Powder Diffraction

Ashfia Huq

Spallation Neutron Source

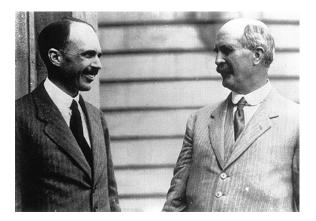
Oak Ridge National Laboratory

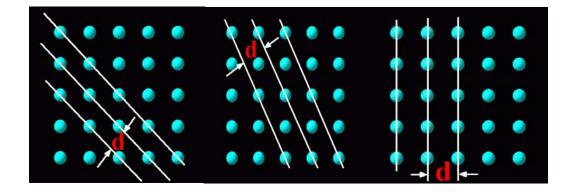




Bragg's law

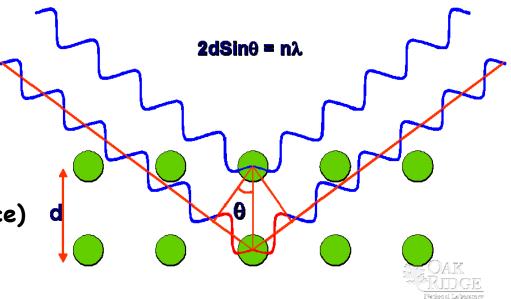
W.H. Bragg (1862-1942) W.L. Bragg (1890-1971)



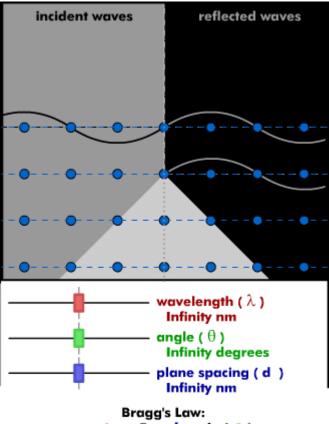


Shared 1915 Nobel Prize

·Zinc Blend (fcc not sc)
·NaCl (not molecular)
·Diamond (two overlapping fcc lattice)



Bragg's law



 $\mathbf{n} \mathbf{x} \lambda = \mathbf{2} \mathbf{x} \mathbf{d} \mathbf{x} \sin(\theta)$ NaN x Infin= 2 xInfinitx sin[nfinity]

WAVES OUT OF PHASE use the sliders to make n an integer



Where are the atoms?

We need wavelength (λ) ~ Object size (for condensed matter that is Å)



X-ray:

 $(\lambda : 10^{-9}m - 10^{-11}m)$

$$\lambda$$
[Å] = 12.398/E_{ph}[keV]

Source:

- Lab diffractometers
- Synchrotron Sources

Neutron:

(thermal λ : 1-4Å)

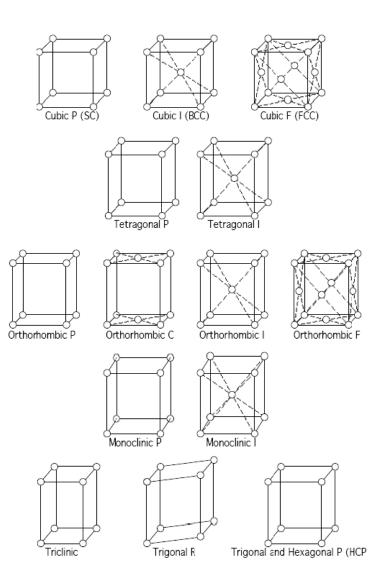
$$E_{n}[meV] = 81.89 / \lambda^{2}[Å]$$

Source:

- Reactors (fission)
- Spallation Source



Crystal Structure = Basis + Lattice



Basis		Lattice				
2	[
1	\otimes					
System	Angles and Dimensions	Lattices in System				
Triclinic	a≠b≠c, α≠β≠γ	P (primitive)				
Monoclinic	a≠b≠c, α=γ=90°≠β	P (primitive)				
		I (body centered)				
Orthorhombic	a≠b≠c, α=β=γ=90°	P (primitive)				
		C (base centered)				
		I (body centered)				
		F (face centered)				
Tetragonal	a=b≠c, α=β=γ=90°	P (primitive)				
		I (body centered)				
Cubic	a=b=c, α=β=γ=90°	P (primitive)				
		I (body centered)				
		F (face centered)				
Trigonal	a=b=c, 120°>α=β=γ≠90°	R (rhombohedral primitive)				
Hexagonal	a=b≠c, α=β=90°, γ=120°	R (rhombohedral primitive)				

3d Bravais Lattices: 14 types in 7 classes

Des McMorrow, Xray Neutron summer school 2008

Bragg Scattering from a crystal

$$F_{hkl} = \sum_{j} f_{j} e^{2\pi i \left(hx_{j} + ky_{j} + lz_{j}\right)}$$

Simple Example:

Simple cubic cell with one atom basis at (000)

$$F_{hkl} = fe^{2\pi i(h.0+k.0+l.0)} = f$$

For bcc lattice : SC with (000, $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$) basis

 $F_{hkl} = f[e^{2\pi i(h.0+k.0+l.0)} + e^{2\pi i(h+k+l).\frac{1}{2}}] = 2f \text{ for even (h+k+l)} = 0 \text{ for odd (h+k+l)}$



Simple Example:

Rewrite Bragg's Law for cubic system:

 $\sin^2\theta = (\lambda/4a)^2(h^2+k^2+l^2)....(1)$

(h ² +k ² +l ²)	0	1	2	3	4	5	6	•	8	9	10	11	12	13	3	14	•	16	17	18	19	20	21	22	. 24
SC																									
bcc																									
fcc																									
diamond																									



In a powder diffraction measurement (Al powder), we measure Bragg angles θ using Cu K_a radiation. The Bragg angles are 19.48°,22.64°, 33.00°,39.68°, 41.83°, 50.35°, 57.05°, 59.42°. Determine the type of lattice and the lattice parameter.

θ	sin ² 0	sin²θ/sin²θ _i	3*sin²θ/sin² θ _i
19.48	0.111	1	3
22.64	0.148	1.333	4
33.00	0.297	2.676	8
39.68	0.408	3.675	11
41.83	0.445	4.009	12
50.35	0.593	5.340	16
57.05	0.704	6.342	19
59.42	0.741	6.676	20

We have indexed the cell to a fcc lattice. λ = 1.5417. Plug in any of these values to eqn (1) to find a = 4.004Å.



Things get a little more complex when you consider systems that are not orthogonal

General Formula:

$$\frac{1}{d_{hkl}^2} = |H_{hkl}|^2 = (h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3).(h\vec{b}_1 + k\vec{b}_2 + l\vec{b}_3)$$

recall (reciprocal lattice vector)

$$\vec{b}_i = 2\pi \frac{\vec{a}_j \times \vec{a}_k}{\vec{a}_i \cdot \vec{a}_j \times \vec{a}_k}$$

$$\frac{1}{d_{hkl}^2} = \frac{1}{(1+2\cos\alpha\cos\beta\cos\gamma - \cos^2\alpha - \cos^2\beta - \cos^2\gamma)} \times \{\frac{h^2\sin^2\alpha}{a^2} + \frac{k^2\sin^2\beta}{b^2} + \frac{l^2\sin^2\gamma}{c^2} + \frac{2hk}{ab}(\cos\alpha\cos\beta - \cos\gamma) - \frac{2kl}{bc}(\cos\beta\cos\gamma - \cos\alpha) - \frac{2hk}{ab}(\cos\gamma\cos\alpha - \cos\beta)\}$$



Space Groups

There are 7 crystal systems:

*Triclinic, all cases not satisfying the requirements of any other system. There is no necessary symmetry other than translational symmetry, although inversion is possible.

*Monoclinic, requires either 1 twofold axis of rotation or 1 mirror plane.

*Orthorhombic, requires either 3 twofold axes of rotation or 1 twofold axis of rotation and two mirror planes.

*Tetragonal, requires 1 fourfold axis of rotation.

*Rhombohedral, also called trigonal, requires 1 threefold axis of rotation.

Hexagonal, requires 1 six fold axis of rotation.

*Isometric or cubic, requires 4 threefold axes of rotation.

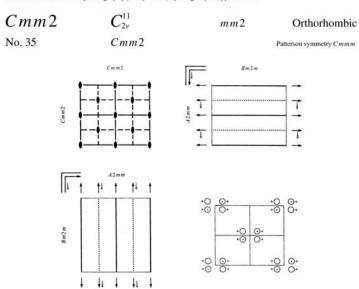
Crystal system	No. of <u>point</u> groups	No. of <u>bravais</u> lattices	No. of <u>space</u> groups
<u>Triclinic</u>	2	1	2
Monoclinic	3	2	13
<u>Orthorhombic</u>	3	4	59
Tetragonal	7	2	68
Rhombohedral	5	1	25
<u>Hexagonal</u>	7	1	27
Cubic	5	3	36
Total	32	14	230
			35 10 F

10 Managed by UT-Battelle for the U.S. Department of Energy

http://en.wikipedia.org/wiki/Crystal_system

Space Groups

International Tables for Crystallography (2006). Vol. A, Space group 35, pp. 238-239.



Origin on mm2

Asymmetric unit $0 \le x \le \frac{1}{4}; \quad 0 \le y \le \frac{1}{2}; \quad 0 \le z \le 1$

Symmetry operations

For (0,0,0)+ set (1) 1	(2) 2	0,0,z	(3) <i>m x</i> ,0, <i>z</i>	(4) m 0, y, z
For $(\frac{1}{2}, \frac{1}{2}, 0)$ + set (1) $t(\frac{1}{2}, \frac{1}{2}, 0)$	(2) 2	1.1.z	(3) $a x, \frac{1}{2}, z$	(4) $b = \frac{1}{2}, y, z$

CONTINUED

No. 35

Cmm2

Generators selected (1); t(1,0,0); t(0,1,0); t(0,0,1); $t(\frac{1}{2},\frac{1}{2},0)$; (2); (3)

Po	sitie	ons					
Multiplicity, Coo Wyckoff letter,		Coord	linates	Reflection conditions			
		ametry		(0,0,0)+	$(\frac{1}{2}, \frac{1}{2}, 0)+$		General:
8	f	1	(1) <i>x</i> , <i>y</i> , <i>z</i>	(2) <i>x</i> , <i>y</i> , <i>z</i>	(3) <i>x</i> , 9, <i>z</i>	(4) <i>x</i> , <i>y</i> , <i>z</i>	$ \begin{array}{l} hkl : \ h+k=2n \\ 0kl : \ k=2n \\ h0l : \ h=2n \\ hk0 : \ h+k=2n \\ h00 : \ h=2n \\ 0k0 : \ k=2n \end{array} $
							Special: as above, plus
4	e	<i>m</i>	0, y, z	$0, \bar{y}, z$			no extra conditions
4	d	. m .	x,0,z	x,0,z			no extra conditions
4	с	2	$\frac{1}{4}, \frac{1}{4}, Z$	$\frac{1}{4}, \frac{3}{4}, z$			hkl: $h=2n$
2	b	<i>m m</i> 2	$0, \frac{1}{2}, z$				no extra conditions
2	a	<i>m m</i> 2	0,0,z				no extra conditions
Sy	mm	etry of s	pecial proje	ctions			
Along [001] c2mm			Along [100] p11		Along [010] p11m		
		$\mathbf{b}' = \mathbf{t}$)		$a' = \frac{1}{2}b$ $b' =$	c	$\mathbf{a}' = \mathbf{c}$ $\mathbf{b}' = \frac{1}{2}\mathbf{a}$
Ori	igin	at 0,0,z			Origin at $x, 0, 0$		Origin at 0, y, 0

Maximal non-isomorphic subgroups

	annu non isomorphic s	angroups
I	[2] C1m1 (Cm, 8) [2] Cm11 (Cm, 8) [2] C112 (P2, 3)	(1; 3)+ (1; 4)+ (1; 2)+
IIa	[2] Pba2 (32) [2] Pbm2 (Pma2, 28) [2] Pma2 (28) [2] Pmm2 (25)	1; 2; (3; 4) + $(\frac{1}{2}, \frac{1}{2}, 0)$ 1; 3; (2; 4) + $(\frac{1}{2}, \frac{1}{2}, 0)$ 1; 4; (2; 3) + $(\frac{1}{2}, \frac{1}{2}, 0)$ 1; 2; 3; 4
III.	1211-22/2 201462	10111-02/d - 201/1-02 462 F

Maximal isomorphic subgroups of lowest index

IIc [2] Cmm2 (c' = 2c) (35); [3] Cmm2 (a' = 3a or b' = 3b) (35)

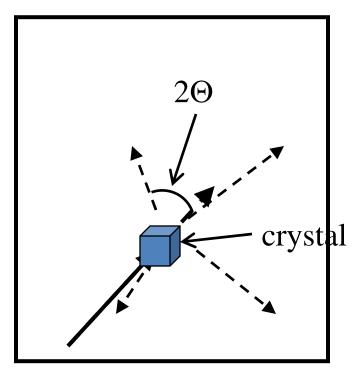
Minimal non-isomorphic supergroups

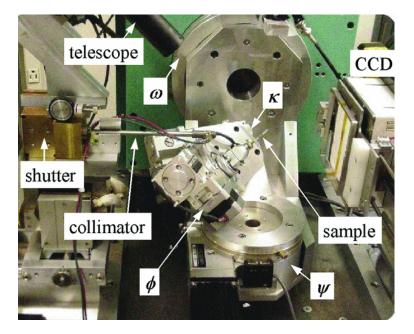
I

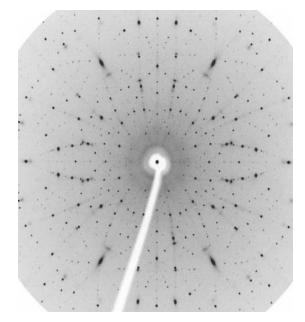
- $[2] Cmmm (65); [2] Cmme (67); [2] P4mm (99); [2] P4bm (100); [2] P4_{2}cm (101); [2] P4_{2}nm (102); [2] P42m (111); [2] P42m (113); [3] P6mm (183)$
- **II** [2] Fmm2(42); [2] $Pmm2(\mathbf{a}' = \frac{1}{2}\mathbf{a}, \mathbf{b}' = \frac{1}{2}\mathbf{b})$ (25)

Single Crystals:

Sample must be correctly oriented in space with respect to the chosen reflection plane.

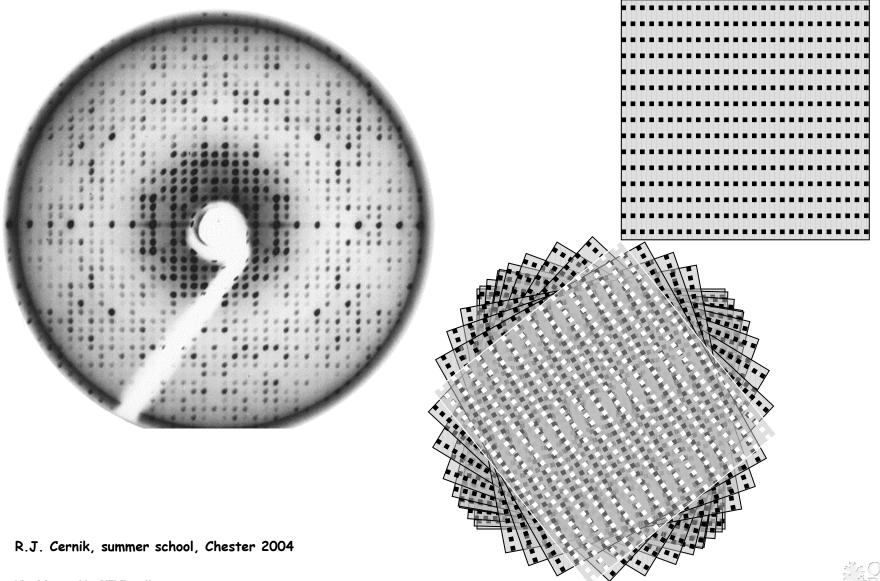








What if you don't have a single crystal?





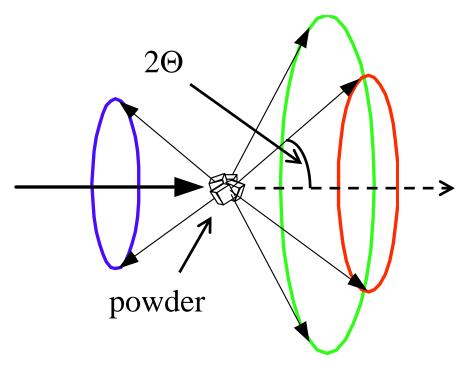


Powder Diffraction

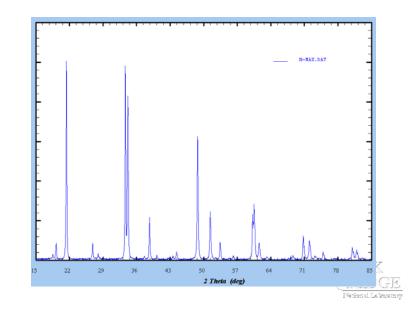
*Powder

 Sizable samples have billions of crystals

In the absence of texture, all crystal orientations are equally represented



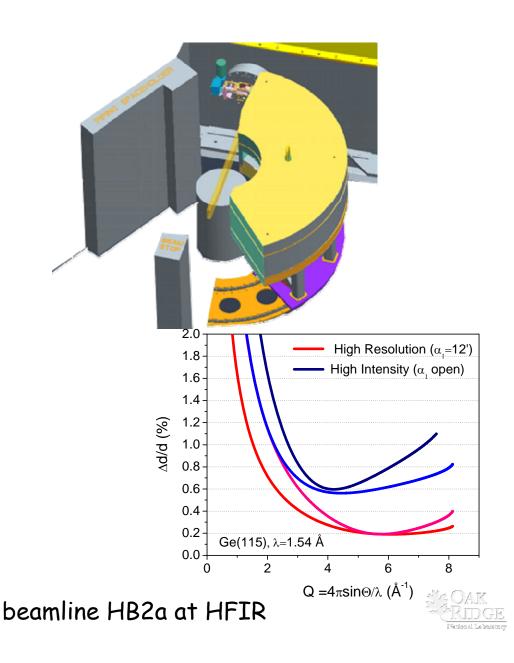




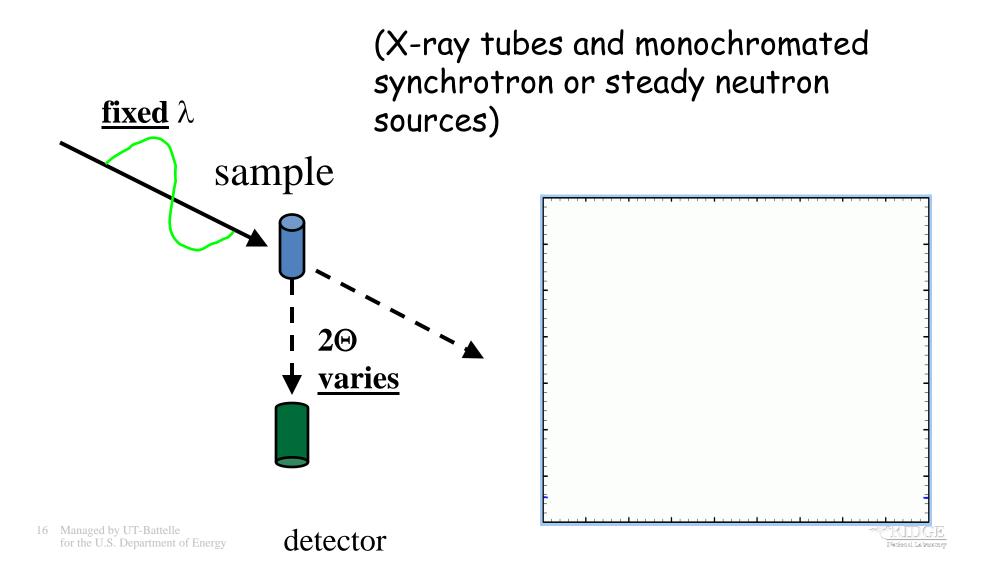
Powder Instruments at user facilities



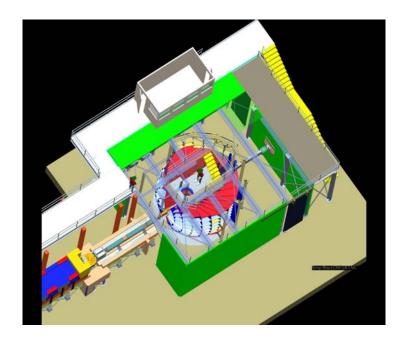
beamline 11BM at APS

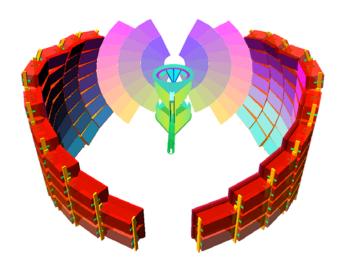


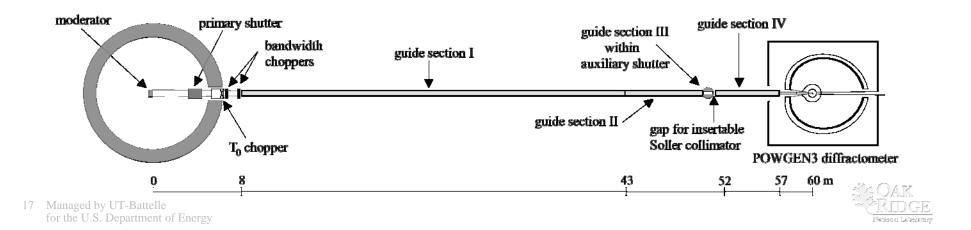
Constant wavelength ($2dsin\Theta = \lambda$)



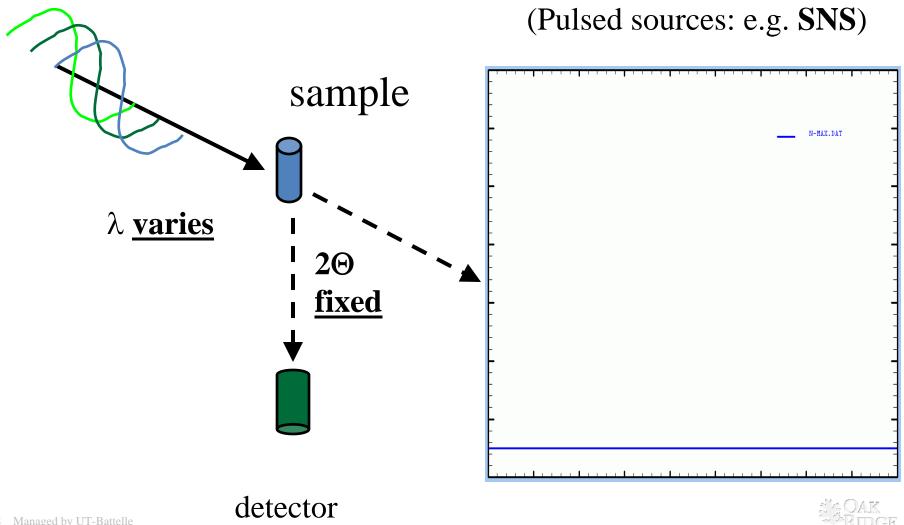
Time of flight instrument (POWGEN)







Time-of-flight ($2dsin\Theta = \lambda$)



Netfonel, Lebnsetner

Time of Flight method

de Broglies relationship:

 λ =h/mv=ht/mL

```
Combine with Bragg to get
```

```
2dsin\theta = ht/mL => t=2dsin\theta.mL/h
```

 t_{hkl} =505.5569Ld_{hkl}sin θ

Resolution:

 $R(d) = \Delta d/d = [(\Delta t/t)^2 + (\Delta L/L)^2 + (\Delta \theta)^2 \cot^2 \theta]^{\frac{1}{2}}$

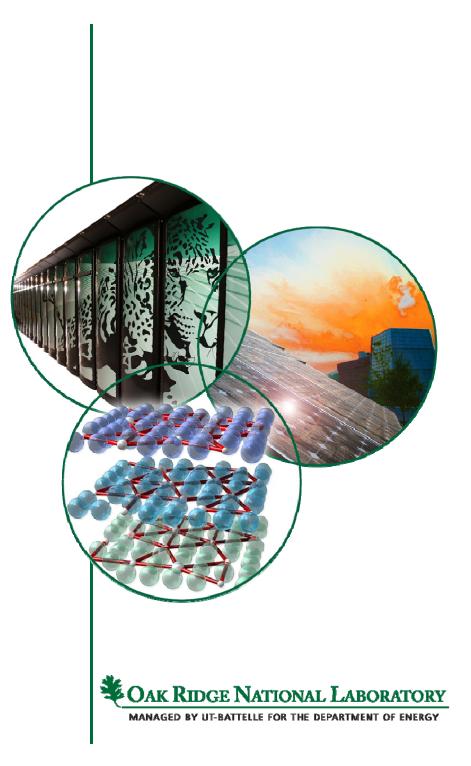
Bragg Intensity:

$$Y_{ph} = F_{ph}^{2}H(T - T_{ph})K_{ph}$$
 where $K_{ph} = E_{ph}A_{h}O_{ph}M_{p}L/V_{ph}$

$$y_{\tau}(i) = D_{\text{expt}} \{ H(t)w(i) \} f_{DS} I_{L}(\lambda) \varepsilon(\lambda) N_{s} \frac{\lambda^{4} \cos \theta \Delta \theta}{4 \upsilon_{uc} \sin^{2} \theta} A_{\tau}(\lambda) E_{\tau}(\lambda) j_{\tau} |F_{\tau}|^{2}$$

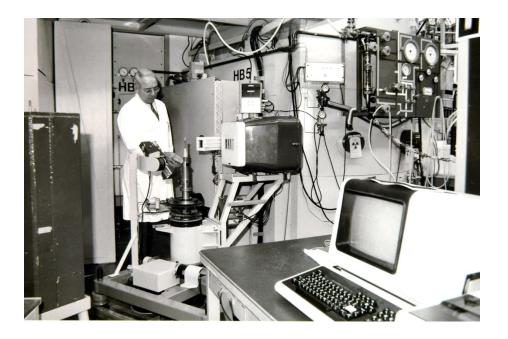


Rietveld Refinement





Hugo Rietveld

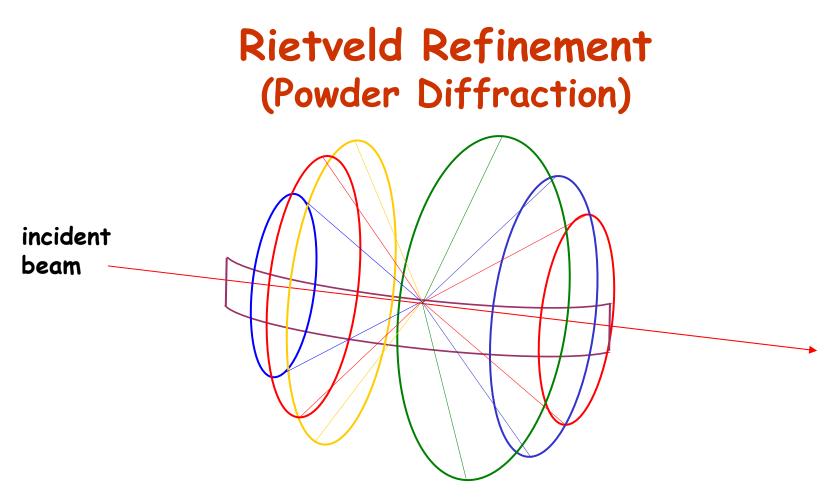


Dr. Rietveld at the neutron powder diffractometer at the High Flux Reactor of the Energy Reseach Foundation ECN in Petten, The Netherlands. (1987)

J. Appl. Cryst. 2, 65, 1969

"A structure refinement method is described which does not use integrated neutron powder intensities, single or overlapping, but employs directly the profile intensities obtained from step-scanning measurements of the powder diagram. Nuclear as well as magnetic structures can be refined, the latter only when their magnetic unit cell is equal to, or a multiple of, the nuclear cell. The least-squares refinement procedure allows, with a simple code, the introduction of linear or quadratic constraints between the parameters."





In a diffraction experiment if the sample is a powder, there will be many grains aligned to diffract the incident beam of neutrons/x-rays. 3D information is reduced to 1D, makes analysis harder than single crystal experiments.



Rietveld Refinement Least Square Method

Model that describes the structure Profile parameters (lattice, line-shape, background etc.) Atomic information

(fractional co-ordinates, thermal parameters fractional occupancy etc.)

No effort is made in advance to allocate observed intensity to particular Bragg reflections nor to resolve overlapped reflections. Consequently, a reasonably good starting model is needed. The method is a structure refinement method and not a structure solution method.



Rietveld Refinement (cont'd)

The contribution of an atom at r_i in real space to a reflection K = (hkl) is given by the structure factor of that reflection

$$F_{hkl} = \sum_{j} N_{j} b_{j} e^{2\pi i K \cdot r_{j}} e^{-M_{j}}$$

(M_j = Debye-Waller factor, $M_j = 8\pi^2 \overline{u_s^2} \sin^2 \theta / \lambda^2$ N_j = site occupancy b_j = scattering length)

Rietveld refinement models the entire pattern as calculate intensities:

$$y_{oi} = s \sum_{K} L_{K} |F_{K}|^{2} f(t_{i} - t_{K}) + y_{bi}$$

(s = scale factor, L_{K} = instrumental and sample factors, f^{K} = profile function, y_{ci} = background)



Rietveld Refinement (cont'd)

The Least Square refinement then adjusts the refinable parameters to minimize the residuals until the best fit is obtained.

$$\chi^2 = \frac{\sum_{i=1}^{N_{obs}} \mathbf{w}_i (\mathbf{I}_{oi} - \mathbf{I}_{ci})^2}{(\mathbf{N}_{obs} - \mathbf{N}_{var})}$$

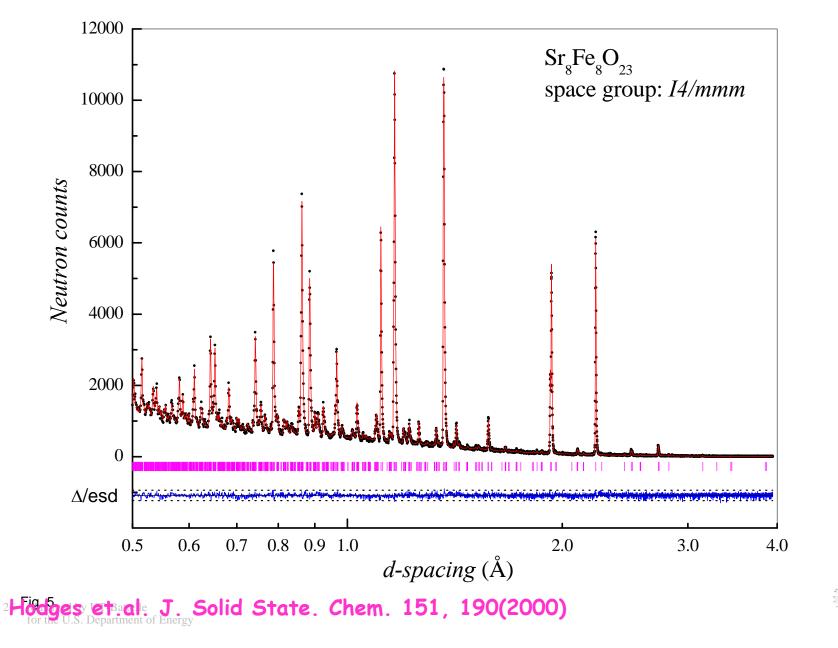
Here $w_i = 1/\sigma_i^2$, is the statistical weight of the *i* th profile observation which is the inverse of the variance of the *i* th observation. I_{oi} and I_{ci} are observed and calculated intensities. From a purely mathematical point of view, R_{wp} is the most meaningful R factor because the numerator is the residual being minimized. So this is the best indicator of the progress of the refinement.

$$R_{wp}^{2} = \frac{\sum_{i=1}^{N_{obs}} \mathbf{w}_{i} (\mathbf{I}_{oi} - \mathbf{I}_{ci})^{2}}{\mathbf{w}_{i} (\mathbf{I}_{oi})^{2}}$$

While numerical criteria are important it is also imperative to use graphical criteria of fit like difference plots.



Typical Rietveld Refinement





Peak Profiles:

CW peak shapes

Convolution of pseudo-Voigt with result of considering the intersection of the Debye Scherrer diffraction cone that is at the scattering angle of 2Θ and a finite height slit positioned below 2Θ by τ

$H(\Delta T)=\int P(\Delta T-\tau)D(\tau)d\tau$

 $\sigma = [U \tan^2 \Theta + V \tan \Theta + W + P/\cos^2 \Theta]^{1/2}$ (Gaussian variance) $\gamma = (X + X_{e/s} \cos \phi) / \cos \Theta + (Y + Y_e \cos \phi + g_L d^2) \tan \Theta$ (Lorentzian, size broadening due to stacking fault)

TOF diffractometers that use cryogenic moderators have more complex behavior for a b and peak position.

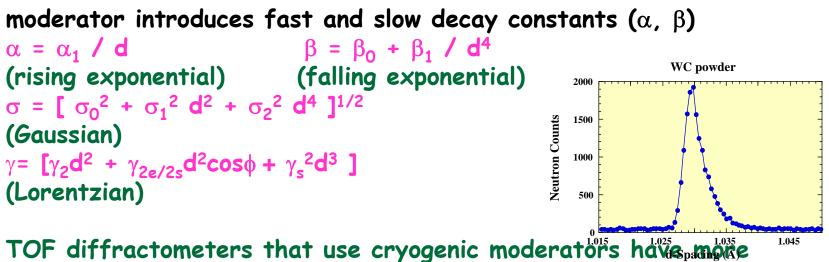


Peak Profiles:

TOF peak shapes

Convolution of rising and falling (back to back) exponentials with pseudo-Voigt (a linear combination of Lorentzian and a Gaussian)

H(∆T)=∫E(∆T-†)P(†)d†



complex behavior for a b and peak position.

28 Ref: R,B. Von Dreele et. al., J. Appl. Cryst. 15, 581 (1982) for the U.S. Department of Energy



Information obtained from Rietveld Refinement:

Phase Fractions

Scale factors relate the weight fractions of p_{th} phase :

$$W_p = \frac{S_{ph}m_p}{\sum_{p=1}^{N_p} S_{ph}m_p}$$

(m_p = unit cell mass for phase p, S_{ph} = Rietveld scale factor)

Sample Broadening

Only affects the Gaussian component of the peak width; contributions from strain S and particle size broadening P can be separated:

S =
$$(1/C)[8ln2(\sigma_1^2 - \sigma_{1i}^2)]^{1/2}100\%$$

 $(\sigma_{1i} = \text{strain-free value for } \sigma_1)$

 $P = (CK)/[8ln2\sigma_2]^{1/2} Å$

(K = Scherrer constant)



Other things to be keep in mind

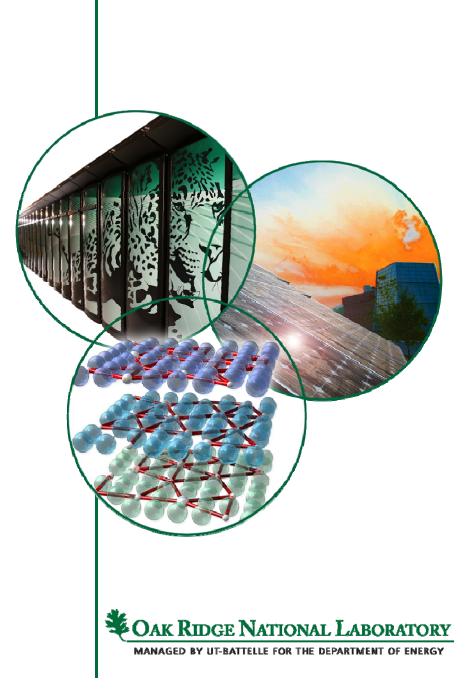
Recall $K_{ph} = E_{ph}A_hO_{ph}M_pL/V_p$

- **E**_{ph}: Extinction correction
- A_h: Absorption correction
- **O**_{ph}: **Preferred orientation correction**
- **M**_p: Reflection multiplicity
- L: Angle dependent correction (Lorentz-polarization)
- V_p: Unit cell volume for the phase



Ab-initio Structure Solution from Powder Diffraction?





"Powder diffraction is of minimal value in crystal structure analysis and is not discussed in this book."

Ladd and Palmer in Structure determination by X-ray crystallography' Plenum Press

but here are some other wise words

>"Heavier-than-air flying machines are impossible" Lord Kelvin, president, Royal Society (1895)

>"I think there is a world market for about five computers." Thomas J. Watson, chairman of the board of IBM(1943)

≻"640K ought to be enough for anybody." Bill Gates, 1981

Bill David, PSI Powder Diffraction Workshop June 2008



Structure solution from powder data:

Given atom positions, it is straightforward to compute the diffraction pattern

$$I_{hkl} = \left| \sum_{\text{atoms } j} f_j \exp(i\vec{Q}_{hkl} \cdot \vec{R}_j) \right|^2$$

Solve a new structure from powder data

- 1. Get data
- 2. Find the lattice
- 3. Space group (internal symmetries) systematic absences, density, guess, luck
- 4. Extract intensities of each individual (*hkl*) peak
- 5. Solve structure
- 6. Refine



Methods of Solution:

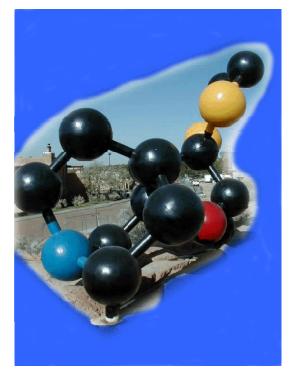
- Adaptation of standard single crystal techniques
- direct space methods based on prior chemical knowledge
- A combination of the two
- Global optimization: simulated annealing, genetic algorithm
- Charge flipping algorithm

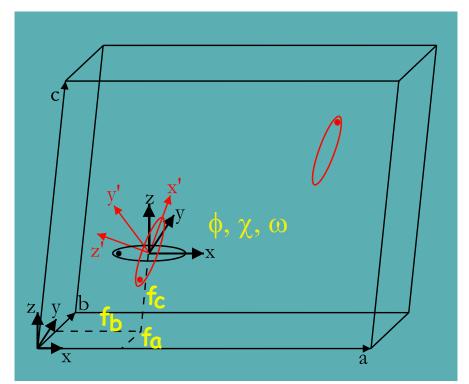


Solution of structures of organic molecules from powder data:

Difficulties: generally weak scatterers, serious overlap for d < 2A, patterns weak at high angles.

Use of known molecular geometry is helpful -- make a model, put it into the lattice, and test it against the data.







Using simulated annealing to search this large space.

Comparing proposed solution with data

Ideas borrowed from simulations of spin glasses:

- 1. Metropolis algorithm to generate ensemble of configurations representative of a given temperature
- 2. Gradually lower temperature to find lowest energy states

Measure agreement of proposed solution with data: R-factor \leftrightarrow energy

- 1. Always accept a move that leads to a better fit to the data
- 2. don't reject all moves that lead to a poorer fit, accept poorer fits with Boltzmann probability

 $exp[-(\chi_{new}^2 - \chi_{old}^2)/T]$



Software and other resources: http://www.ccp14.ac.uk/

Indexing:

• Crysfire: <u>http://www.ccp14.ac.uk/tutorial/crys/</u>

Rietveld:

- GSAS: <u>http://www.ccp14.ac.uk/solution/gsas/</u>
- Fullprof: <u>http://www.ccp14.ac.uk/ccp/web-mirrors/fullprof/</u>
- Rietan, Topas, Expo, JANA, Jade etc.

Structure Solution:

- DASH: <u>http://www.ccdc.cam.ac.uk/products/powder_diffraction/dash/</u>
- FOX: <u>http://vincefn.net/Fox/</u>
- PSSP:
- Topas: http://www.dur.ac.uk/john.evans/topas_academic/topas_main.htm



Reference material

Elements of X-ray diffraction (B.D. Cullity)

- □Introduction to X-ray Powder Diffractometry (R. Jenkins & R.L. Snyder)
- Modern Powder Diffraction (edited by Bish & Post)
- The Rietveld Method (edited by R.A. Young)
- Neutron Diffraction (G.E. Bacon)
- Theory of neutron scattering from condensed matter (S.W. Lovesey)
- Structure determination from powder diffraction data (Edited by W.I.F. David, K. Shankland, L.B. McCusker and Ch. Baerlocher)

