Practical Applications of Reliability Theory

George Dodson
Spallation Neutron Source
Topics

• Reliability Terms and Definitions

• Reliability Modeling as a tool for evaluating system performance
  – In the design phase what are the tradeoffs of cost vs. reliability performance?
  – In the operational phase, does the performance meet expectations?

• Analysis of the failure rate of systems or components
  – How do systems fail?
  – Is the failure rate “reasonable”?

• Analytical calculation for the number of Spares
  – What kinds of spares are there?
  – What is a “reasonable” number of spares?
Reliability Terms

- Mean Time To Failure (MTTF) for non-repairable systems
- Mean Time Between Failures for repairable systems (MTBF)
- Reliability Probability (survival) $R(t)$
- Failure Probability (cumulative density function) $F(t)=1-R(t)$
- Failure Probability Density $f(t)$
- Failure Rate (hazard rate) $\lambda(t)$
- Mean residual life (MRL)
Important Relationships

\[ R(t) + F(t) = 1 \]

\[ f(t) = \lambda(t) \exp\left(-\int_0^t \lambda(u) du\right) = dF(t) / dt \quad F(t) = \int_0^t f(u) du, \]

\[ R(t) = 1 - F(t) = \exp\left(-\int_0^t \lambda(u) du\right) \quad \lambda(t) = f(t) / R(t) \]

Where \( \lambda(t) \) is the failure rate function
MTBF

The MTBF is widely used as the measurement of equipment's reliability and performance. This value is often calculated by dividing the total operating time of the units by the total number of failures encountered. This metric is valid only when the data is exponentially distributed. This is a poor assumption which implies that the failure rate is constant if it is used as the sole measure of equipment's reliability.
Modeling

• There are essentially 2 types of models
  – Static
    • \( \lambda(t) \) is constant
    • Easy, if only life were this simple
  – Dynamic
    • \( \lambda(t) \) has a complex functional form

• To build a model:
  – Create a logical structure of components
  – Specify the reliability of each component
    • Drill down the structure as deep as you need to and/or have data
SNS Static Model ($\lambda(t)$ is constant)
Uses Markov Chains

<table>
<thead>
<tr>
<th>System</th>
<th>Equipment/Failure Mode</th>
<th>Failure Rate (1/h x $10^{-6}$)</th>
<th>Failure Rate Source</th>
<th>MTBF (h)</th>
<th>Percent of Anticipated Failures</th>
<th>Effective MTBF for Unanticipated Failures (1/h x $10^{-6}$)</th>
<th>Effective Failure Rate (1/h x $10^{-6}$)</th>
<th>No. of Equipments</th>
<th>No. of Spares Included</th>
<th>Type of Redundancy</th>
<th>Repair (o=off, c=on-line, 1=on-line, 2=sup mod)</th>
<th>Effective Total Failure Rate (1/h x $10^{-5}$)</th>
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ONE DTL RF STATION TOTAL

<table>
<thead>
<tr>
<th>Equipment/Failure Mode</th>
<th>Repair Personnel Required</th>
<th>Mean Time To Repair MTTR (h)</th>
<th>MTTR Source</th>
<th>Other Delays MTTR (h)</th>
<th>Switch-over Time MTTS (h)</th>
<th>Mean Down Time MDT (h)</th>
<th>Maximum Acceptable Total MDT/MTBF eff (x 10^6)</th>
<th>Estimated Average Total Annual Repair Time (h)</th>
<th>Steady State Availability</th>
<th>Mission Time (h)</th>
<th>Reliability for mission time</th>
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</table>
Dynamic Model
Uses of the Model

• Design Phase
  – Model is a simple “what if” tool for evaluating performance to compare the projected system reliability with the customer’s expectations.

• Operational Phase
  – Validate model parameters with measured performance. Are you getting what you expected?
  – If not, questions to ask include, was the system:
    • Designed wrong
    • Built wrong
    • Installed wrong
    • Operated wrong
    • Maintained wrong
    • In a “sick” location
Time Distributions (Models) of the Failure Rate Function

• **Exponential Distribution**

\[ f(t) = \lambda e^{-\lambda t} \]

- Very commonly used, even in cases to which it does not apply (simple);
- Applications: Electronics, mechanical components etc.

• **Normal Distribution**

\[ f(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} \]

- Very straightforward and widely used;
- Applications: Electronics, mechanical components etc.

• **Lognormal Distribution**

\[ f(t) = \frac{1}{\sqrt{2\pi\sigma_t}} e^{-\frac{(\ln t - \mu)^2}{2\sigma_t^2}} \]

- Very powerful and can be applied to describe various failure processes;
- Applications: Electronics, material, structure etc.

• **Weibull Distribution**

\[ f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} e^{-\left( \frac{t}{\eta} \right)^\beta} \]

- Very powerful and can be applied to describe various failure processes;
- Applications: Electronics, mechanical components, material, structure etc.
Exponential Model

- **Definition: Constant Failure Rate**

\[ \lambda(t) = f(t) / R(t) = \lambda \]

\[ f(t) = \lambda \exp(-\lambda t) \quad \lambda > 0, \ t \geq 0 \]

\[ R(t) = \exp(-\lambda t) = 1 - F(t) \]

\[ R(x \mid t) = P_r(T > t + x \mid T > t) = \frac{e^{-\lambda(t+x)}}{e^{-\lambda t}} = e^{-\lambda x} = R(x) \]
Exponential Model Cont.

- Statistical Properties

\[ MTTF = \frac{1}{\lambda} \quad \rightarrow \quad R(MTTF) = e^{-\lambda \times MTTF} \]
\[ = e^{-1} = 0.367879 \]

\[ Var(T) = \frac{1}{\lambda^2} \]

Median life = \( (\ln 2) \frac{1}{\lambda} = 0.693147 \times MTTF \)
Weibull Model

- **Definition**

\[ f(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \exp \left[ -\left( \frac{t}{\eta} \right)^{\beta} \right] \quad \beta > 0, \quad \eta > 0, \quad t \geq 0 \]

\[ R(t) = \exp \left[ -\left( \frac{t}{\eta} \right)^{\beta} \right] = 1 - F(t) \]

\[ \lambda(t) = f(t) / R(t) = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \]

- **\( \beta \)** is the Shape Parameter and
- **\( \eta \)** is the Characteristic Lifetime (1/e) survival
Weibull Model Continued:

- **Statistical Properties**

\[ MTTF = \eta \int_0^{\infty} t^{1/\beta} e^{-t} \, dt = \eta \Gamma \left( 1 + \frac{1}{\beta} \right) \]

\[ Var = \eta^2 \left[ \Gamma \left( 1 + \frac{2}{\beta} \right) - \left( \Gamma \left( 1 + \frac{1}{\beta} \right) \right)^2 \right] \]

\[ \text{Median life} = \eta \left( (\ln 2)^{1/\beta} \right) \]
Versatility of Weibull Model

Failure Rate:

\[ \lambda(t) = \frac{f(t)}{R(t)} = \frac{\beta}{\eta} \left( \frac{t}{\eta} \right)^{\beta-1} \]

Failure Rate: 
- **Constant Failure Rate Region**
  - \( \beta = 1 \)
- **Early Life Region**
  - \( 0 < \beta < 1 \)
- **Wear-Out Region**
  - \( \beta \geq 1 \)
Graphical Model Validation

- Use a Q-Q Plot for checking normality
  - Plot Probability Quantiles (\(\ln(\ln(1/\text{median rank})/\text{vs. Model Distribution Quantiles}
  - The result should be linear in \(\ln(\text{time})\) in the middle of the plot

- Estimate \(Q(t_i) = 1 / \hat{F}(t_i)\) at \(t_i\) using Bernard’s Formula

\[
\hat{F}(t_i) = \frac{i - 0.3}{n + 0.4}
\]

For \(n\) observed failure time data \((t_1, t_2, \ldots, t_i, \ldots, t_n)\)
Example: Q-Q of Weibull Distribution and Weibull Fit (works well)

- $T \sim \text{Weibull}(1, 4000)$ Generate 50 data points
Example: Q-Q Weibull Distribution and T-Normal Fit (obviously wrong functional form)

- $T \sim \text{Normal}(100, 400)$ Generate 50 data points

Weibull model is invalid
Analysis of the Failure Rate of Systems or Components

With a relatively modest failure data set you can:

– Determine what your failure rate is at any given time
– Watch this rate change with time, through Infant Failures and into Random Failures
– Predict the onset of Terminal Failures
– Alerts you to watch more closely for the predictive symptoms of failure
– Determine the cost-effectiveness of proactive replacement before failure occurs
– Closely watch your Spares (number of spares, time to repair or acquire replacements, cost)
Many people use Microsoft Excel on a daily basis. Yet few people realize the extent of Excel's analytical capabilities. Fewer still put these capabilities to work for process improvement, product improvement and profit. Most Excel users are aware of the common formulas and charts. But with some creativity, users can produce tools like control charts, Pareto charts and box-and-whisker plots (see "Using Excel for Data Analysis," *Quality Digest*, October 1997). And with a little guidance, users can employ more advanced statistical tools.
1) Order by number of cycles (in accelerators, hours to failure)
2) Calculate the “Median Rank”=((B2-0.3)/(10+0.4))
3) Be sure that the Analysis ToolPak Add-In is loaded into Excel.
While on the page you just created, from the menu bar, select Tools and Data Analysis. Scroll down and highlight "Regression" and click OK. A data-entry window will pop up.
3.For "Input X Range," type: $F$1:$F$11.
4.Click to add a checkmark in the box for "Labels."
5.For "Output Options," select "New Worksheet Ply."
6.Click to add a checkmark in the box for "Line Fit Plots."
7.Click OK. Excel will perform the regression and place the output on a new worksheet.
Note the:

Shape Parameter $\beta > 1$  Terminal Mortality
Characteristic Lifetime, where $1/e = 62.3\%$ have failed.
Beta = 0.39 (Infant Failures)
Alpha = 287
Adjusted R square = 0.93

The λ for an Exponential model = 475!!
# VME Crate Power Supplies (2009)

## VME hours to failure

<table>
<thead>
<tr>
<th>VME hours to failure</th>
<th>Rank</th>
<th>Median Ranks</th>
<th>1/(1-Median Rank)</th>
<th>ln(ln(1/(1-Median Rank)))</th>
<th>ln(Design A Cycles)</th>
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## SUMMARY OUTPUT

### Regression Statistics

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### ANOVA

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<th>MS</th>
<th>F</th>
<th>Significance</th>
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### Coefficient Table

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<th>t Stat</th>
<th>P-value</th>
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<th>Upper 95%</th>
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<th>Upper 95%</th>
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## Residual Output

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<tr>
<th>Observation</th>
<th>Predicted ln(ln(1/(1-Median Rank)))</th>
<th>Residuals</th>
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Beta = 0.97  
Alpha = 78 Yrs.
Spares

Classes of Spares

In all evaluations of Mean Time to Repair, there are assumptions on the availability of spares for systems, structures and components. In most cases, the assumption is that there is a spare of some sort available to install. There are a number of classes of spares. They include;

• A “true spare” consisting of a “like for like or equivalent” “on the shelf, tested and ready to go “, “plug compatible” replacement unit.

• A “like for like or equivalent” that is installed in some other system that is not required for operation of the accelerator systems e.g. a Test Stand.

• A system structure or component that must be modified to be used as a spare.

Only a “true spare” will not contribute to down time. In both other classes, demounting and modification of the replacement will necessarily contribute to downtime.
Beyond the “larger of 10% or 2” rule of thumb, the evaluation of the baseline number of spares should include a calculational basis which considers:

1. Number of installed units
2. Mean Time Between Failures (estimated at first, then validated against experience)
3. Mean Time to Repair or Replace in the calculation.

The result will be a Mean Time to Out of Stock as a function of the number of spare units.

- Spares Calculator code is available – validated against MIL Spec - U.S. Navy, Reliability Engineering Handbook, NAVAIR 01-1A-32
Spare Calculator 2010 Professional

Powerful and Easy to Learn Spare Parts Forecasting Software

Spare Calculator 2010 Professional is a powerful and accurate tool that lets you work out the risk of equipment stock-out so that you can plan your contracts and logistics operations accordingly. What’s more, the package is easy to use so there’s no need for specialist knowledge, complex manuals or time consuming training. Whether you supply, procure or manage spare equipment, Spare Calculator makes an essential purchase.

Key Facts

1. Easy to use spare parts forecasting software.
   Use Spare Calculator Professional to determine the optimum level of spare parts inventory. Trade-off spare parts, costs, Stock-Out-Risk (SOR) and Mean-Time-Between-Stock-Out (MTBSO).

2. Justify investments, eliminate outages and eradicate redundant stock.
   Justify investments by convincing customers, managers and shareholders to invest in the correct level of spare parts. Eliminate outages by finding the right level of inventory in seconds. Eradicate redundant stock by identifying slow moving stock and plan alternative strategies.

3. Based on de facto industry standard models.
   Based on de facto industry standard models and proven in literally thousands of real-life and simulated trials, Spare Calculator Professional offers a scientific way to forecast spare parts.

4. Streamline budgets and costs.

User Comment

“Spare Calculator has given us a scientific way to work out how many spare parts we need. We tested Spare Calculator on a random sample of 20 line items used in our 12 operations spread across Asia. The results were amazingly accurate. We now have an enterprise licence and use Spare Calculator in all of our procurement and operational planning projects.”

Customers Include

- Rogers Wireless
- Alcatel-Lucent
- General Dynamics
Spares – How Many

• Use the MTBSO to evaluate what Comfort Level you can afford to have.

• Caveat –
  – This calculation assumes a random distribution and is not accurate for NEW systems where a large number of identical are all installed at the same time.
Summary:

For a given set of performance data and an appropriate model, analysis of the data can accurately yield MTBF, MTTR for components and systems. The analysis can also yield information on where components and systems are in the lifetime curve so that you can make decisions about when to replace components and how many you should have in inventory (particularly important in long-lead-time components).

These data can be used to validate your RAMI Model of your accelerator systems.
Issues in Modeling

• “… no model is absolutely correct. In particular, however, some models are more useful than others.” –

• The model should be sufficiently simple to be handled by available mathematical and statistical methods, and be sufficiently realistic such that the deducted results are of practical use.
Backup Slides
Most of these distribution functions are not Symmetric, so:
Median, Mode and Mean are not the same

Median - \( t_m : R(t_m) = 50\% \)

Mode - \( t_{mode} : \max f(t) \)
Example of a Non-Constant Failure Rate Curve: The “Bathtub” Curve

- **Early Life Region**
- **Constant Failure Rate Region**
- **Wear-Out Region**
Infant Mortality or Burn-In:

According to MIL-STD-883C, burn-in is a test performed to screen or eliminate marginal components with inherent defects or defects resulting from manufacturing process.
Use of Burn-In

- One of environment stress screening (ESS) techniques

\[ MRL(t) = \int_0^\infty R(x \mid t)dx = \frac{1}{R(t)} \int_0^\infty R(x+t)dx \]

- Example: for

\[ R(t) = \frac{a^2}{(a+t)^2} \quad t \geq 0 \]

\[ MTTF = \int_0^\infty R(t)dt \quad \text{MTTF} \]

\[ MRL(T_0) = \frac{1}{R(T_0)} \int_0^\infty R(x+T_0)dx \]

\[ = -a^2 \bigg|_0^\infty = a \]

\[ = a + T_0 > a \]
An important assumption for effective maintenance is that components will eventually have an Increasing Failure Rate. Maintenance can return the component to the Constant Failure Region.
Components will eventually enter the Wear-Out Region where the Failure Rate increases, even with an effective Maintenance Program. You need to be able to detect the onset of Terminal Mortality.
Exponential Distribution (Model)

Constant Failure Rate

Single/Multiple Failure Modes
Example

• The higher the failure rate is, the faster the reliability drops with time
Weibull Distribution (Model) and Model Validation

Walldi Weibull 1887-1979
Photo by Sam C. Saunders
• **Waloddi Weibull**, a Swedish inventor and engineer invented the Weibull distribution in 1937. The U.S. Air Force recognized the merit of Weibull’s methods and funded his research to 1975.

• **Leonard Johnson** at General Motors improved Weibull’s methods. He suggested the use of median rank values for plotting.

• The engineers at Pratt & Whitney found that the Weibull method worked well with extremely small samples, even for 2 or 3 failures.
• Failure Probability Density is related to the Failure Probability by:

\[ f(x) = \int_{0}^{x} f(s) \, ds \]

\[ f(x) = \frac{d(F(x))}{dx} \]

• Reliability Function is related to the Failure Probability Density by:

\[ R(t) = 1 - F(t) = \int_{t}^{\infty} f(u) \, du \]

2 is better than 1?
Failure Rate Function

• Increasing failure rate (IFR) v.s. decreasing failure rate (DFR)

\[ \lambda(t) > 0 \quad \text{or} \quad \lambda(t) < 0 \quad \text{respectively} \]

• Examples

\[ \lambda(t) = c \quad \text{where} \ c \ \text{is a constant} \]

\[ \lambda(t) = at \quad \text{where} \ a > 0 \]

\[ \lambda(t) = \frac{1}{t + 1} \quad \text{for} \ t > 0 \]
Q-Q Plot for the Normal Distribution

- $T \sim \text{Normal}(100, 400)$ Generate 50 data points
Formal Statistical Test Procedures

• Test for assumption in a more statistical way

  • $\chi^2$ Goodness-of-Fit test
  • Bartlett’s test for Exponential
  • Mann’s test for Weibull
  • Komogorov-Smirnov (KS) test
Graphical Model Validation

• Weibull Plot

\[ F(t) = 1 - R(t) = 1 - \exp \left[ - \left( \frac{t}{\eta} \right)^\beta \right] \]

\[ \Rightarrow \ln \ln \frac{1}{1 - F(t)} = \beta \ln t - \beta \ln \eta \quad \text{is linear function of } \ln(\text{time}). \]

• Estimate \( \hat{F}(t_i) \) at \( t_i \) using Bernard’s Formula

For \( n \) observed failure time data \( (t_1, t_2, ..., t_i, ... t_n) \)

\[ \hat{F}(t_i) = \frac{i - 0.3}{n + 0.4} \]