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**Office of
Science**

U.S. DEPARTMENT OF ENERGY

A U.S. Department of Energy laboratory
managed by The University of Chicago

Synchrotron Radiation Properties and Production

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Scientific User Facilities

National School for Neutron and X-ray Scattering

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X-rays Were Discovered in 1895



- X-rays were discovered (accidentally) in 1895 by Wilhelm Konrad Roentgen.
- Roentgen won the first Nobel Prize in 1901 “for the discovery with which his names is linked for all time: the... so-called Roentgen rays, as he himself called them, X-rays...”



X-rays Are A Part of Our Culture....



"Guaranteed since 1891." At that time, the company must have used a different name since x-rays weren't discovered until 1895.

RATDOG AFTER-PARTY!
WITH **XRAYGUN**
WINESKIN & THE JOHN GINTY BAND*
*WITH MEMBERS OF RATDOG SITTING-IN
FRI, APRIL 7
ACE OF CLUBS
www.aceofclubsnyc.com
9 GREAT JONES
NYC
9PM SHARP
FOR TICKETS:
www.ticketweb.com

XRAYGUN
www.myspace.com/xraygun

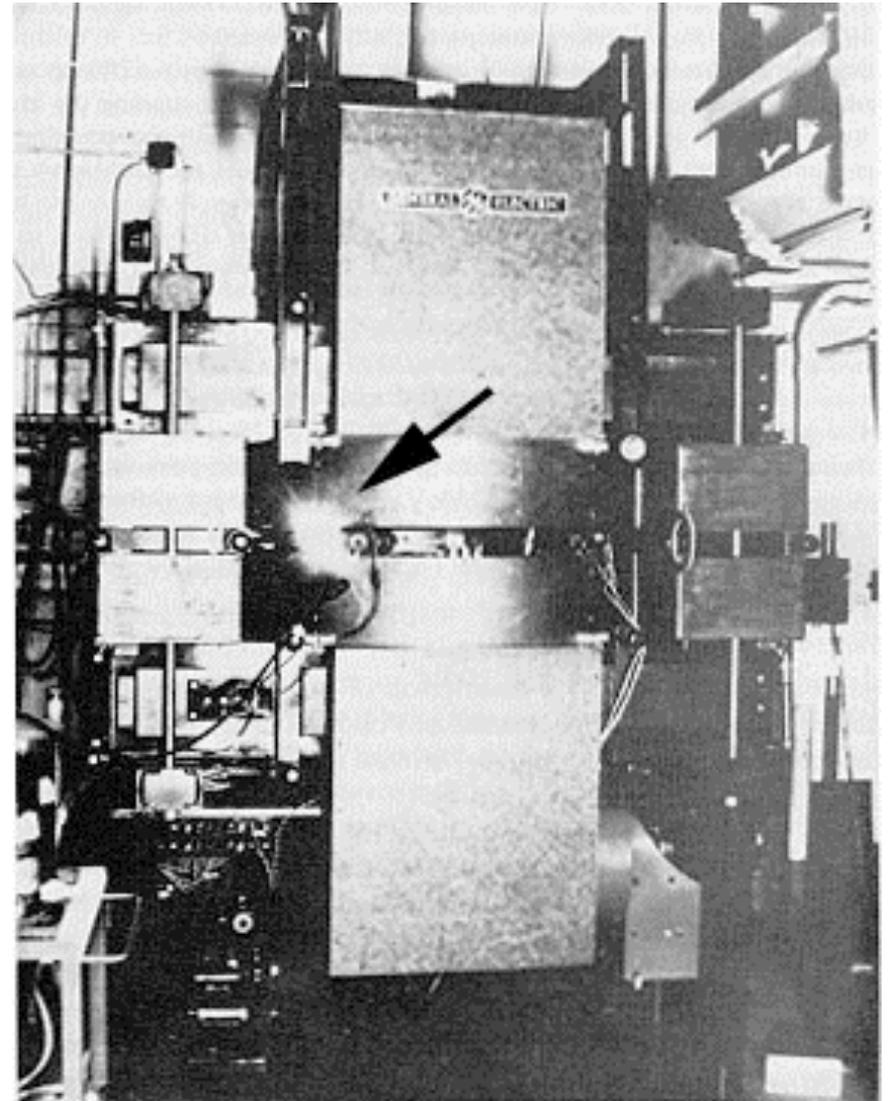


Synchrotron Radiation - Some Background

Synchrotron radiation (SR) - radiation from charged particles traveling in circular orbits - was first observed (accidentally) from a 70 MeV synchrotron 1947.

On April 24, [1947] Langmuir and I [Herbert Pollack] were running the machine and as usual were trying to push the electron gun and its associated pulse transformer to the limit. Some intermittent sparking had occurred and we asked the technician to observe with a mirror around the protective concrete wall. He immediately signaled to turn off the synchrotron as "he saw an arc in the tube." The vacuum was still excellent, so Langmuir and I came to the end of the wall and observed. At first we thought it might be due to Cerenkov radiation, but it soon became clearer that we were seeing Ivanenko and Pomeranchuk [i.e., synchrotron] radiation.

Excerpted from Handbook on Synchrotron Radiation, Volume 1a, Ernst-Eckhard Koch, Ed., North Holland, 1983.



From Synchrotrons to Storage Rings

THE REVIEW OF SCIENTIFIC INSTRUMENTS

Letters to the Editor

Prompt publication of brief reports of NEW ideas in measurement and instrumentation or comments on papers appearing in this Journal may be secured by addressing them to this department. No proof will be sent to the authors. Communications should not exceed 500 words in length. The Board of Editors does not hold itself responsible for the opinions expressed by the correspondents.

Use of Synchrotron Orbit-Radiation in X-Ray Physics*

L. G. PARRATT

Cornell University, Ithaca, New York

(Received December 1, 1958)

IT is well known by now that intense x-radiation is emitted by the centripetally accelerated electrons in the orbit of a high-energy synchrotron.¹ Comparison is made here of the prospective usefulness of this radiation in the range of wavelengths 0.1 to 20 Å with the x-rays obtainable from a conventional x-ray tube.

I. Calculations of the spectral distribution for three typical synchrotron energies and orbit radii are shown in Fig. 1.^{2,3} The spectral power is shown averaged over one-half of an acceleration cycle in which the electron energy is taken as proportional to the square of a sinusoid.⁴ The power is expressed in ergs per second per 4π solid angle per x unit per electron, and must be trimmed

- Synchrotrons were first used as sources of SR. However, the particles' constantly changing energy was not attractive and the advent of storage rings provided a far more attractive source.
- We now use the name synchrotron radiation to describe radiation that is emitted from charged particles traveling at relativistic speeds, regardless of the accelerating mechanism and shape of the trajectory.
- Although synchrotron radiation can cover the entire electromagnetic spectrum, we are interested in radiation in the x-ray regime.

$$(\lambda[\text{Å}] = 12.4 / E[\text{keV}])$$

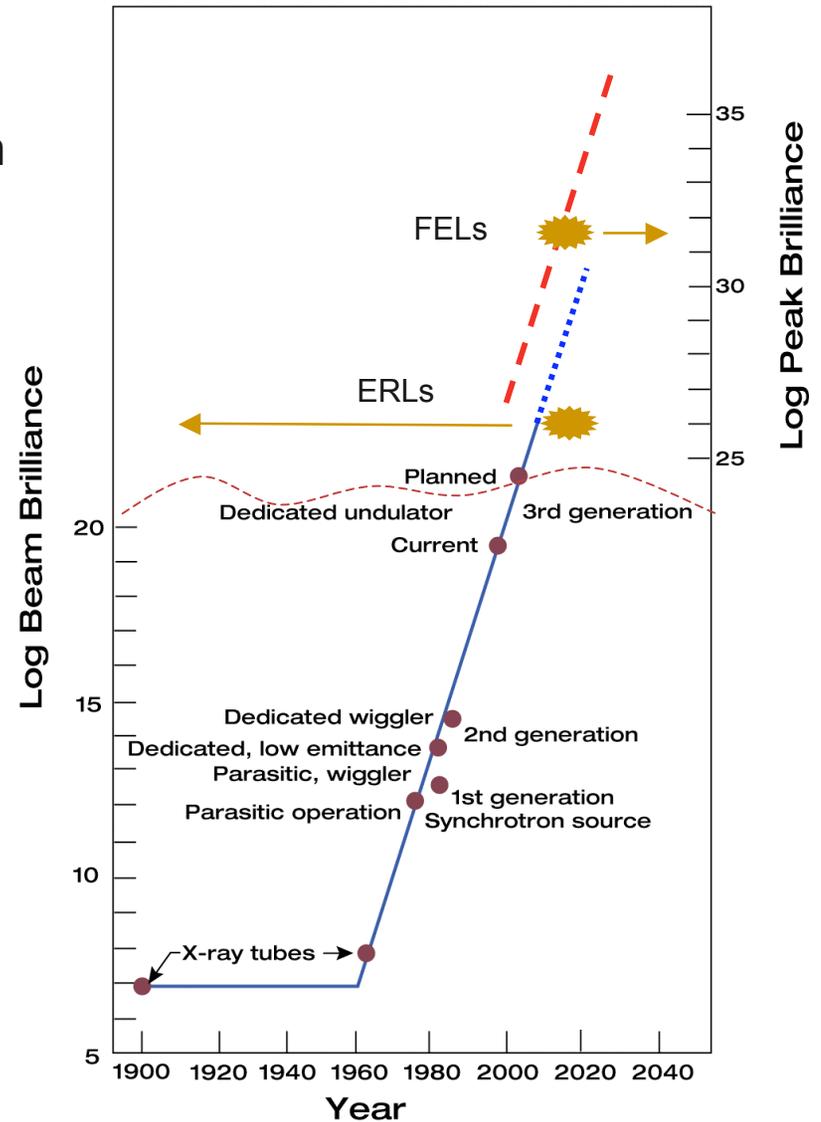
Square brackets indicate the units to be used in the calculation.

The Evolution of Synchrotron Radiation Sources

Synchrotron radiation (from VUV to X-ray and now even IR) has been used a research tool for nearly 50 years.

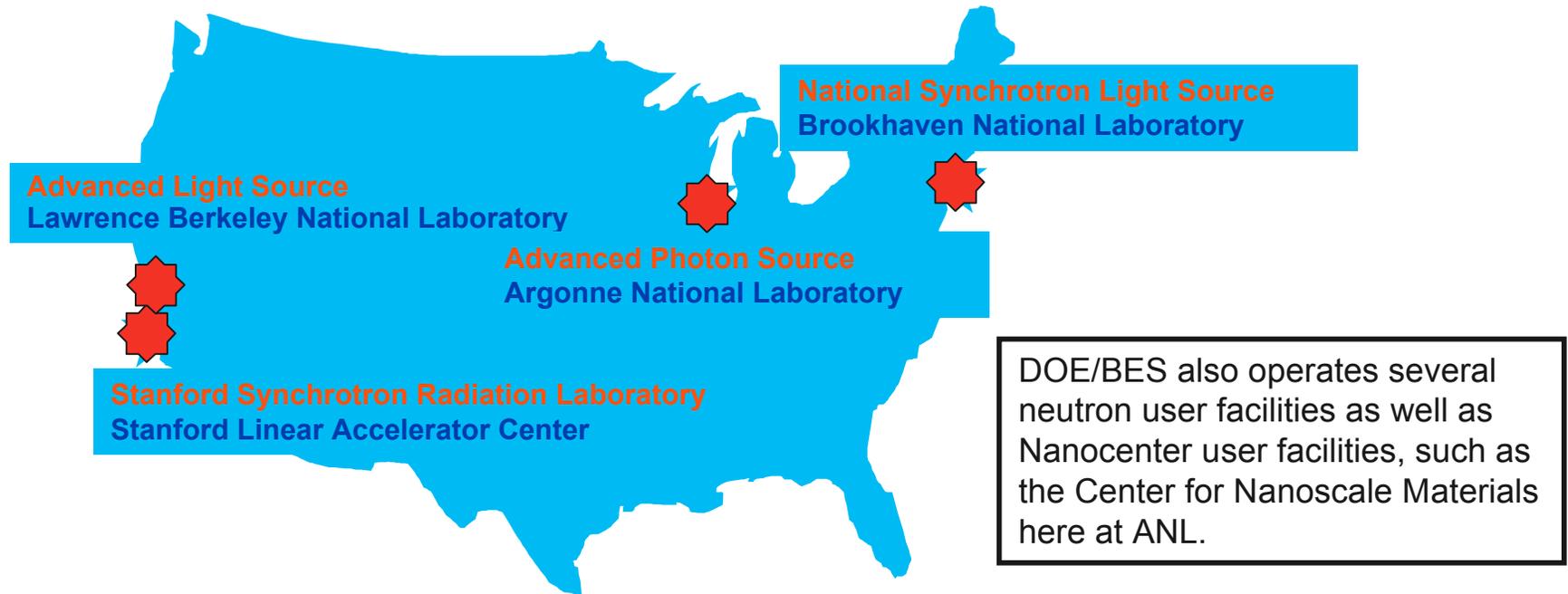
- 1st Generation Sources
Ran parasitically on accelerations for high energy physics (CHESS)
- 2nd Generation Sources
Built to optimize synchrotron radiation from the bending magnets (NSLS)
- 3rd Generation Sources
Built to optimize synchrotron radiation from insertion devices (APS)
- 4th Generation Sources (not covered here)
Energy Recovery Linacs (ERLs)
X-ray Free Electron Lasers (X-FELs)

History of (8-keV) X-Ray Sources



A Vital National Resource for Science and Technology

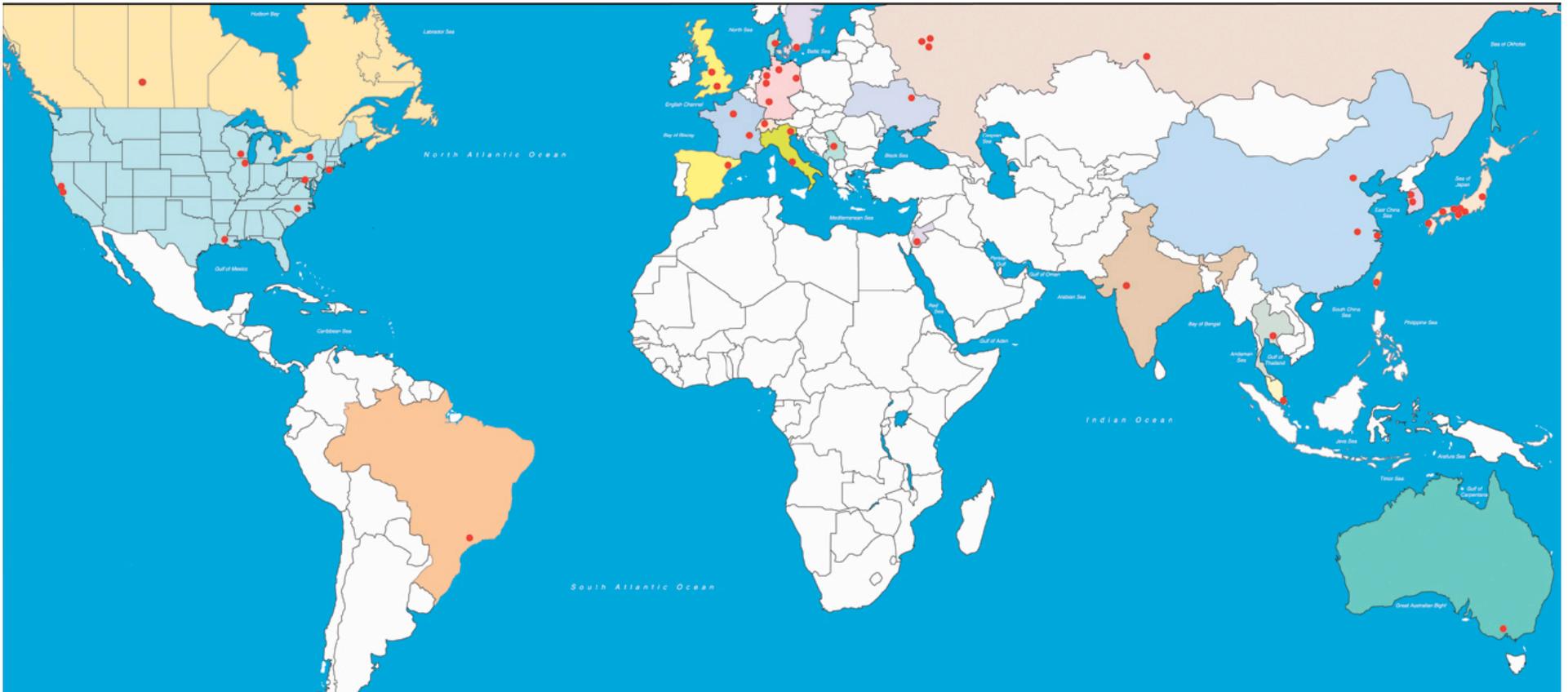
The **Advanced Photon Source (APS)** is a fully optimized, insertion-device-based, third generation x-ray source for the production of high intensity (brightness) x-ray beams.



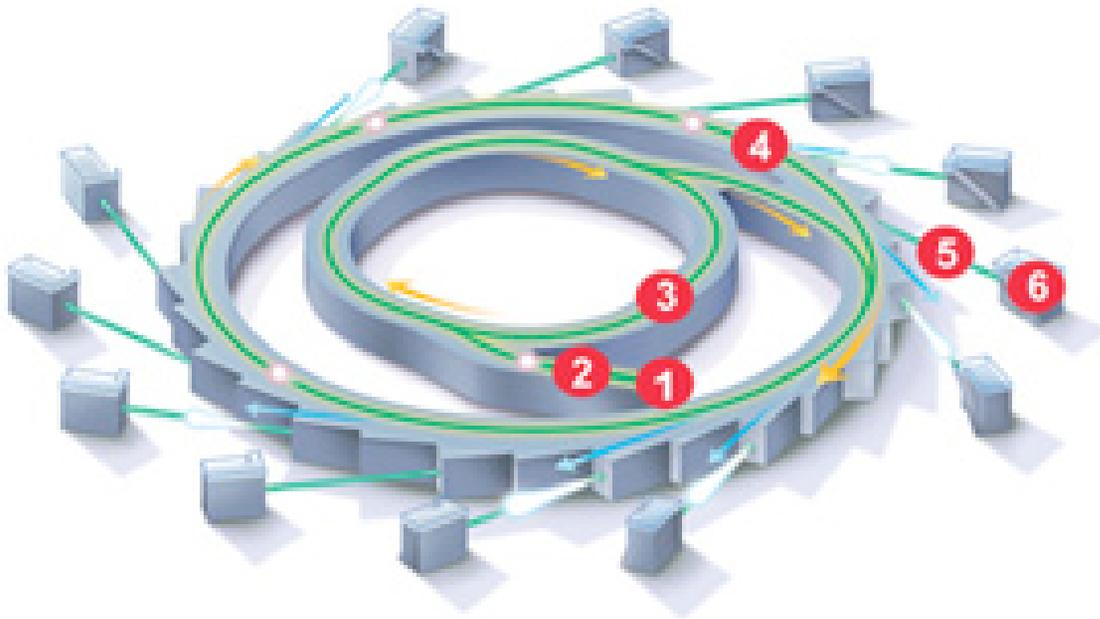
APS is funded by the Department of Energy Office of Basic Energy Science (DOE/BES).

Over **3500 unique researchers** from around the world came to Argonne last year to perform experiments at the APS.

Synchrotron Radiation Facilities Around the World



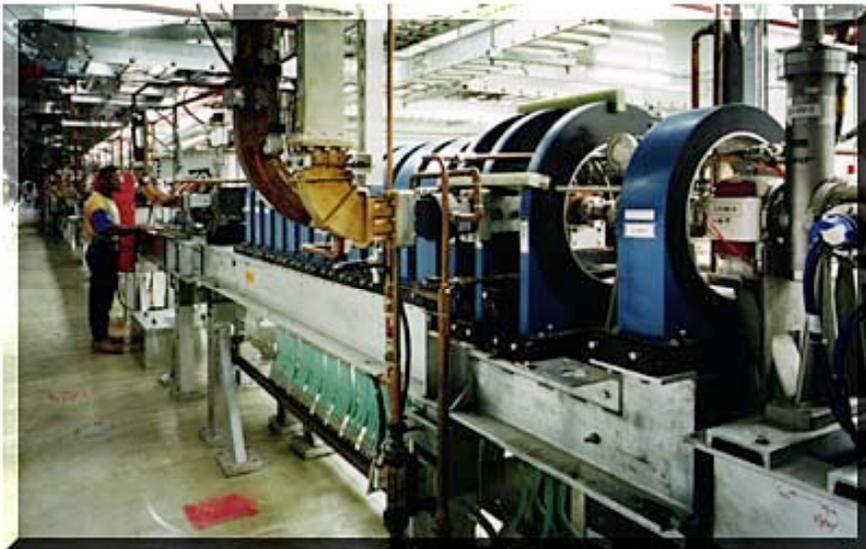
Typical SR Facility Complex



1. Electron gun
2. Linear Accelerator LINAC
3. Booster Synchrotron
4. Storage Ring (SR)
5. Beamline
6. Experiment station

(Courtesy: Australian Synchrotron,
Illustrator: Michael Payne)

APS Linear Accelerator and Booster

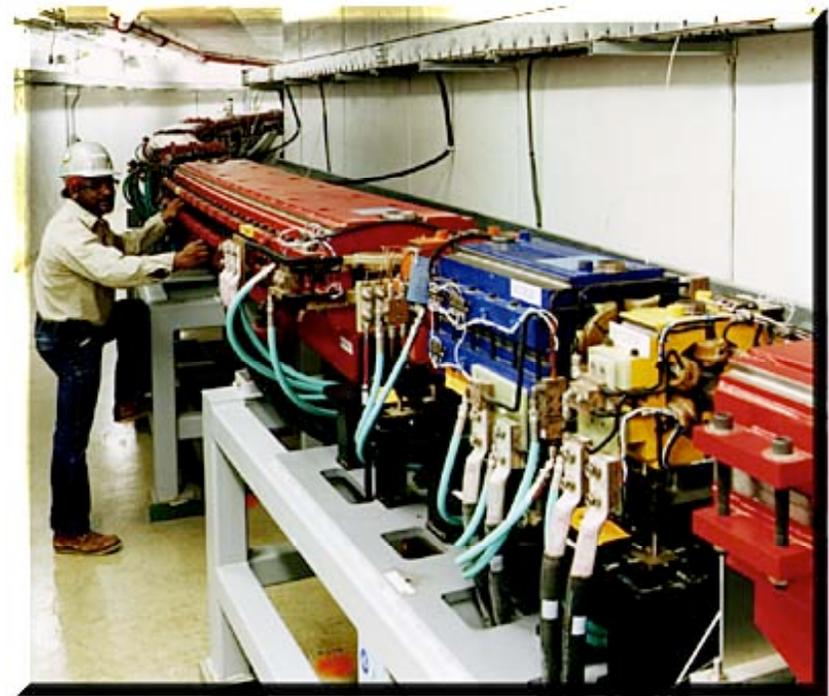


Booster

- Accelerates electrons from 450 MeV to 7 GeV
- Operates at 2 Hz
- 368 m circumference

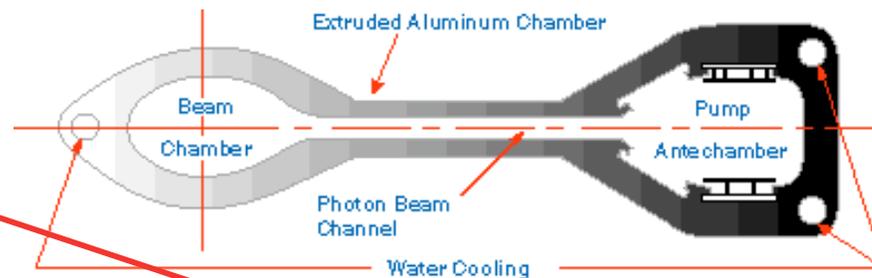
LINAC

- Accelerates electrons from 0 to 450 MeV
- Operates at 48 Hz
- 50 meters in length

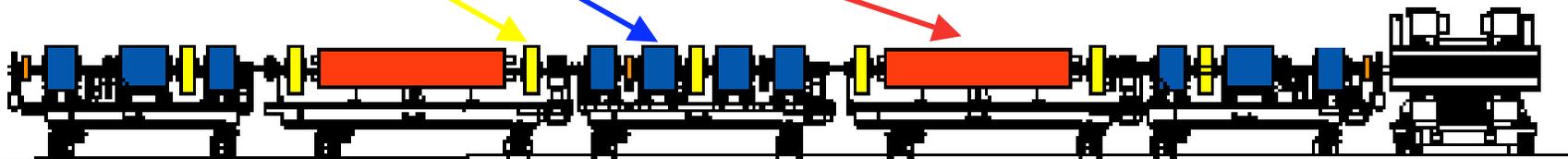


Storage Ring

- Present operation:
 - 7 GeV and 100 mA
 - lifetimes > 12 hrs
 - filling time 1.1 minutes
 - bunch length 70 psec
- 40 straight sections each 5 meters long
 - 5 for accelerator purposes
 - 35 available as radiation sources
- 1104 m circumference (3.68 microsecond period)
 - 80 dipoles
 - 240 quads
 - 20 skew quads
 - 280 sextupoles



Cross-section of the Al storage ring vacuum chamber



The Advanced Photon Source Accelerator Complex



Radiated Power from Charges at Relativistic Velocities

The classical formula for the radiated power from an accelerated electron is:

$$P = \frac{2e^2}{3c^3} a^2$$

Where P is the power and a the acceleration. For a circular orbit of radius r , in the non-relativistic case, a is just the centripetal acceleration, v^2/r . In the relativistic case:

$$a = \frac{1}{m_o} \frac{dp}{d\tau} = \frac{1}{m_o} \gamma \frac{d\gamma m_o v}{dt} = \gamma^2 \frac{dv}{dt} = \gamma^2 \frac{v^2}{r}$$

Where $\tau = t/\gamma =$ proper time, $\gamma = 1/\sqrt{1-\beta^2} = E/m_o c^2$ and $\beta = v/c$

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

Boxes with lines like this indicate an important equation.

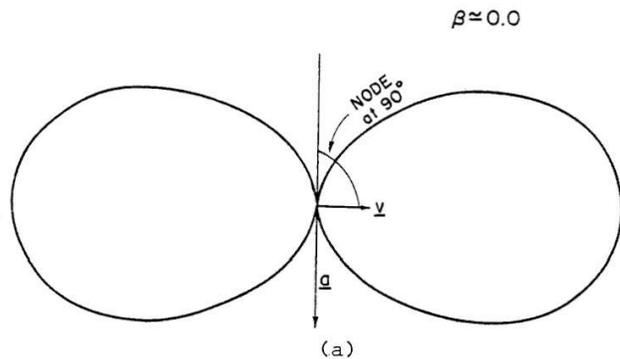
Dependence on Mass and Energy of Radiated Power

$$P = \frac{2e^2}{3c^3} \frac{\gamma^4 v^4}{r^2} = \frac{2ce^2}{3r^2} \frac{E^4}{m_o^4 c^8}$$

There are two points about this equation for total radiated power:

1. Scales inversely with the mass of the particle to the 4th power (protons radiate considerably less than an e⁻ with the same total energy, E.)
2. Scales with the 4th power of the particle's energy (a 7 GeV storage ring radiates 2400 times more power than a 1 GeV ring with the same radius)

Radiation Patterns When $v \ll c$



When $v \ll c$, ($\beta \approx 0$), the shape of the radiation pattern is a classical dipole pattern.

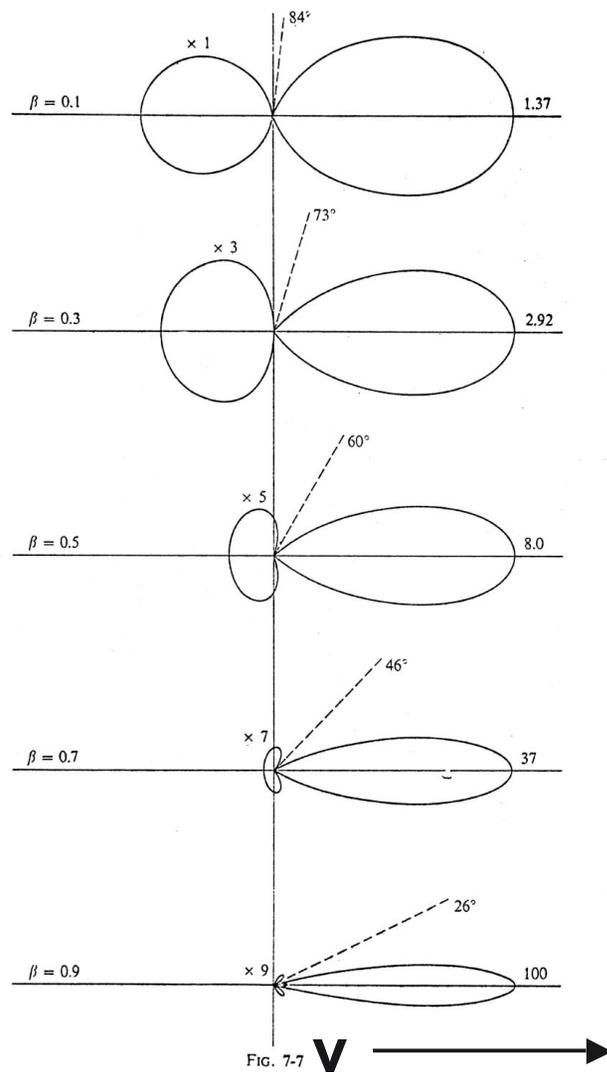
Recall that special relativity says that angles transform as:

$$\tan \theta_{lab} = \frac{\sin \theta'}{\gamma(\cos \theta' + \beta)}$$

And so as you crank up β , the radiation pattern begins to deform (in the lab frame).

At the APS with $E = 7 \text{ GeV}$,
 $\gamma = E/m_0c^2 = 7 \text{ GeV}/0.511 \text{ MeV}$
 $\gamma = 1.4 \times 10^4$
 $1/\gamma = 73 \times 10^{-6}$

Radiation Patterns When V Approaches C



So as β approaches 1 :

- The shape of the radiation pattern is changing; it is more forward directed
- The size of the radiation pattern is changing; it is getting bigger

So at $\beta \approx 1$, the node at $\theta' = 90^\circ$ (in the frame of the radiating particle) transforms to:

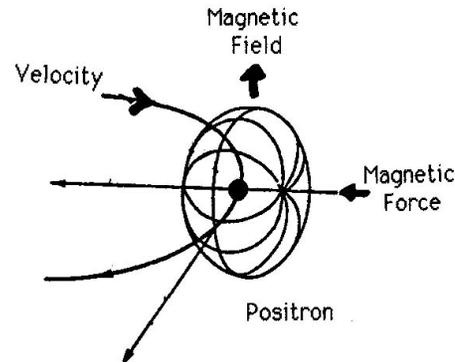
$$\tan \theta_{lab} = \frac{\sin \theta'}{\gamma(\cos \theta' + \beta)} = \frac{1}{\gamma\beta} \approx \frac{1}{\gamma}$$

Radiation from Highly-Relativistic Particles

RADIATION FROM CHARGED PARTICLE (CONSTANT B-FIELD)

NON-RELATIVISTIC CASE: $v/c \ll 1$

Dipole Radiation



In fact, the opening angle in both the horizontal and vertical directions, is given approximately by:

$$\theta = 1/\gamma,$$

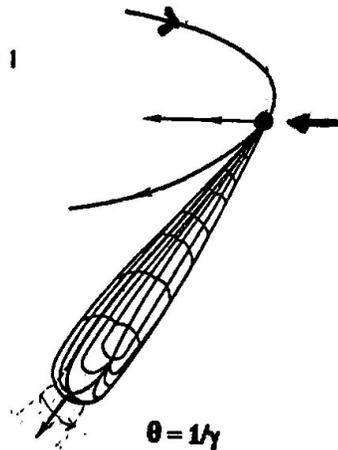
when $b \approx 1$.

HIGHLY RELATIVISTIC CASE: $v/c = \beta \approx 1$

$$\gamma = E/m_0c^2$$

$$m_0c^2 = 0.511 \text{ MeV}$$

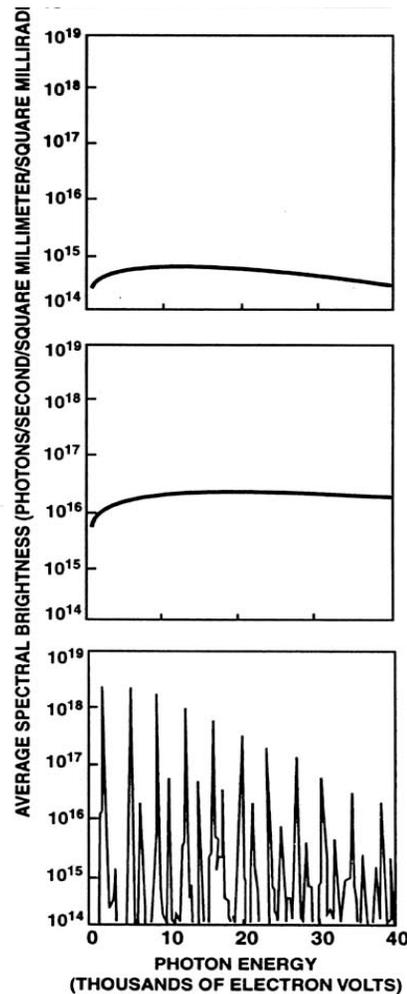
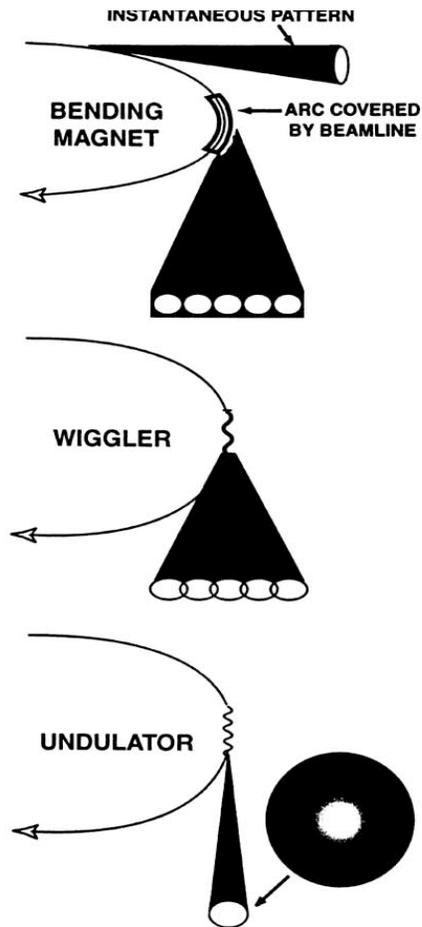
E(GeV)	γ	$\theta(\mu\text{rad})$
1	1957	511
3	5871	170
7	13699	73



Relativistic velocities are good!!

- radiation forward directed
- radiated power $\propto E^4$

Radiation Sources at 3rd Generation Facilities



There are two different sources of radiation at 3rd generation sources:

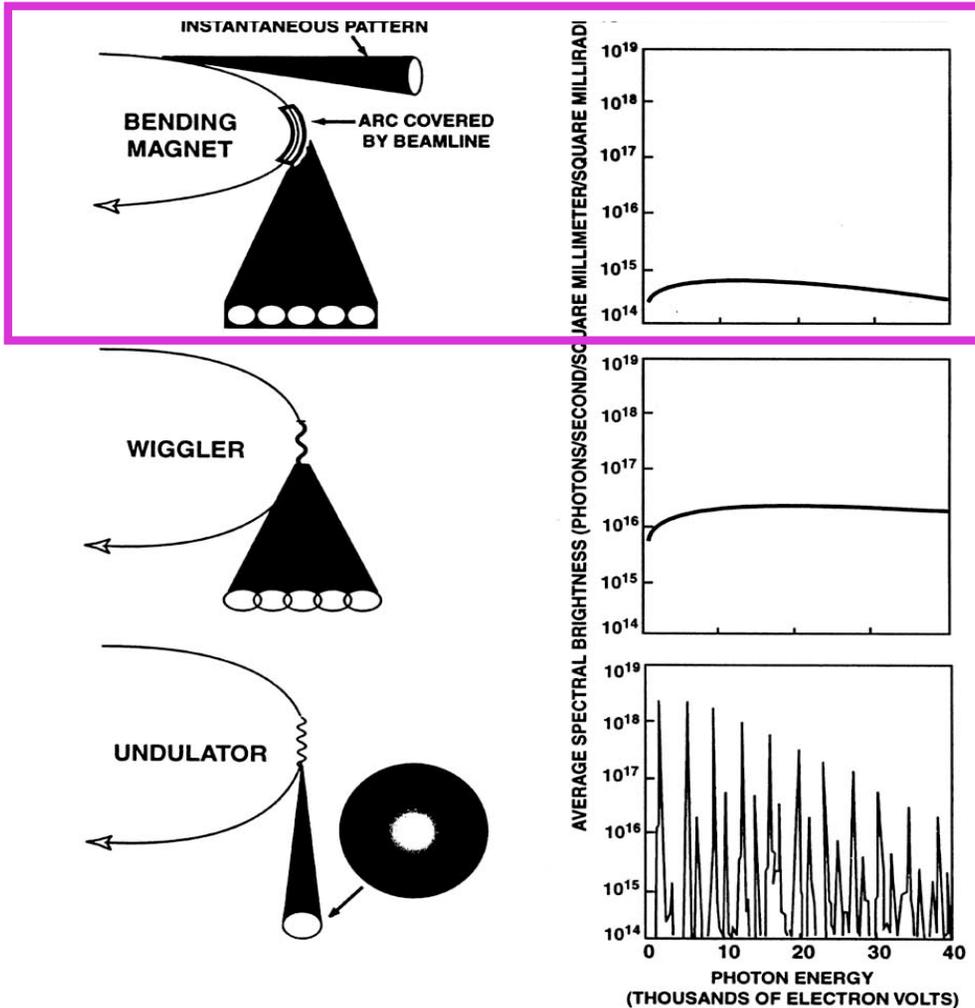
- bending magnets (BMs)
- insertion devices (IDs); periodic arrays of magnets inserted between the BMs (wigglers or undulators)

The important parameters to know about each one is:

- Spectral distribution
- Flux (number of x-rays/sec - 0.1%bw)
- Brightness (flux/source size-source divergence)
- Polarization (linear, circular)

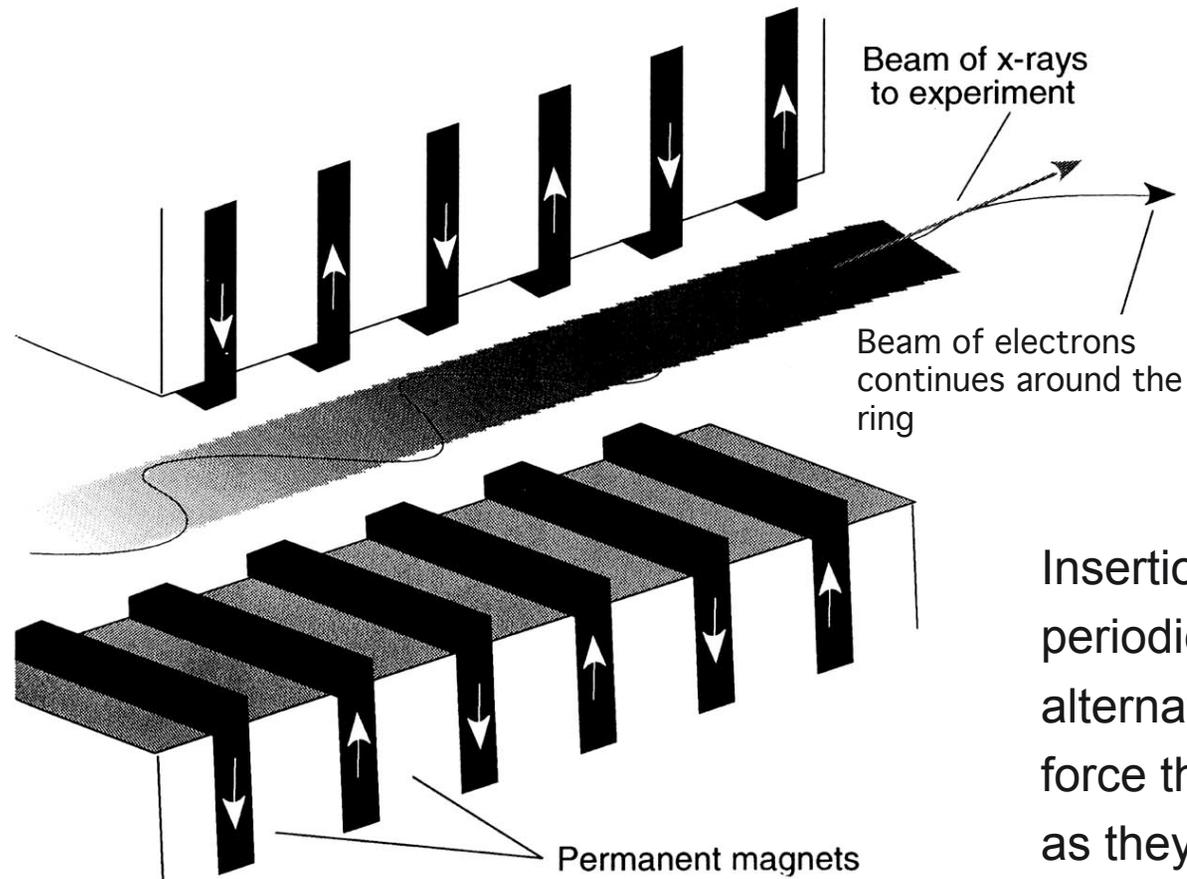
BM Spectral Distribution

Bend Magnet Radiation (see Appendix 1)



- Spectrum characterized by the critical energy:
 $E_c = 3hc\gamma^3/4\pi r.$
- Flux “typically” 10^{13} photons/sec/0.1% BW /mrad from 3rd generation source
- Vertical opening angle $1/\gamma$. For the APS:
 $1/\gamma = 73 \times 10^{-6}$ radians
- Horizontal opening angle determined by apertures
- In the plane of the orbit, the polarization is linear and parallel to the orbital plane.
- Out of the plane, there is a component perpendicular to the orbit and out of phase by $\pi/2$ with respect to the parallel component and so the off-axis beam is elliptically polarized.

Insertion Devices



Insertion devices (IDs) are periodic magnetic arrays with alternating field directions that force the particles to oscillate as they pass through the device.

Characterizing Insertion Devices

IDs are characterized by the so-called field index or deflection parameter, K (See Appendix 2):

$$K = eB_0\lambda_{ID}/2\pi m_0c = 0.0934 \lambda_{ID}[\text{cm}] B_0[\text{kG}]$$

where λ_{ID} is the period of the insertion device and B_0 the peak magnetic field. (The length of the ID, L , is equal to the number of periods, N , times the length of the period, i.e., $L = N\lambda_{ID}$.)

The maximum deflection angle of the particle beam, θ_{\max} , is given by:

$$\theta_{\max} = \pm(K/\gamma)$$

and the amplitude of the oscillation of the particles, x_{\max} , by:

$$x_{\max} = (K/\gamma)(\lambda_{ID}/2\pi)$$

APS Undulator A has a period of 3.3 cm and operates with $K \approx 1$, therefore:

$$\theta_{\max} \approx 1/\gamma \quad \text{and} \quad x_{\max} \approx 0.38 \text{ microns.}$$

Wigglers and Undulators

Wigglers:

- $K \gg 1$
- $\theta_{\max} = (K/\gamma) \gg 1/\gamma$, i.e. the angular deflection of the particle beam is much greater than the natural opening angle of the radiation ($1/\gamma$).
- radiation spectrum looks like $2N$ dipole sources (N = number of periods)

Undulators:

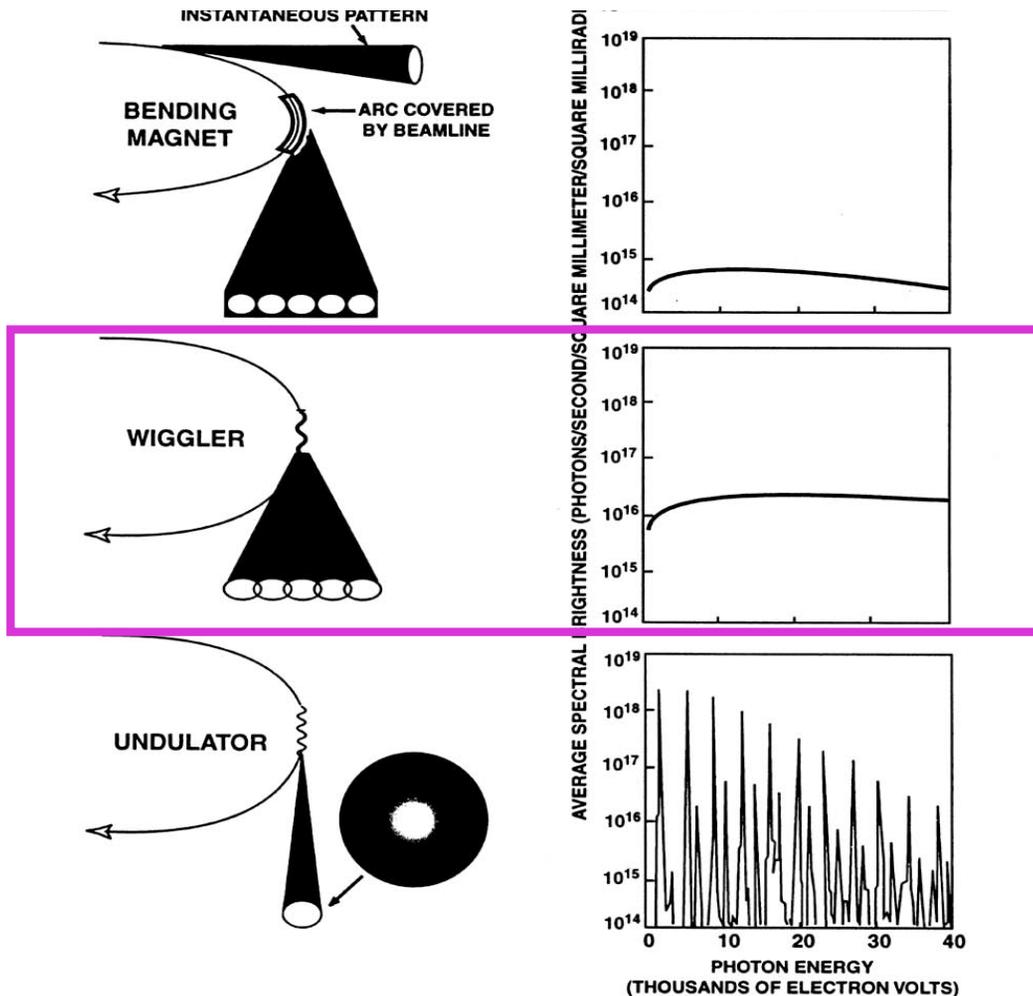
- $K \approx 1$
- θ_{\max} is comparable to the natural opening angle of the radiation ($1/\gamma$) and so the radiation from each pole overlaps causing interference effects in the spectral distribution.
- radiation spectrum does not look like $2N$ dipole sources

APS 2.4 m long Undulator A ($\lambda_{ID} = 3.3$ cm)



Wiggler Radiation Pattern and Spectrum

Wiggler Radiation
(see Appendix 3)



- like BM radiation where each pole is a “source”
- spectrum characterized by the critical energy (which may be different than BM critical energy)
- flux “typically” 10^{14} to 10^{15} photons/sec/0.1% BW/mrad
- vertical radiation opening angle $1/\gamma$ (73×10^{-6} radians for APS)
- Presently, there are NO planar wigglers installed at the APS. Wigglers with fields in both the x and y directions) produce elliptically polarized radiation. These are sometimes called elliptical multipole wigglers (EMWs).

Undulator Radiation

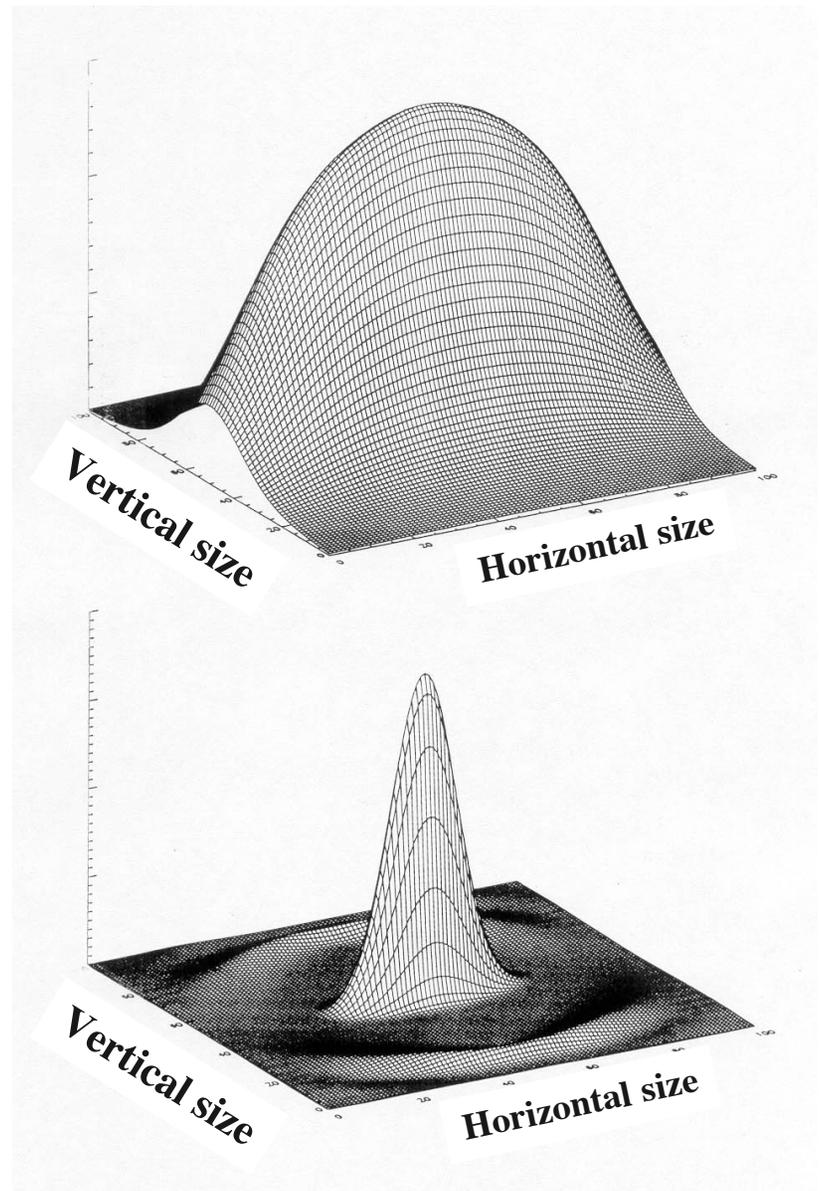
Undulator radiation is the coherent super-position of radiation from each pole of the undulator. Interference from different parts of the particle's trajectory in the undulator causes the radiation to be squeezed into discrete spectral lines and into a narrower emission angle.

Constructive interference occurs at wavelengths given by:

$$\lambda_n^{\text{x-ray}} = (\lambda_{\text{ID}}/2\gamma^2n)(1 + K^2/2 + \gamma^2\theta^2),$$

where n is the harmonic number. (Only odd harmonics are observed on axis due to symmetry arguments.)

This bundle of radiation from the odd harmonics in the center of the power envelope is called the "central cone".



The power profile of the beam from APS undulator A operating a $K = 2.25$ (first harmonic, $E_1 = 4$ keV) is shown at the top. The total power is 3927 watts and the peak power density is 150 W/mm^2 .

How Do You Get 1Å X-rays from a 3 cm Period Magnetic Field?

Where does the $1/\gamma^2$ come from in the equation: $\lambda_n^{\text{x-ray}} = (\lambda_{\text{ID}}/2\gamma^2 n)(1 + K^2/2)$?

1) Consider the electron in its rest frame:

- It does not see a static magnetic field from the undulator, but rather a time-varying B-field and associated E-field (due to the relativistic transformation of the magnetic field of the device).
- The period of the E and B field are **Lorentz contracted** so that: $\lambda_{\text{e-frame}} = \lambda_{\text{ID}} / \gamma$ and so the electron oscillates (and hence radiates) with that same period driven by the EM fields.

2) Back in the lab frame:

- Due to the fact that the electron is traveling towards us, the radiation emitted by the electron is Doppler shifted to higher frequencies (shorter wavelengths). The **relativistic Doppler shift** goes as $\sqrt{1-\beta} / \sqrt{1+\beta} \approx 1/2\gamma$, and so the wavelength observed in the lab is:

$$\lambda_{\text{lab}} \approx (\lambda_{\text{ID}}/\gamma)(1/2\gamma) = (\lambda_{\text{ID}}/2\gamma^2)$$

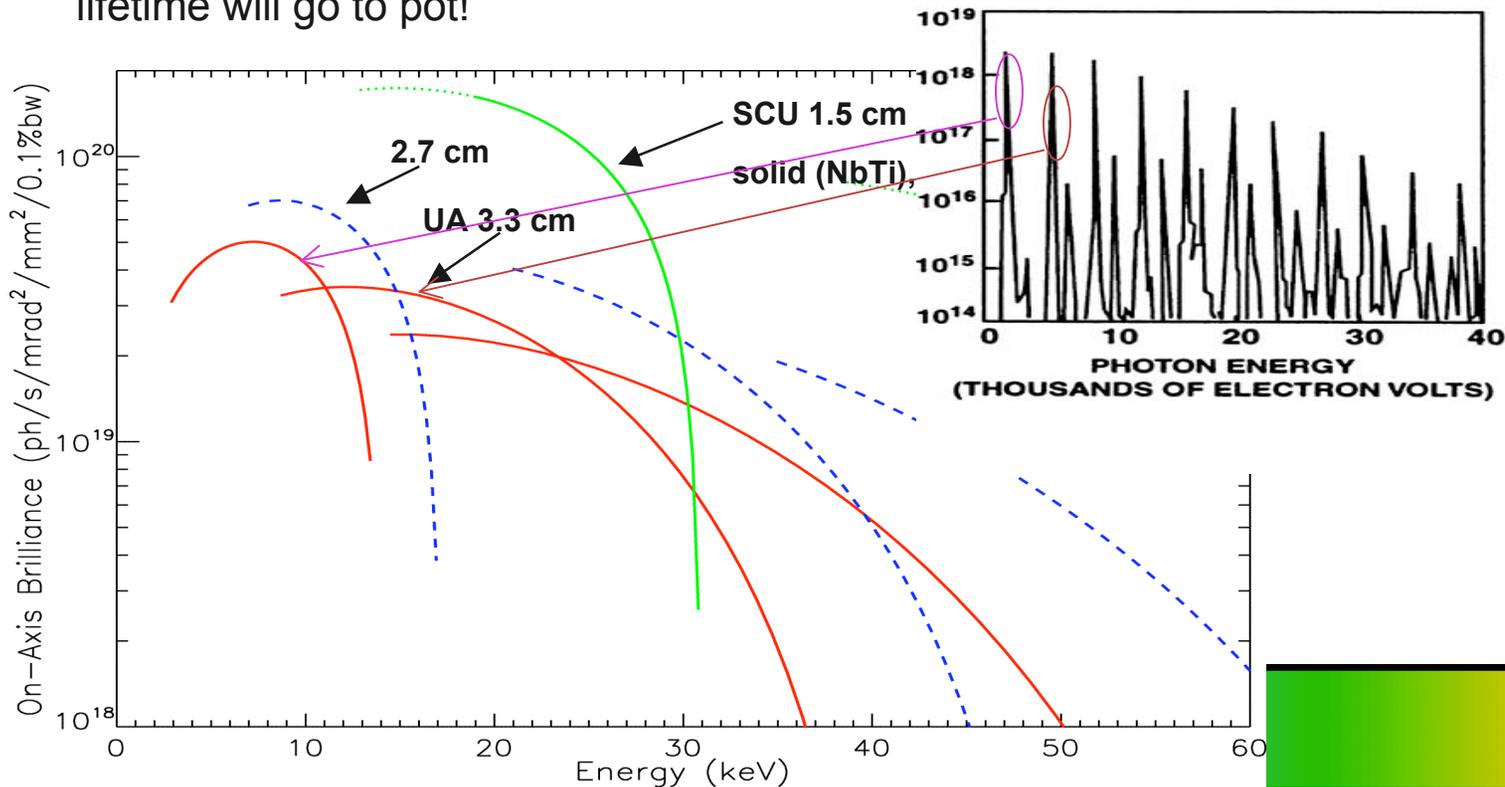
Tuning the Peaks of Undulator Radiation

On-axis ($\theta = 0$), we can write:

$$\lambda_n^{\text{x-ray}} = (\lambda_{\text{ID}}/2\gamma^2n)(1 + K^2/2)$$

$\lambda_n^{\text{x-ray}}$, can be adjusted by B, i.e. by varying K ($= 0.0934 \lambda_{\text{ID}}[\text{cm}] B_0[\text{kG}]$).

This can be achieved by varying the current in the windings of electromagnetic devices or by varying the separation between the upper and lower poles (“the gap”) in permanent magnet devices. You can’t reduce the gap too much since you will cut into the particle beam and lifetime will go to pot!

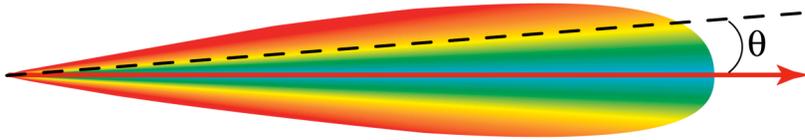


Undulator Radiation Energy Spread and Angular Distribution

The energy spread of the interference peak (central cone) is given by:

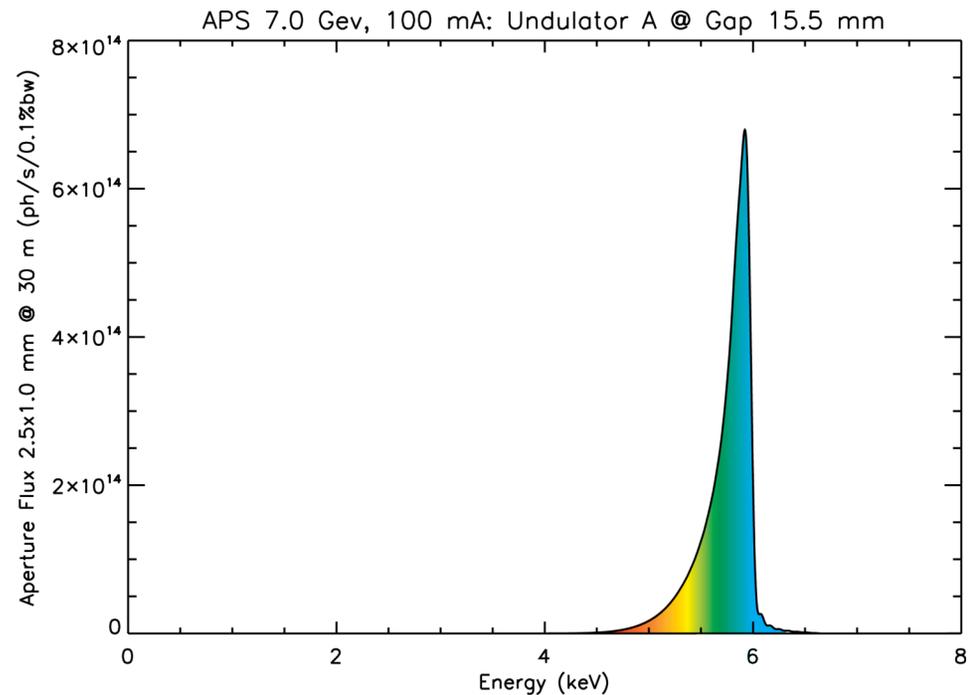
$$\Delta E/E = \Delta \lambda/\lambda \approx 1/nN \quad (\text{like a grating!}).$$

For a given K-value (gap), the wavelength at angle θ is $\lambda_1 = (\lambda_{ID}/2\gamma^2)(1 + K^2/2 + \gamma^2\theta_1^2)$



The central cone opening angle, θ_1 , for the odd harmonics is given by:

$$\theta_1/2 = (\lambda^{\text{x-ray}}/2L)^{1/2}$$

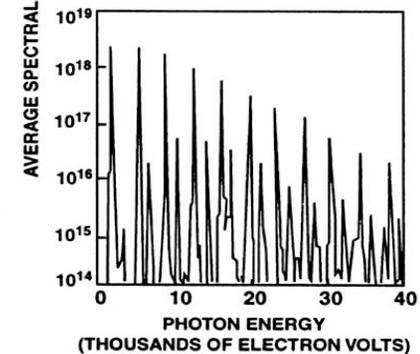
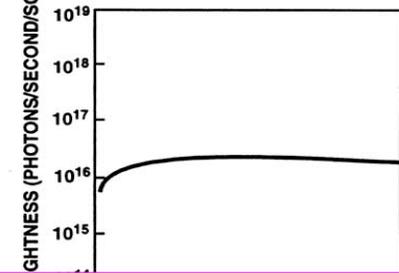
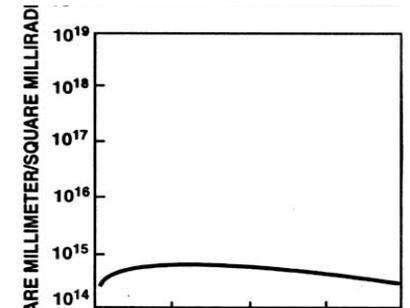
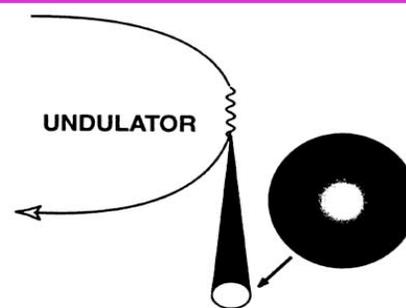
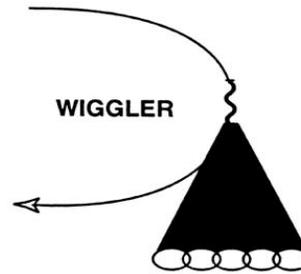
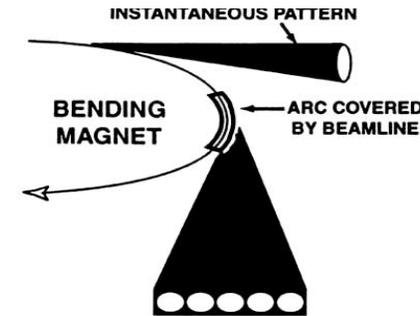


Undulator Radiation Patterns and Spectra

Undulator Radiation (see Appendix 4)

- undulators defined as IDs with horizontal deflection angle $\approx 1/\gamma$, i.e., $K \approx 1$
- spectrum peaked at x-ray specific x-ray energies, but peaks are tunable by varying K ($K = 0.94 B[T] \lambda_{ID}[cm]$)
- at the peaks (harmonics) the horizontal and vertical opening angles of the radiation is given by:

$$(\lambda_{x\text{-ray}} / 2L)^{1/2}$$
- to get the true opening angle, need to consider the opening angle of the emitting particles - more later on!

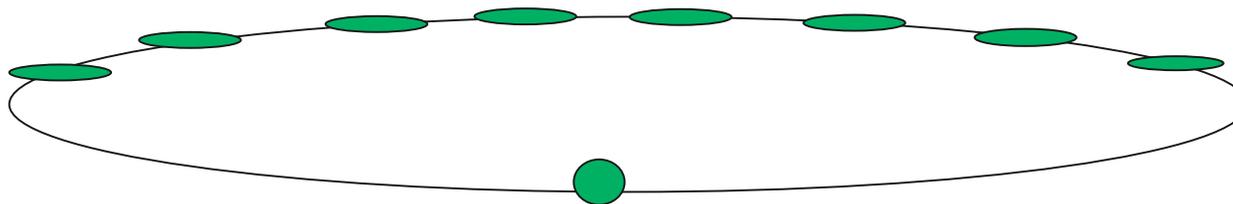


Time Structure of the Radiation

- Particles are grouped together by the action of the RF cavities into bunches.
At the APS:
 - natural bunch length (i.e. in the limit of zero current) 35 psec FWHM
 - typically about 100 psec FWHM
- 1104 m circumference (3.68 microsecond period)
 - harmonic number 1296 i.e. 1296 evenly spaced “RF buckets” around the ring
 - minimum spacing is 2.8 nsec
- Details of the time structure depends on the fill pattern, i.e. which buckets have electrons in them.

Typical APS Filling Patterns

- 324 equally spaced bunches
 - approximately 11 nsec between bunches
 - approximates a continuous source
- 24 equally spaced bunches
 - approximately 154 nsec between bunches
 - compromise between continuous source and pulsed source
- 1 + 7x8 (hybrid mode)
 - a single bunch followed by 8 groups of 7 bunches
 - timing experiments



Schematic of APS
hybrid mode

Transverse Properties of Particle Beams

- Up until now, we have calculated the radiation properties from a **single electron**.
- However in a storage ring, the radiation is emitted from an **ensemble of electrons** with some finite size and divergence distribution.
- For our needs, a particle in a storage ring is completely specified if we know where it is (x,y,z) and where it is going (p_x, p_y, p_z) . All this information can be represented by a point in a six-dimensional phase space with the coordinates of that space given above.
- Both the transverse and longitudinal properties of the particle beam in a storage ring are the **equilibrium** properties of the particle beam.

Transverse Properties and Photon Beam Brightness

Why do we need to know about the transverse particle beam properties?

- Although the flux from BM and ID sources can be determined without detailed knowledge of the source size and divergence, one very important characteristic of the beam, namely **brightness**, requires a more detailed knowledge of the particle beam's size and divergence.

Brightness has units of:

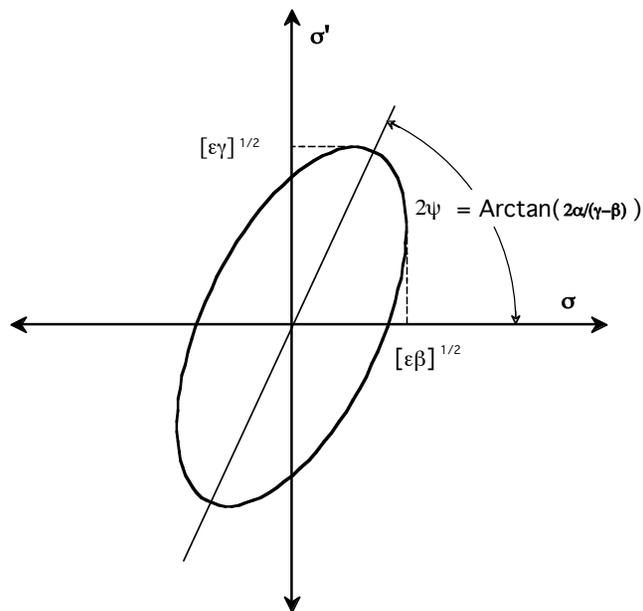
photons/sec/0.1% BW/source area/source solid angle

$$\text{Flux} / 4\pi^2 \Sigma_h \Sigma_v \Sigma_h' \Sigma_v'$$

where Σ_i (Σ_i') is the **effective** one sigma value of the source size (divergence) in the i^{th} direction. The effective source size and divergence has **contributions from both the particle beam and the radiation** itself.

Some Definitions

- A mono-energetic, charged particle, under the action of a linear force such as is found in a storage ring, **traces a contour in phase space that is an ellipse.**



In the above diagram, ϵ is the emittance and α , β , and γ describe the shape of the ellipse at any point in the storage ring. α , β , and γ are sometimes called the Twiss parameters.

- $\sigma_{H,V}(s)$ and $\sigma'_{H,V}(s)$ are the one-sigma values of the transverse position and divergence, respectively, at some position, s , around the storage ring.
- The area of the ellipse is proportional to a parameter of the beam called the **emittance** (units are **length x angle**).
- Liouville's Theorem states that for a system such as described here, the phase space volume should remain a constant. In other words **the area of the phase space ellipse (emittance) is constant even if the shape of the ellipse changes (periodically) as one goes around the particle's trajectory.**

APS Parameters

APS runs with $\varepsilon_H = 3 \times 10^{-9}$ m-rad and a coupling (ratio of vertical emittance to horizontal emittance) of 0.9% , therefore

$$\varepsilon_V = 0.025 \times 10^{-9} \text{ m-rad.}$$

The particle beam source size and divergence at the straight sections are:

$$\sigma_H = 270 \text{ microns}$$

$$\sigma_H' = 11 \text{ microradians}$$

$$\sigma_V = 9 \text{ microns}$$

$$\sigma_V' = 3 \text{ microradians}$$

Diffraction Limited Source Size and Divergence

- The effective phase space of the radiation source (Σ_i and Σ_i') has contributions from size and divergence of the particle beam generating the radiation and the intrinsic source size and divergence of the radiation itself. **Is there are limit to how small the effective phase space area (i.e., emittance) can be?** Yes, you are still bound by the Heisenberg Uncertainty Principle. Recall:

$$\Delta x \Delta p_x \geq \hbar/2$$

$$\frac{p_x}{p_z} = \Theta_x \text{ or } \frac{\Delta p_x}{p_z} = \Delta \Theta_x \text{ and } p_z = \hbar k = \hbar(2\pi/\lambda)$$

$$\text{so: } \Delta x \Delta p_x = \Delta x \Delta \Theta_x p_z = \Delta x \Delta \Theta_x [\hbar(2\pi/\lambda)] \geq \hbar/2$$

$$\Delta x \Delta \Theta_x \geq \lambda/4\pi$$

- This is the so-called **diffraction limit**. For central cone of the undulator:

$$\sigma_r' \text{ or } (\Delta \Theta) = \sqrt{[\lambda/2L]} \text{ and so}$$

$$\sigma_r \text{ or } (\Delta x) = \sqrt{[\lambda L/8\pi^2]}$$

Comparison of Radiation and Beam Properties

- If Gaussian distributions are assumed for both the particle beam and the radiation itself, resultant source size and divergence is the quadrature sum of the two components, namely:

$$\Sigma_i = \sqrt{[\sigma_r^2 + \sigma_i^2]} \quad \text{and} \quad \Sigma_i' = \sqrt{[\sigma_r'^2 + \sigma_i'^2]}.$$

- We can now calculate the beam brightness since

$$\mathcal{B} = \text{Flux} / 4\pi^2 \Sigma_H \Sigma_V \Sigma_H' \Sigma_V'$$

APS Undulator A has a length of 2.4 meters. For 1Å radiation the natural opening angle is:

$$\sigma_r' = \sqrt{[\lambda/2L]} = 4.5 \text{ microradians.}$$

The corresponding source size of the radiation is:

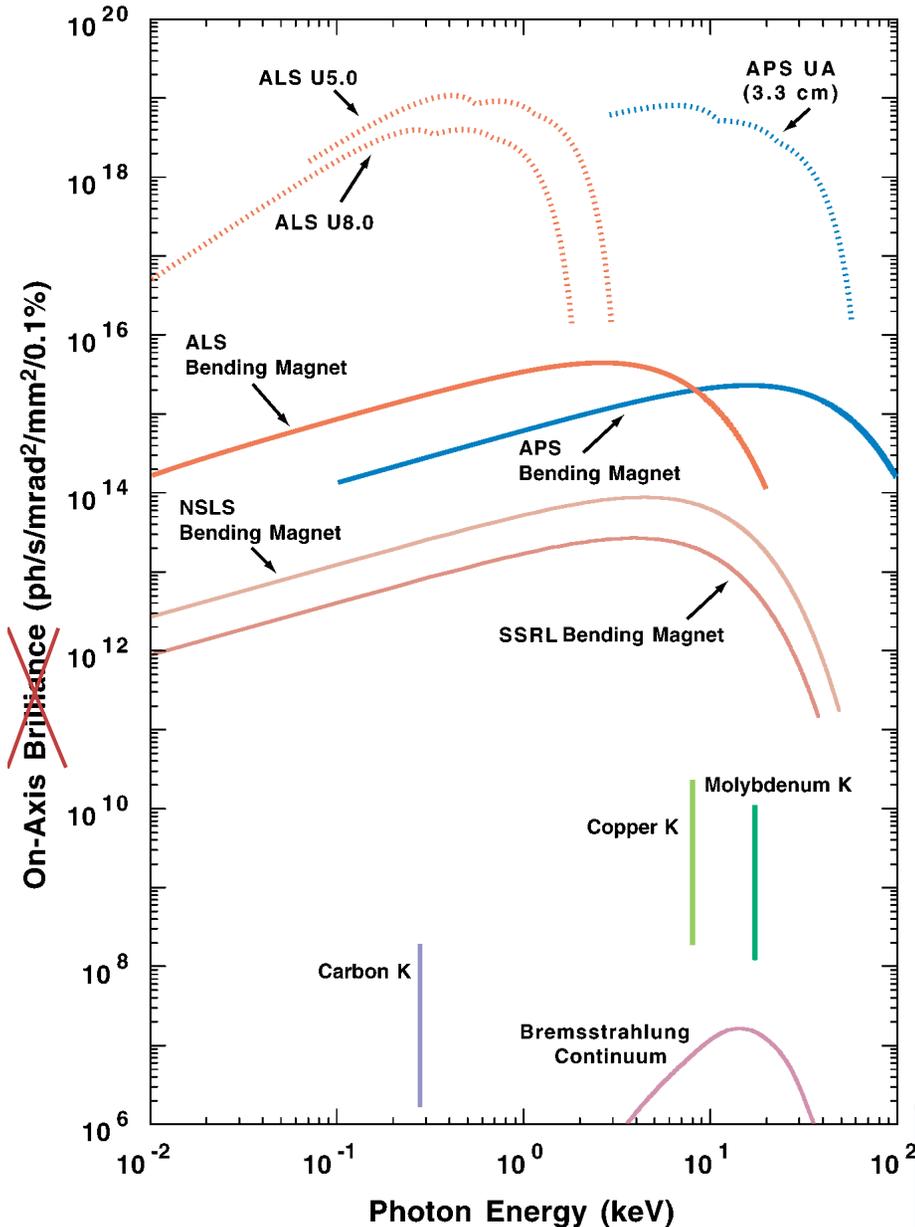
$$\sigma_r = \sqrt{[\lambda L/8\pi^2]} = 1.7 \text{ microns.}$$

Compare this with the vertical size and divergence of the APS beam:

$$\sigma_V' = 3 \text{ microradians}$$

$$\sigma_V = 9 \text{ microns}$$

Brightness vs. Photon Energy for Various SR Facilities



- The high brightness beams at 3rd generation sources (ALS and APS, for example) results in an x-ray beam with **partial transverse (or spatial) coherence**.
- This beam property can be an important parameter in some experiments such as photon correlation spectroscopy, x-ray holography, imaging, etc.

What's in Store for the Future?

- **What parameters would users like to see “enhanced”?**
 - Increased brightness (i.e., larger coherent fraction = $F_{\text{coherent}} / F_{\text{total}}$)
 - Shorter pulses (from 100 ps to 1 ps or less)
- **We are about at the limits of what storage rings can do:**
 - Particle beam emittance (source size x divergence):
 - *Horizontal emittance:* 3×10^{-9} m-rads
 - *Average brightness:* 10^{19} - 10^{20} x-rays/sec-0.1% bw-mm²-mrad²
 - Particle beam longitudinal properties:
 - *Bunch length:* 20 mm (70 psec)
- **We need to move away from “equilibrium” storage-ring-based light sources and use the low emittance and short pulses that can be generated by linear accelerators.**

See talk by Sol Gruner to learn more about the next (4th) generation of light sources.

Appendix 1: BM Spectral Distribution

The spectral/angular distribution of "synchrotron radiation" was worked out by J. Schwinger in 1949. Schwinger found the spectral distribution from an accelerating particle, under the influence of a constant magnetic field, was a smoothly varying function of photon energy and that the spectrum could be parameterized by a critical energy, E_c .

$$E_c = 3hc\gamma^3/4\pi r.$$

Here h is Planck's constant and r is the radius of curvature of the trajectory. Note that the **critical energy scales as γ^3** . In practical units, the critical energy can be written as:

$$E_c[\text{keV}] = 2.218 E^3[\text{GeV}] / \rho[\text{m}] = 0.06651 B[\text{kG}] E^2[\text{GeV}]$$

At the APS the bending magnets have a field strength of 5.99 kilogauss and the ring operates at $E = 7 \text{ GeV}$. The critical energy of the radiation emitted from the BM is:

$$E_c[\text{keV}] = 0.06651 B[\text{kG}] E^2[\text{GeV}]$$

or

$$E_c = 0.06651(5.990)(7^2) = \underline{19.5 \text{ keV}} \text{ or } \underline{0.64 \text{ \AA}}.$$

Appendix 1: BM Angular Distribution and Flux

- The opening angle of the entire radiation field (i.e., the power) is approximately:

$$\theta_x = \theta_y \approx 1/\gamma,$$

where θ_x is the horizontal angle and θ_y is the vertical angle.

- The collimation in the horizontal direction is lost and what is observed is just the vertical opening angle.
- Flux from a bending magnet is usually quoted as **flux per unit horizontal angle** (integrated over the vertical angle). There are no "simple" closed-form solutions for the photon flux, F , as a function of wavelength from a bending magnet however there are numerous series approximations that can easily be evaluated with a calculator/computer.

From a bending magnet ($B = 5.99$ kG) at the APS operating at $E = 7$ GeV and $I = 100$ mA (at the critical energy, integrated over all vertical angles) we get:

$$dF/d\theta_x \approx 10^{13} \text{ photons/sec} - 0.1\% \text{ BW} - \text{mrad } \theta_x$$

Appendix 2: Where did “K” come from?

$$F_x = ma_x = \gamma m_0 \dot{v}_x = e\vec{v} \times \vec{B} = ecB_0 \sin\left(\frac{2\pi z}{\lambda_{TD}}\right)$$

$$\dot{v}_x = \frac{ecB_0}{\gamma m_0} \sin\left(\frac{2\pi z}{\lambda_{TD}}\right) \quad z = ct$$

$$v_x = -\frac{ecB_0}{\gamma m_0} \frac{\lambda_{TD}}{2\pi c} \cos\left(\frac{2\pi ct}{\lambda_{TD}}\right) = -\frac{eB_0}{\gamma m_0} \frac{\lambda_{TD}}{2\pi} \cos\left(\frac{2\pi ct}{\lambda_{TD}}\right)$$

$$x = \frac{eB_0}{\gamma m_0 c} \left[\frac{\lambda_{TD}}{2\pi}\right]^2 \sin\left(\frac{2\pi ct}{\lambda_{TD}}\right) = \left[\frac{eB_0}{m_0} \frac{\lambda_{TD}}{2\pi c}\right] \frac{1}{\gamma} \left[\frac{\lambda_{TD}}{2\pi}\right] \sin\left(\frac{2\pi z}{\lambda_{TD}}\right) = K \frac{1}{\gamma} \left[\frac{\lambda_{TD}}{2\pi}\right] \sin\left(\frac{2\pi z}{\lambda_{TD}}\right)$$

$$x_{\max} = K \frac{1}{\gamma} \left[\frac{\lambda_{TD}}{2\pi}\right] \quad \text{and} \quad \left[\frac{dx}{dz}\right]_{\max} = \frac{K}{\gamma} \quad \text{where} \quad K = \left[\frac{eB_0}{2\pi} \frac{\lambda_{TD}}{m_0 c}\right]$$

Appendix 3: Wiggler Radiation Spectral Distribution

Spectral Distribution:

Wiggler radiation is the incoherent superposition of radiation from each pole of the wiggler. As with the bending magnet, the spectral distribution of the emitted radiation from a wiggler is smoothly varying as a function of photon energy and is characterized using the critical energy as a parameter.

The APS has built a wiggler with a magnetic field, B_o , of 10 kilogauss, a period of 8.5 cm, and a length of 2.4 meters ($N=28$).

$$K = 7.9, \quad E_c = 32.6 \text{ keV}, \quad \theta_{max} = 577 \text{ microradians}, \quad x_{max} = 8 \text{ microns}$$

Wiggler Flux:

The flux from a wiggler can be calculated by multiplying the bending magnet equations by $2N$ where N is the number of periods (and by using the appropriate critical energy!).

The APS Wiggler ($B_o = 10 \text{ kG}$, $\lambda_{ID} = 8.5 \text{ cm}$, $L = 2.4 \text{ meters}$; $N=28$, has a flux (at the critical energy integrated over all vertical angles) of:

$$dF/d\theta_x = 4.8 \times 10^{14} \text{ photons/sec} - 0.1 \% \text{ BW} - \text{mrad } \theta_x$$

Appendix 4: Undulator Flux

To determine the flux from an undulator, we *integrate over both the vertical and horizontal angular distributions of the central cone* and so the flux will have units of:

x-rays/sec-0.1% bw.

As with bending magnets, there is not a closed form expression for the flux, but it can be approximated by:

where:

$$F_n = 0.72 \times 10^{11} N Q_n I [\text{mA}] \text{ ph/sec-0.1\% bw}$$
$$Q_n(K) \approx 1 \text{ for } K > 1.$$

For APS Undulator A ($N = 72$) with the storage ring running at 7 GeV and 100 milliamps, typical flux values for the first harmonic ($n=1$) are:

$$F = 3\text{-}5 \times 10^{14} \text{ ph/sec-0.1\% bw.}$$