and Cross Sections

Excitations in solids

Triple Axis and Chopper TOF Techniques

Practical concerns







<sup>235</sup>U + n → daughter nuclei + 2-3 n + gammas

neutrons:

no charge s=1/2 <mark>massive: mc<sup>2</sup>~1 GeV</mark>

### How do we produce neutrons



### Fission

- chain reaction
- continuous flow
- 1 neutron/fission



### **Spallation**

- no chain reaction
- pulsed operation
- 30 neutrons/proton





# Neutron interaction with matter

#### • Properties of the neutron

- Mass m<sub>n</sub>=1.675 x 10<sup>-27</sup> kg
- Charge 0
- Spin-1/2, magnetic moment  $\mu_n = -1.913 \mu_N$

### Neutrons interact with...

- Nucleus
  - Crystal structure/excitations (eg. phonons)
- Unpaired e<sup>-</sup> via dipole scattering
  - Magnetic structure/excitations (eg. spin waves)





**Nuclear scattering** 

NXS School Magnetic dipole scattering



# Wavelength-energy relations

#### Neutron as a wave ...

• Energy (E), velocity (v), wavenumber (k), wavelength ( $\lambda$ )

$$k = \frac{m_n v}{h} = \frac{2\pi}{\lambda}$$

$$E = \frac{h^2 k^2}{2m_n} = \frac{h^2}{2m_n} \left(\frac{2\pi}{\lambda}\right)^2 = \frac{81.81 \text{meV} \cdot \text{\AA}^2}{\lambda^2}$$

$$E = k_B T = \left(0.08617 \text{meV} \cdot \text{K}^{-1}\right) T$$



 $\lambda$  ~ interatomic spacing  $\rightarrow$  E ~ excitations in condensed matter

	Energy (meV)	Temperature (K)	Wavelength (Å)
Cold	0.1 – 10	1 – 120	4 – 30
Thermal	5 – 100	60 - 1000	1 – 4
Hot	100 – 500	1000 - 6000	0.4 – 1



### **Incident Beam:**

- monochromatic
- "white"
- "pink"

### **Scattered Beam:**

- Resolve its energy
- Don't resolve its energy
- Filter its energy

### Fermi's Golden Rule within the 1<sup>st</sup> Born Approximation

$$W = 2\pi / h | < f | V | i > |^{2} \rho (E_{f})$$
  
$$\delta \sigma = W / \Phi = (m/2\pi h^{2})^{2} k_{f} / k_{i} | < f | V | i > |^{2} \delta \Omega$$

 $\delta^2 \sigma / \delta \Omega \, \delta E_f = k_f / k_i \, \sigma_{coh} / 4\pi \, N \, S_{coh} (\mathbf{Q}, \omega)$ 

+ 
$$k_f/k_i \sigma_{incoh}/4\pi$$
 N  $S_{incoh}(\mathbf{Q}, \omega)$ 

### $\sigma_{coh}$ and $\sigma_{incoh}$

### parametrize the strength of the scattering from the nuclei





## Nuclear correlation functions

 $d\Omega dE_{f}$ 

 $4\pi k_i$ 



# Nuclear (lattice) excitations

Neutron scattering measures simultaneously the wavevector and energy of **collective excitations**  $\rightarrow$  dispersion relation,  $\omega(\mathbf{q})$  In addition, **local excitations** can of course be observed

#### • Commonly studied excitations

- Phonons
- Librations and vibrations in molecules
- Diffusion
- Collective modes in glasses and liquids

#### Excitations can tell us about

- Interatomic potentials & bonding
- Phase transitions & critical phenomena (soft modes)
- Fluid dynamics
- Momentum distributions & superfluids (eg. He)
- Interactions (eg. electron-phonon coupling)



## **Atomic diffusion**

For long times compared to the collision time, atom diffuses



Auto-correlation function

$$G_{s}(r,t) = \left\{ 6\pi \left\langle r^{2}(t) \right\rangle \right\}^{-3/2} \exp \left( -\frac{r^{2}}{6 \left\langle r^{2}(t) \right\rangle} \right)$$

$$S(Q,\omega) = \frac{1}{\pi h} \exp\left(\frac{h\omega}{2k_BT}\right) \frac{DQ^2}{\omega^2 + (DQ^2)^2}$$

100 θ=60\* Ω<sub>0</sub>=0·82 *θ* =20° 60 Q=0.28 20  $\left\{ \sigma_{\text{inc}} / \sigma_{\text{inc}} + \sigma_{\text{conf}} \right\} S(Q, \omega) (1/\beta \hbar)$ 60 θ=30\* Q=0.42 20 θ=45 Q= 0.62 30 20 θ=90° = 1·16 10 2-0 x 10<sup>12</sup> 0 1.0 -1-0 2.0 -1.0 0 0 ω (rad s-1) Cocking, J. Phys. C 2, 2047 (1969)..

Liquid Na

## **Molecular vibrations**

- Large molecule, many normal modes
- Harmonic vibrations can determine interatomic potentials



C60 molecule



Prassides et al., Nature 354, 462 (1991).

**Crystalline Materials: Structure is Periodic with Period a** 

Work in Reciprocal Space – Momentum space, most natural for understanding diffraction and scattering

Mapping Momentum – Energy (Q-E) space



#### **Bragg diffraction:**

#### Elastic scattering : $|\mathbf{k}_i| = |\mathbf{k}_f|$



#### **Bragg diffraction:**





а

Elastic scattering :  $|\mathbf{k}_i| = |\mathbf{k}_f|$ 

#### **Bragg diffraction:**





а

Elastic scattering :  $|\mathbf{k}_i| = |\mathbf{k}_f|$ 

### **Elementary Excitations in Solids**



MOMENTUM, Q

• Lattice Vibrations (Phonons)

• Spin Fluctuations (Magnons)



#### **Energy vs Momentum**

• Forces which bind atoms together in solids



## Phonons

• Normal modes in periodic crystal  $\rightarrow$  wavevector

$$\mathbf{u}(l,t) = \frac{1}{\sqrt{NM}} \sum_{j\mathbf{q}} \boldsymbol{\varepsilon}_{j}(\mathbf{q}) \exp(i\mathbf{q}\cdot\mathbf{l}) \hat{B}(\mathbf{q}j,t)$$

• Energy of phonon depends on **q** and polarization

Loragitverimenhodele





FCC structure





## **Phonon intensities**







## More complicated structures









## Spin excitations

#### • Spin excitations

- Spin waves in ordered magnets
- Paramagnetic & quantum spin fluctuations
- Crystal-field & spin-orbit excitations

#### • Magnetic inelastic scattering can tell us about

- Exchange interactions
- Single-ion and exchange anisotropy (determine Hamiltonian)
- Phase transitions & critical phenomena
- Quantum critical scaling of magnetic fluctuations
- Other electronic energy scales (eg. CF & SO)
- Interactions (eg. spin-phonon coupling)



### Spin waves



Perring et al., Phys. Rev. Lett. 77, 711 (1996).

Antiferromagnetic 80 [[20]] [00**5**] [٤00] [¦soç] [505] 70 MAGNON ENERGY (meV) 60 50 Tm FeO<sub>z</sub> 40 =102.5°K 07 ± 0.58° K 30 =-1.88±0.23°K 20 =-24.51 ±0.14°K J'= 0.0°K 10  $[0,0,\frac{1}{2}]$  [0,0,0]  $[\frac{1}{2},0,0]$   $[\frac{1}{2},0,\frac{1}{2}]$ [0,0,0]  $\left[\frac{1}{2}, 0, \frac{1}{2}\right]$ Shapiro et al., Phys. Rev. B 10, 2014 (1974).



McQueeney et al., Phys. Rev. Lett. 99, 246401 (2007).



## Scattering experiments





# **Kinematic limitations**

- Many combinations of  $k_i, k_f$  for same  $Q, \omega$ 
  - Only certain configurations are used (eg. E<sub>f</sub>-fixed)
- Cannot "close triangle" for certain Q,ω due to kinematics







Kinematic limits, E<sub>i</sub>=160 meV





### **Brockhouse's Triple Axis Spectrometer**



Collimator\_

-Shielding drum

Monochromator

crystal



#### **Two Axis Spectrometer:**

- 3-axis with analyser removed
- Powder diffractometer
- Small angle diffractometer
- Reflectometers





#### **Soller Slits: Collimators**

## Define beam direction to +/- 0.5, 0.75 etc. degrees







#### Single crystal monochromators:

### Bragg reflection and harmonic contamination





 $n\lambda = 2d \sin(\theta)$ 

Get:  $\lambda$ ,  $\lambda/2$ ,  $\lambda/3$ , etc.





Bragg's Law:  $n\lambda = 2d sin(\theta)$ 

 $|\mathbf{k}| = 2\pi / \lambda$ 

#### Volume of Q – E space sampled

~ k<sup>3</sup> cot (θ)

"Efficiency" of monochromator / analyser varies strongly with k,  $\boldsymbol{\theta}$
### **Bragg Diffraction:**

### **Two Axis Diffraction: No E<sub>f</sub> discrimination**





**Two different ways of performing constant-Q scans** 

### **Constant** k<sub>f</sub>:

•  $k_f$  ,  $\theta_A$  do not change; therefore analyser "efficiency" is constant

•  $k_i$ ,  $\theta_M$  do change, but monitor detector normalizes to incident neutron flux

 Monitor detector (low) efficiency goes like ~ 1/v ~ 1/k<sub>i</sub>



### **Recall that our cross-section was:**

 $\delta \sigma = W / \Phi = (m/2\pi h^2)^2 k_f / k_i |< f | V | i>|^2 \delta \Omega$ 

Which gave us (keeping only the coherent scattering)

 $\delta^2 \sigma / \delta \Omega \, \delta E_f = k_f / k_i \, \sigma_{coh} / 4\pi \, N \, S_{coh} (\mathbf{Q}, \omega)$ 

However, we are measuring the incident flux with an efficiency of  $1/k_i$ 

So, for constant k<sub>f</sub>, this means we measure:

 $\delta^2 \sigma / \delta \Omega \delta E_f = k_f / k_i \sigma_{coh} / 4\pi N S_{coh} (\mathbf{Q}, \omega) / 1 / k_i \sim S_{coh} (\mathbf{Q}, \omega)$ 

### Mapping Momentum – Energy (Q-E) space



**2π/**a









### MOMENTUM, Q

#### **Elementary Excitations in Solids**

• Lattice Vibrations (Phonons)

• Spin Fluctuations (Magnons)



#### **Energy vs Momentum**

• Forces which bind atoms together in solids







Constant Q, Constant E 3-axis technique allow us to Put Q-Energy space on a grid, And scan through as we wish

Map out elementary excitations In Q-energy space (dispersion Surface)



picomotor

# Inelastic x-ray scattering

- Ei=20 keV, need 1 meV resolution
- $\Delta E/E_i \simeq 10^{-7} !!!$





piezo





 $\ensuremath{\varphi}\xspace$ -scan of monochromator 1 meV  $\Rightarrow \ensuremath{\mu}\xspace$ rad

T-scan of monochromator  $1 \text{ meV} \Rightarrow 0.02 \text{ K}$ 

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# Samples

#### • Samples need to be BIG

- ~ gram or cc
- Counting times are long (mins/pt)



Co-aligned CaFe<sub>2</sub>As<sub>2</sub> crystals





# Monochromators

• Selects the incident wavevector





- Reflectivity
- focusing
- high-order contamination
   eg. λ/2 PG(004)

Mono	d(hkl)	uses
PG(002)	3.353	General
Be(002)	1.790	High k <sub>i</sub>
Si(111)	3.135	No λ/2



### Detectors

### Gas Detectors

- $n + {}^{3}He \rightarrow {}^{3}H + p + 0.764 \text{ MeV}$
- Ionization of gas
- e<sup>-</sup> drift to high voltage anode
- High efficiency

Neutron Kinetic Energy [meV] 103 102 Conversion Efficiency [%] 60 <sup>3</sup>He (n, p) <sup>3</sup>H 20Detector Depth = 1.5 cm 10 6 8 Neutron Wavelength [Å]

- Beam monitors
- Low efficiency detectors for measuring beam flux

# Resolution

### • Resolution ellipsoid

- Beam divergences
- Collimations/distances
- Crystal mosaics/sizes/angles



Resolution convolutions

 $\mathbf{I}(\mathbf{Q}_0,\omega_0) = \int \mathbf{S}(\mathbf{Q}_0,\omega_0) R(\mathbf{Q}-\mathbf{Q}_0,\omega-\omega_0) d\mathbf{Q} d\omega$ 



# **Resolution focusing**

- Optimizing peak intensity
- Match slope of resolution to dispersion



### Neutrons have mass so higher energy means faster – lower energy means slower



We can measure a neutron's energy, wavelength by measuring its speed



# Time-of-flight methods



Spallation neutron source



Pharos – Lujan Center

Effectively utilizes time structure of pulsed neutron groups

$$t = \frac{d}{v} = \left(\frac{m}{h}d\right)\lambda$$







A single (disk) chopper pulses the neutron beam. A second chopper selects neutrons within a narrow range of speeds.



Counter-rotating choppers (close together), with speed  $\bullet$ , behave like single choppers with speed 2 $\bullet$ . They can also permit a choice of pulse widths.

Additional choppers remove "contaminant" wavelengths and reduce the pulse frequency at the sample position.

### The DCS has seven choppers, 4 of which have 3 "slots"



### Disk 4B





# Fermi Choppers

- Body radius ~ 5 cm
- Curved absorbing slats
  - B or Gd coated
  - ~mm slit size
- f = 600 Hz (max)
- Acts like shutter,  $\Delta t \approx \mu s$





Figure 1. ISIS MAPS chopper and slit package assembly - exploded view





### T-zero chopper

- Background suppression
- Blocks fast neutron flash







# Position sensitive detectors

- <sup>3</sup>He tubes (usu. 1 meter)
- Charge division
- Position resolution ~ cm
- Time resolution ~ 10 ns





#### MAPS detector bank





# Sample environment

- Temperature, field, pressure
- Heavy duty for large sample environment
  - CCR
  - He cryostats
  - SC magnets





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### 

# Guides

- Transport beam over long distances
- Background reduction
- Total external reflection
  - Ni coated glass
  - Ni/Ti multilayers (supermirror)





# Size matters



### • Length = resolution

- Instruments ~ 20 40 m long
- E-resolution ~ 2-4%  $E_i$
- More detectors
  - SEQUOIA 1600 tubes, 144000 pixels
  - Solid angle coverage 1.6 steradians
- Huge data sets
- 0.1 1 GB



SEQUOIA detector vacuum vessel





# Data visualization

- Large, complex data from spallation sources
- Measure  $S(\mathbf{Q}, \omega) 4D$  function



Ye et al., Phys. Rev. B, 75 144408 (2007).



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# Order



The appearance of spin waves indicates that the field-induced state is long range ordered

#### Weak magnetic field // [110] induces LRO:

appearance of long-lived spin waves at low T and moderate H





# References

#### **General neutron scattering**

G. Squires, "Intro to theory of thermal neutron scattering", Dover, 1978. S. Lovesey, "Theory of neutron scattering from condensed matter", Oxford, 1984. R. Pynn, http://www.mrl.ucsb.edu/~pynn/.

#### **Polarized neutron scattering**

Moon, Koehler, Riste, Phys. Rev 181, 920 (1969).

#### **Triple-axis techniques**

Shirane, Shapiro, Tranquada, "Neutron scattering with a triple-axis spectrometer", Cambridge, 2002.

#### **Time-of-flight techniques**

B. Fultz, http://www.cacr.caltech.edu/projects/danse/ARCS\_Book\_16x.pdf

Why map reciprocal space?

Crystallography! Variation of Bragg peak intensities = where the atoms are





# Kinematics

- Essentially elastic scattering
- No kinematic limits

$$Q \approx 2k_i \sin \theta$$

$$h = h c (k_i - k_f)$$



# TOF vs. 3-axis

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- epithermal (up to 2 eV)
- Total spectra (esp. powder samples)
- Absolute normalization
- Low-dimensional systems
- Hardware inflexible
- Software intensive



- High flux of thermal neutrons
- Focused studies in Q,ω (soft modes, gaps, etc.)
- Three-dimensional systems
- Hardware intensive
- Software inflexible





### IXS vs. INS

• SAMPLE SIZE



- Simple scattering geometry (k<sub>i</sub>≈k<sub>f</sub>)
- Resolution function simpler (most angles fixed, E-scans only)
- No spurions
   (high-order refs. keV, no incoherent scat.)
- Can only do lattice excitations





# Computation







Fe<sub>3</sub>O<sub>4</sub> spin waves





### **Incident Beam:**

- monochromatic
- "white"
- "pink"

### **Scattered Beam:**

- Resolve its energy
- Don't resolve its energy
- Filter its energy


## Absolute normalization

- Absolute normalization
  - Using incoherent scattering from vanadium
  - $-\sigma/4\pi = 404 \text{ mbarns/Sr}$



Fermi's Golden Rule within the 1<sup>st</sup> Born Approximation

$$W = 2\pi / h | < f | V | i > |^{2} \rho (E_{f})$$
  
$$\delta\sigma = W / \Phi = (m/2\pi h^{2})^{2} k_{f} / k_{i} | < f | V | i > |^{2} \delta\Omega$$

$$\begin{split} \delta^{2}\sigma / \delta\Omega \, \delta \mathsf{E}_{\mathsf{f}} &= \, \mathsf{k}_{\mathsf{f}}/\mathsf{k}_{\mathsf{i}} \, \, \sigma_{\mathsf{coh}}/4\pi \, \, \mathsf{N} \, \, \mathsf{S}_{\mathsf{coh}}(\mathbf{Q}, \, \omega) \\ &+ \, \mathsf{k}_{\mathsf{f}}/\mathsf{k}_{\mathsf{i}} \, \sigma_{\mathsf{incoh}}/4\pi \, \, \mathsf{N} \, \, \mathsf{S}_{\mathsf{incoh}}(\mathbf{Q}, \, \omega) \end{split}$$



## Spin correlation functions





The cross-section is proportional to the magnetic susceptibility, i.e. it is the response of the system to spatially & time varying magnetic field



## Paramagnetic scattering

$$\left\langle S_{j}^{\alpha}S_{j'}^{\beta}\right\rangle = 0 \left(j \neq j'\right)$$

Single ion scattering

$$\left\langle S_{j}^{z}(0)S_{j}^{z}(t)\right\rangle = \left\langle \left(S_{j}^{z}\right)^{2}\right\rangle e^{-\Gamma t} = \frac{1}{3}\left\langle \left(S_{j}\right)^{2}\right\rangle e^{-\Gamma t} = \frac{1}{3}S\left(S+1\right)e^{-\Gamma t}$$

$$\frac{\operatorname{Im}\left\{\chi^{zz}(0,\omega)\right\}}{\pi h\omega} = \frac{g^2 S(S+1)\mu_B^2}{3k_B T} \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (h\omega)^2}$$

- Inverse width,  $1/\Gamma$ , gives relaxation time
- Note crystal field excitation

$$\chi_0 = \int_{-\infty}^{\infty} \frac{\operatorname{Im}\left\{\chi^{zz}(0,\omega)\right\}}{\pi \hbar \omega} d\omega = \frac{g^2 S(S+1)\mu_B^2}{3k_B T}$$



McQueeney et al., Phil. Mag. B 81, 675 (2001).



### Scattering experiments



Momentum Transfer (Å<sup>-1</sup>)



### **Triple-axis instruments**



High flux isotope reactor - ORNL



HB-1A 3-axis spectrometer



- Hardware flexibility
- Constant-Q (or E) scans
  - Ideally suited for single-xtals



## Time-of-flight methods



Spallation neutron source



Pharos – Lujan Center

- Hardware inflexible
- Effective for powders
- Complicated **Q**,E-scans a challenge for single-xtals

Detector

Sample

Time

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Fermi chopper





## INS data

- Intensities as a function of  ${\bm Q}$  and  $\omega$ 









# Other triple-axis stuff

0.9

0.8

transmission 70.0

0.3

0.5

1 1.5 2 2.5 3 3.5 4 4.5 wavelength(Å)

#### • Soller Collimators

- Define beam divergence
- Q, $\omega$  resolution function

#### • Filters

- Xtal Sapphire: fast neutron background
- Poly Be: low-energy (5 meV) band pass
- PG: higher order contamination

#### Masks

- Beam definition
- Background reduction



# PG filter



#### Magic numbers

- Best filter for rejection of  $\lambda/2$  contamination
- $E_{f} = 13.7, 14.7, 30.5, 41 \text{ meV}$





### **Reciprocal space**



## Spurions



- Bragg incoherent Bragg
  - Eg.  $k_i 2k_f$ 
    - ħ $\omega$  = 41.1 meV
    - E<sub>f</sub> = 13.7 meV
      E<sub>i</sub> = 54.8 meV
      4E<sub>f</sub> = 54.8 meV
    - Incoherent elastic scattering visible from analyzer  $\lambda/2$
- incoherent Bragg Bragg
  - Sample 2 $\theta$  in Bragg condition for  $k_f\text{-}k_f$
  - Even for inelastic config, weak incoherent from mono





### **Resolution effects**





#### Introduction to Inelastic Neutron Scattering

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Brockhouse Institute for Materials Research



### Lattice





Cubic lattice