Making Good Decisions with Your Reliability Data – A Formalized Approach

Ken Baggett Ph.D.
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Abstract

All organizations want to find ways to improve their system’s reliability. Collecting and interpreting data can often be a complex and confusing process. This is especially true when both qualitative and quantitative data are used in the analysis method. Simple cost-benefit analysis techniques often result in management ignoring other important factors such as “ease of installation” or “availability of spares”. The Analytic Hierarchy Process (AHP) provides a methodology to evaluate the benefits of the alternatives, initially independent of cost, utilizing a pairwise comparison technique. This technique allows the analyst to evaluate a problem’s alternatives using specific criteria while also gauging (and maximizing) the decision maker’s consistency. A simple example will be used to explain the use of the methodology in the reliability domain.
Overview of AHP

• The Analytic Hierarchy Process (AHP) is a multi-criteria decision making (MCDM) methodology used by decision analysts to rank alternatives

• It offers several advantages to the process:
  – Well tested:
    • 30-35 years of thorough (successful) testing by thousands of organizations from around the globe
  – Intuitive
    • Easy to explain to your audience
  – Promotes discussion about priorities of criteria
    • "Is A or B more important, and by how much?"
  – Validates Input
    • The model offers a way to evaluate the constancy of your criteria preferences
Who uses AHP?

- Industries:
  - Banking
  - Manufacturing
  - Utilities
  - Insurance
  - Medical Devices
  - Pharmaceuticals
  - Information Technology
  - Telecommunications
  - Government

Many organizations are using this method, can it help us improve?

[1]
Select the best maintenance policy for an accelerator system:

- Harps, Valves, Fast Valves, etc.
- Management needs to develop a method for selecting the best maintenance policy (alternative) for the equipment.
  - Corrective, Preventative, Predictive, etc.
- The manager wants to establish a system to use to analyze different systems based on the way the lab places importance on evaluation criteria:
  - Safety
  - Machine Importance
  - Maintenance Cost
  - Failure Frequency
  - Downtime Length
  - Component Access
AHP Process

Step 1
• Define decision criteria in the form of a hierarchy of objectives.

Level 1 – Overall Goal

Level 2 – Criteria

Level 3 – Alternatives
Step 2 - Weight the criteria, sub-criteria and alternatives as a function of their importance for the corresponding element of the higher level.

- AHP uses *pairwise comparisons* to determine weights and ratings so that the analyst can concentrate on just two factors at one time.

- Example:
  - How important is the “Maintenance Cost” criteria with respect to the “Failure Frequency” criteria, in terms of the “Maintenance Program Selection” (i.e. the problem goal)?
    - equally important?
    - moderately more important?
    - something else?
  - The responses are quantified and translated into a score
### The Fundamental Scale for Pairwise Comparisons

<table>
<thead>
<tr>
<th>Intensity of Importance</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equal importance</td>
<td>Two elements contribute equally to the objective</td>
</tr>
<tr>
<td>3</td>
<td>Moderate importance</td>
<td>Experience and judgment slightly favor one element over another</td>
</tr>
<tr>
<td>5</td>
<td>Strong importance</td>
<td>Experience and judgment strongly favor one element over another</td>
</tr>
<tr>
<td>7</td>
<td>Very strong importance</td>
<td>One element is favored very strongly over another; its dominance is demonstrated in practice</td>
</tr>
<tr>
<td>9</td>
<td>Extreme importance</td>
<td>The evidence favoring one element over another is of the highest possible order of affirmation</td>
</tr>
</tbody>
</table>

Intensities of 2, 4, 6, and 8 can be used to express intermediate values. Intensities 1.1, 1.2, 1.3, etc. can be used for elements that are very close in importance.
AHP Steps – The Priority Vector

Step 3 – Calculate the priority vector

• The priority vector is the weights for the model
  – Also called the normalized eigenvector, or Perron Vector

• A priority vector must reproduce itself on a ratio scale
  – it should remain invariant under multiplication by a positive constant $c$,

  AND

  – it should be invariant under hierarchic composition for its own judgment matrix so that one does not keep getting new priority vectors from that matrix

• A simple matrix squaring method can be used to accomplish this (more to come)

• Ratios preserve the strength of preferences
Pairwise Comparison of Criteria

- Pairwise comparisons are made for each of the criteria based on the values given in the AHP scale
  - Here, “Safety” is rated 6x more important than “Maintenance Cost”
    - (between strong and very strong)

<table>
<thead>
<tr>
<th>Pairwise Comparisons</th>
<th>Safety</th>
<th>Machine Importance</th>
<th>Maint Cost</th>
<th>Failure Freq</th>
<th>Downtime Length</th>
<th>Component Access</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety</td>
<td>-</td>
<td>2/1</td>
<td>6/1</td>
<td>3/1</td>
<td>3/1</td>
<td>7/1</td>
</tr>
<tr>
<td>Machine Importance</td>
<td>1/2</td>
<td>-</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4/1</td>
</tr>
<tr>
<td>Maint Cost</td>
<td>1/6</td>
<td>1/3</td>
<td>-</td>
<td>1</td>
<td>1/5</td>
<td>1/1</td>
</tr>
<tr>
<td>Failure Freq</td>
<td>1/3</td>
<td>1/2</td>
<td>1/1</td>
<td>-</td>
<td>0.2</td>
<td>1/1</td>
</tr>
<tr>
<td>Downtime Length</td>
<td>1/3</td>
<td>1/1</td>
<td>5/1</td>
<td>5/1</td>
<td>-</td>
<td>3/1</td>
</tr>
<tr>
<td>Component Access</td>
<td>1/7</td>
<td>1/4</td>
<td>1/1</td>
<td>1/1</td>
<td>1/3</td>
<td>-</td>
</tr>
</tbody>
</table>
Pairwise Comparison of Criteria

- Pairwise comparisons are often inconsistent in practice
  - Since D3 = 6, the DM feels Safety is $6x$ as important as Maintenance Cost
  - Since E3 = 3, the DM feels Safety is $3x$ as important as Failure Frequency
  - When compared directly (E5):
    - Maintenance Cost is rated **equally** as important as Failure Frequency
    - The DM should believe that Maintenance Cost is $6 / 3 = 2$ times as important as Failure Frequency if they were consistent!

- AHP uses a **consistency index** to ensure we are consistent enough to produce meaningful results.

<table>
<thead>
<tr>
<th>Pairwise Comparisons</th>
<th>Safety</th>
<th>Machine Importance</th>
<th>Maint Cost</th>
<th>Failure Freq</th>
<th>Downtime Length</th>
<th>Component Access</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety</td>
<td>-</td>
<td>2/1</td>
<td>6/1</td>
<td>3/1</td>
<td>3/1</td>
<td>7/1</td>
</tr>
<tr>
<td>Machine Importance</td>
<td>1/2</td>
<td>-</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>4/1</td>
</tr>
<tr>
<td>Maint Cost</td>
<td>1/6</td>
<td>1/3</td>
<td>-</td>
<td>1</td>
<td>1/5</td>
<td>1/1</td>
</tr>
<tr>
<td>Failure Freq</td>
<td>1/3</td>
<td>1/2</td>
<td>1/1</td>
<td>-</td>
<td>0.2</td>
<td>1/1</td>
</tr>
<tr>
<td>Downtime Length</td>
<td>1/3</td>
<td>1/1</td>
<td>5/1</td>
<td>5/1</td>
<td>-</td>
<td>3/1</td>
</tr>
<tr>
<td>Component Access</td>
<td>1/7</td>
<td>1/4</td>
<td>1/1</td>
<td>1/1</td>
<td>1/3</td>
<td>-</td>
</tr>
</tbody>
</table>
Checking Consistency

- Normalize the evaluation matrix
  - Divide the values in each column by the sum of the column ($A_{\text{norm}}$)

- Approximate $w_{\text{max}}$ for each row by calculating the average of the rows of the normalized matrix

- Compute $A \cdot w_{\text{max}}$

- Compute the eigenvalue approximation ($\lambda_{\text{max}}$)

  \[
  \frac{1}{n} \sum_{i=1}^{n} \frac{(A \cdot w)_i}{w_i} = \left( \frac{1}{6} \right) \left\{ \frac{2.459}{0.386} + \frac{1.185}{0.190} + \frac{0.370}{0.060} + \frac{0.466}{0.077} + \frac{1.412}{0.227} + \frac{0.376}{0.060} \right\} = 6.215
  \]

- Compute the Consistency Index

  \[
  CI = \frac{(\lambda_{\text{max}}) - n}{n - 1} = \frac{6.215 - 6}{5} = 0.043
  \]
Evaluate the Constancy

- Compare CI to the random index (RI) for the appropriate value of n, shown in table below
  - RI gives values if the pairwise comparison matrix values were picked at random
  - CI/RI < 0.10, consistency satisfactory
  - CI/RI > 0.10, serious inconsistency exists (won't get meaningful results)
  - In the example, CI/RI = 0.043 / 1.24 = 0.035 < 0.10,

  ✓ no serious inconsistency

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.51</td>
</tr>
</tbody>
</table>

- For a perfectly consistent decision maker:
  - $\lambda_{\text{max}} = n$
  - CI = 0
Analyzing the Data

• I use the eigenvector method to determine the priority vector for each pairwise comparison matrix
  – These are the weights that will be used to select the best alternative
    • Other approximation methods also exist

• Short method for obtaining the ranking:
  1. Raise the pairwise matrix to powers that are successively squared each time
  2. Sum the rows and normalize the row sums
  3. Stop when the difference between iterations is sufficiently small (4 decimal places usually sufficient)
## Painwise Comparisons (the matrix A)

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Safety</td>
<td>Machine Importance</td>
<td>Maint Cost</td>
<td>Failure Freq</td>
<td>Downtime Length</td>
<td>Component Access</td>
<td>Priority Vector</td>
<td>Normalized</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Safety</td>
<td>1.000</td>
<td>2.000</td>
<td>6.000</td>
<td>3.000</td>
<td>3.000</td>
<td>7.000</td>
<td>22.000</td>
<td>0.3649</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Machine Importance</td>
<td>0.500</td>
<td>1.000</td>
<td>3.000</td>
<td>2.000</td>
<td>1.000</td>
<td>4.000</td>
<td>11.500</td>
<td>0.1907</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Maint Cost</td>
<td>0.167</td>
<td>0.333</td>
<td>1.000</td>
<td>3.000</td>
<td>1.000</td>
<td>2.000</td>
<td>3.700</td>
<td>0.0614</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Failure Freq</td>
<td>4.095</td>
<td>5.783</td>
<td>25.000</td>
<td>21.000</td>
<td>21.000</td>
<td>22.392</td>
<td>84.010</td>
<td>0.1211</td>
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<td></td>
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<tr>
<td>Downtime Length</td>
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<td>1.952</td>
<td>1.574</td>
<td>1.574</td>
<td>1.574</td>
<td>1.574</td>
<td>1.574</td>
<td>0.0014</td>
<td></td>
<td></td>
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<tr>
<td>Component Access</td>
<td>1.022</td>
<td>1.952</td>
<td>6.274</td>
<td>5.595</td>
<td>1.745</td>
<td>6.000</td>
<td>22.585</td>
<td>0.0095</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Eigenvectors

- **Round 1**
  - Safety: 258.158
  - Machine Importance: 494.056
  - Maint Cost: 1576.019
  - Failure Freq: 1371.151
  - Downtime Length: 455.301
  - Component Access: 1553.619

- **Round 2**
  - Safety: 124.154
  - Machine Importance: 237.808
  - Maint Cost: 759.827
  - Failure Freq: 661.655
  - Downtime Length: 219.215
  - Component Access: 747.968

- **Round 3**
  - Safety: 38.651
  - Machine Importance: 74.140
  - Maint Cost: 237.496
  - Failure Freq: 206.985
  - Downtime Length: 68.447
  - Component Access: 233.384

### Eigenvector Vector

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety</td>
<td>0.393</td>
</tr>
<tr>
<td>Machine Importance</td>
<td>0.189</td>
</tr>
<tr>
<td>Maint Cost</td>
<td>0.059</td>
</tr>
<tr>
<td>Failure Freq</td>
<td>0.075</td>
</tr>
<tr>
<td>Downtime Length</td>
<td>0.224</td>
</tr>
<tr>
<td>Component Access</td>
<td>0.060</td>
</tr>
</tbody>
</table>

### Normalize Delta

- **Round 1**
  - Safety: 0.3927
  - Machine Importance: -0.0061
  - Maint Cost: 0.1892
  - Failure Freq: -0.0002
  - Downtime Length: 0.0591
  - Component Access: 0.0011

- **Round 2**
  - Safety: 0.3927
  - Machine Importance: -0.0061
  - Maint Cost: 0.1892
  - Failure Freq: -0.0002
  - Downtime Length: 0.0591
  - Component Access: 0.0011

- **Round 3**
  - Safety: 0.3927
  - Machine Importance: -0.0061
  - Maint Cost: 0.1892
  - Failure Freq: -0.0002
  - Downtime Length: 0.0591
  - Component Access: 0.0011
AHP Hierarchy

Select a Maintenance Program (Fast Valves)

Criteria Weights

- Safety: 0.393
- Machine Importance: 0.189
- Maintenance Cost: 0.059
- Failure Frequency: 0.075
- Downtime Length: 0.224
- Component Access: 0.060

Preventive Maintenance
Corrective Maintenance
Predictive Maintenance
Now that the criteria have been weighted, we repeat the process for the alternatives

- This needs to be done for each goal (in this case, determine the best maintenance plan for fast valves)
- Rate the DM preference for each alternative with respect to the goal

Ranking when we only consider “maintenance cost” may be very different than ranking when we only consider “downtime length”

- Complexity of considering all criteria at the same time isn’t really possible,
- We need to break the problem up into small, manageable chunks, evaluate our preferences, and put the puzzle back together
## Ranking the Alternatives

### Ranking for the criteria “Safety”

<table>
<thead>
<tr>
<th></th>
<th>Safety</th>
<th>Prev</th>
<th>Corr</th>
<th>Pred</th>
<th>Vector</th>
<th>Normalized Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Round 1 Eigenvector</strong></td>
<td></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td><strong>Round 2 Eigenvector</strong></td>
<td></td>
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<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Round 3 Eigenvector</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **Prev**
- **Corr**
- **Pred**
- **Vector**
- **Normalized Delta**

---

**Note:** The table presents the ranking for the criteria “Safety” with eigenvectors for different rounds.
### Ranking the Alternatives

<table>
<thead>
<tr>
<th>Maint Cost</th>
<th>Vector</th>
<th>Normalized Delta</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Prev</strong></td>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td><strong>Corr</strong></td>
<td>5.000</td>
<td>1</td>
</tr>
<tr>
<td><strong>Pred</strong></td>
<td>0.333</td>
<td>0.111</td>
</tr>
<tr>
<td></td>
<td>6.333333</td>
<td>1.311111</td>
</tr>
<tr>
<td><strong>Round 1 Eigenvector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.733333</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>1.222222</td>
<td>0.288889</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Round 2 Eigenvector</strong></td>
<td></td>
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</tr>
<tr>
<td></td>
<td>28.06667</td>
<td>6.653333</td>
</tr>
<tr>
<td></td>
<td>118.3333</td>
<td>28.06667</td>
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<tr>
<td></td>
<td>11.08889</td>
<td>2.62963</td>
</tr>
<tr>
<td></td>
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</tr>
<tr>
<td><strong>Round 3 Eigenvector</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2362.36</td>
<td>560.1775</td>
</tr>
<tr>
<td></td>
<td>9962.458</td>
<td>2362.36</td>
</tr>
<tr>
<td></td>
<td>933.6291</td>
<td>221.388</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
## Ranking the Alternatives

### Decision Score

\[
\text{Decision Score} = \sum_i w_i \cdot (\text{decision score on objective } i)
\]

For the system “Fast Valves” “Predictive Maintenance” has the highest score and is the preferred choice.

Multiply the vectors for each alternative with the criteria weights to get the final priorities.
AHP Hierarchy

Select a Maintenance Program (Fast Valves)

Safety 0.393
Machine Importance 0.189
Maintenance Cost 0.059
Failure Frequency 0.075
Downtime Length 0.224
Component Access 0.060

Preventive Maintenance 0.304
Corrective Maintenance 0.201
Predictive Maintenance 0.495

2nd Choice
3rd Choice
1st Choice
The AHP method is used throughout the world, in many different contexts, to help with complex decision making.

The methodology assists decision makers in developing consistent judgements about their evaluation criteria.

Both qualitative and quantitative data can be integrated into the technique.

The model can be refined as more data becomes available.

Once the decision analysis is complete, a prioritized ranking of alternatives defines the best choice based on the decision maker’s input and provides a structured, defendable methodology to explain how the choice was made.
QUESTIONS?
References

• [3] Baggett, K. “AHP for MCDA Applications”, Old Dominion University [power point slides]. 2015
Step 1: Normalize the matrix

For each of matrix (call it \( A \)'s) columns do the following.
- Divide each entry in column \( i \) of \( A \) by the sum of the entries in column \( i \)
- This yields a new matrix (\( A_{\text{norm}} \))
  - the sum of the entries in \( A_{\text{norm}} \) sum to 1.
Step 2: Estimate $W_{\text{max}}$

- Find an approximation to $w_{\text{max}}$ (to be used as our estimate of $\underline{w}$).
- Estimate $w_i$ as the average of the entries in row $i$ of $A_{\text{norm}}$. 

![Excel Table]

$A_{\text{norm}}$:

<table>
<thead>
<tr>
<th></th>
<th>Safety</th>
<th>Machine Importance</th>
<th>Maint Cost</th>
<th>Failure Freq</th>
<th>Downtime Length</th>
<th>Component Access</th>
</tr>
</thead>
<tbody>
<tr>
<td>Safety</td>
<td>0.404</td>
<td>0.393</td>
<td>0.353</td>
<td>0.231</td>
<td>0.523</td>
<td>0.412</td>
</tr>
<tr>
<td>Machine Importance</td>
<td>0.202</td>
<td>0.197</td>
<td>0.176</td>
<td>0.154</td>
<td>0.174</td>
<td>0.235</td>
</tr>
<tr>
<td>Maint Cost</td>
<td>0.067</td>
<td>0.066</td>
<td>0.059</td>
<td>0.077</td>
<td>0.035</td>
<td>0.059</td>
</tr>
<tr>
<td>Failure Freq</td>
<td>0.135</td>
<td>0.098</td>
<td>0.059</td>
<td>0.077</td>
<td>0.035</td>
<td>0.059</td>
</tr>
<tr>
<td>Downtime Length</td>
<td>0.135</td>
<td>0.197</td>
<td>0.294</td>
<td>0.385</td>
<td>0.174</td>
<td>0.176</td>
</tr>
<tr>
<td>Component Access</td>
<td>0.058</td>
<td>0.049</td>
<td>0.059</td>
<td>0.077</td>
<td>0.058</td>
<td>0.059</td>
</tr>
</tbody>
</table>

$w_i = \text{average of row } i$
### Step 3: Estimate the Weights

#### Step 3: Compute $A_{\text{norm}} \cdot w^T$
- $(w$ represents a quick estimate of the DM’s weights)
- For our example we obtain:

$$
A \cdot w^T = \begin{pmatrix}
0.404 & 0.393 & 0.353 & 0.231 & 0.523 & 0.412 \\
0.202 & 0.197 & 0.176 & 0.154 & 0.174 & 0.235 \\
0.067 & 0.066 & 0.059 & 0.077 & 0.035 & 0.059 \\
0.135 & 0.098 & 0.059 & 0.077 & 0.035 & 0.059 \\
0.135 & 0.197 & 0.294 & 0.385 & 0.174 & 0.176 \\
0.058 & 0.049 & 0.059 & 0.077 & 0.058 & 0.059
\end{pmatrix} \cdot \begin{pmatrix}
0.386 \\
0.190 \\
0.060 \\
0.077 \\
0.227 \\
0.060
\end{pmatrix} = \begin{pmatrix}
2.459 \\
1.185 \\
0.370 \\
0.466 \\
1.412 \\
0.376
\end{pmatrix}
$$

#### Step 4: Compute the eigenvalue approximation $\lambda_{\text{max}}$

$$
\frac{1}{n} \sum_{i=1}^{n} \frac{\text{i-th entry in } A \cdot w^T}{i \text{-th entry in } w^T} = \left( \frac{1}{6} \right) \left[ \frac{2.459}{0.386} + \frac{1.185}{0.190} + \frac{0.370}{0.060} + \frac{0.466}{0.077} + \frac{1.412}{0.227} + \frac{0.376}{0.060} \right] = 6.215
$$

- **Step 5: Compute the Consistency Index**

$$
CI = \frac{(\lambda_{\text{max}}) - n}{n - 1} = \frac{6.215 - 6}{5} = 0.043
$$
Step 6 – Evaluate the Constancy

- Compare CI to the random index (RI) for the appropriate value of $n$, shown in table below.

<table>
<thead>
<tr>
<th>$n$</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>RI</td>
<td>0</td>
<td>0.58</td>
<td>0.90</td>
<td>1.12</td>
<td>1.24</td>
<td>1.32</td>
<td>1.41</td>
<td>1.45</td>
<td>1.51</td>
</tr>
</tbody>
</table>

- For a perfectly consistent decision maker:
  - $\lambda_{\text{max}} = n$
  - $\text{CI} = 0$

- The values of RI give the average value of CI if the entries in $A$ were chosen at random
Consistency

- If CI is sufficiently small, the DM’s comparisons are probably consistent enough to give useful estimates of the weights for his/her objective function.

- If CI/RI < 0.10, the degree of consistency is satisfactory, but if CI/RI > 0.10 serious inconsistency may exist, and the AHP may not yield meaningful results.

- In the example, CI/RI = 0.043 / 1.24 = 0.035 < 0.10, → no serious inconsistency.