

Outline

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- III. Nuclear inelastic scattering
 - A. Correlation functions
 - B. Examples

Nuclear

Magnetic

IV. Magnetic inelastic scattering

Interaction of the neutron

General inelastic scattering

- A. Correlation functions
- B. Examples



Neutron interaction with matter



Properties of the neutron

- Mass $m_n = 1.675 \times 10^{-27} \text{ kg}$
- Charge0
- Spin-1/2, magnetic moment $\mu_n = -1.913 \mu_N$

Neutrons interact with...

- Nucleus
 - Crystal structure/excitations (eg. phonons)
- Unpaired e⁻ via dipole scattering
 - Magnetic structure/excitations (eg. spin waves)







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Wavelength-energy relations



• Energy (E), velocity (v), wavenumber (k), wavelength (λ)

$$k = \frac{m_n v}{h} = \frac{2\pi}{\lambda}$$
$$E = \frac{h^2 k^2}{2m_n} = \frac{h^2}{2m_n} \left(\frac{2\pi}{\lambda}\right)^2 = \frac{81.81 \text{meV} \cdot \text{\AA}^2}{\lambda^2}$$
$$E = k_B T = \left(0.08617 \text{meV} \cdot \text{K}^{-1}\right) T$$



 λ ~ interatomic spacing \rightarrow E ~ excitations in condensed matter

	Energy (meV)	Temperature (K)	Wavelength (Å)
Cold	0.1 – 10	1 – 120	4 – 30
Thermal	5 – 100	60 – 1000	1 – 4
Hot	100 – 500	1000 - 6000	0.4 – 1

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Dynamical (time) scales





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Shorter length scales school

Inelastic scattering

• Scattering process that changes the energy of the neutron

- Conservation of energy and momentum
 - $\mathbf{h}\boldsymbol{\omega} = \boldsymbol{E}_i \boldsymbol{E}_f \qquad \mathbf{Q} = \mathbf{k}_i \mathbf{k}_f$
- Scattering triangle





Elastic scattering
$h\omega = 0$
$\left \mathbf{k}_{i}\right = \left \mathbf{k}_{f}\right $
$Q = 2k_i \sin \theta$







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Inelastic scattering

• Scattering process that changes the energy of the neutron

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 - $\mathbf{h}\boldsymbol{\omega} = \boldsymbol{E}_i \boldsymbol{E}_f \qquad \mathbf{Q} = \mathbf{k}_i \mathbf{k}_f$
- Scattering triangle





Inelastic scattering $h\omega > 0$ $|\mathbf{k}_i| \neq |\mathbf{k}_f|$ $Q^2 = k_i^2 + k_f^2 - 2k_i k_f \cos 2\theta$





Nuclear (lattice) excitations



Neutron scattering measures simultaneously the wavevector and energy of **collective excitations** \rightarrow dispersion relation, $\omega(\mathbf{q})$ In addition, **local excitations** can of course be observed

• Commonly studied excitations

- Phonons
- Librations and vibrations in molecules
- Diffusion
- Collective modes in glasses and liquids

Excitations can tell us about

- Interatomic potentials & bonding
- Phase transitions & critical phenomena (soft modes)
- Fluid dynamics
- Momentum distributions & superfluids (eg. He)
- Interactions (eg. electron-phonon coupling)

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Atomic diffusion

For long times compared to the collision time, atom diffuses



Auto-correlation function $G_{s}(r,t) = \left\{ 6\pi \left\langle r^{2}(t) \right\rangle \right\}^{-3/2} \exp \left(-\frac{r^{2}}{6 \left\langle r^{2}(t) \right\rangle} \right)$

 $S(Q,\omega) = \frac{1}{\pi h} \exp\left(\frac{h\omega}{2k_BT}\right) \frac{DQ^2}{\omega^2 + (DQ^2)^2}$





Diatomic molecule





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Diatomic molecule





Molecular vibrations



- Large molecule, many normal modes
- Harmonic vibrations can determine interatomic potentials





Prassides et al., Nature 354, 462 (1991).

Phonons



• Normal modes in periodic crystal \rightarrow wavevector

$$\mathbf{u}(l,t) = \frac{1}{\sqrt{NM}} \sum_{j\mathbf{q}} \varepsilon_j(\mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{l}) \hat{B}(\mathbf{q}j,t)$$

• Energy of phonon depends on **q** and polarization

Doagisudiselmoote





FCC structure



FCC Brillouin zone ¹⁴

Phonon intensities





Guthoff et al., Phys. Rev. B 47, 2563 (1993).



Spin correlation functions





$$\frac{d^{2}\sigma}{d\Omega dE_{f}} = \frac{k_{f}}{k_{i}} \left[\frac{1}{2} \gamma r_{0}gF(Q) \right]^{2} \sum_{\alpha\beta} \left(\delta_{\alpha\beta} - \hat{Q}_{\alpha}\hat{Q}_{\beta} \right) \sum_{jj'} e^{i\mathbf{Q}\cdot(\mathbf{R}_{j'}-\mathbf{R}_{j})} \frac{1}{2\pi\hbar} \int dt e^{-i\omega t} \left\langle S_{j}^{\alpha}(0)S_{j'}^{\beta}(t) \right\rangle$$
Scattering Dipole interaction Spin-spin correlation function
$$\frac{d^{2}\sigma}{d\Omega dE_{f}} \bigg|_{inel} = \frac{k_{f}}{k_{i}} \left[\frac{1}{2} \gamma r_{0}gF(Q) \right]^{2} \sum_{\alpha\beta} \left(\delta_{\alpha\beta} - \hat{Q}_{\alpha}\hat{Q}_{\beta} \right) \left(1 - e^{\hbar\omega/kT} \right)^{-1} \frac{1}{\pi(g\mu_{B})^{2}} \operatorname{Im} \left\{ \chi^{\alpha\beta}(\mathbf{Q}, \omega) \right\}$$

The cross-section is proportional to the magnetic susceptibility, i.e. it is the response of the system to spatially & time varying magnetic field

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Spin excitations



• Spin excitations

- Spin waves in ordered magnets
- Paramagnetic & quantum spin fluctuations
- Crystal-field & spin-orbit excitations

• Magnetic inelastic scattering can tell us about

- Exchange interactions
- Single-ion and exchange anisotropy (determine Hamiltonian)
- Phase transitions & critical phenomena
- Quantum critical scaling of magnetic fluctuations
- Other electronic energy scales (eg. CF & SO)
- Interactions (eg. spin-phonon coupling)

Paramagnetic scattering



$$\left\langle S_{j}^{\alpha}S_{j'}^{\beta}\right\rangle = 0 \left(j \neq j'\right)$$

Single ion scattering

$$\left\langle S_{j}^{z}(0)S_{j}^{z}(t)\right\rangle = \left\langle \left(S_{j}^{z}\right)^{2}\right\rangle e^{-\Gamma t} = \frac{1}{3}\left\langle \left(\mathbf{S}_{j}\right)^{2}\right\rangle e^{-\Gamma t} = \frac{1}{3}S\left(S+1\right)e^{-\Gamma t}$$

$$\frac{\operatorname{Im}\left\{\chi^{zz}(0,\omega)\right\}}{\pi h\omega} = \frac{g^2 S(S+1)\mu_B^2}{3k_B T} \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (h\omega)^2}$$

- Inverse width, $1/\Gamma$, gives relaxation time
- Note crystal field excitation

$$\chi_0 = \int_{-\infty}^{\infty} \frac{\operatorname{Im}\left\{\chi^{zz}(0,\omega)\right\}}{\pi \hbar \omega} d\omega = \frac{g^2 S(S+1)\mu_B^2}{3k_B T}$$



McQueeney et al., Phil. Mag. B 81, 675 (2001).

Spin waves





Linear spin waves

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21

[00ξ]

0.00 0.25 0.50 0.75 1.00

Fe₃O

150

125

Energy (meV)

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25

McQueeney et al., Phys. Rev. Lett. 99, 246401 (2007).



Perring et al., Phys. Rev. Lett. 77, 711 (1996).







Spin waves

Scattering experiments





5

4

6

0

Momentum Transfer (Å⁻¹)

- 1

23456

2 3

0

Triple-axis instruments





High flux isotope reactor - ORNL



HB-1A 3-axis spectrometer



- Hardware flexibility
- Constant-**Q** (or E) scans
 - Ideally suited for single-xtals

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Time-of-flight methods





Spallation neutron source



Pharos – Lujan Center

- Hardware inflexible
- Effective for powders
- Complicated Q,E-scans a challenge for single-xtals





INS data

• Intensities as a function of ${\bf Q}$ and ω









Reciprocal space



Kinematic limitations



- Many combinations of k_i,k_f for same Q,ω
 - Only certain configurations are used (eg. E_f-fixed)
- Cannot "close triangle" for certain Q,ω due to kinematics
 LaFeO₃ (G-type)





Triple-axis instruments



Workhorse INS instrument

- **k**_i, **k**_f defined by Bragg scattering
- Sample goniometer
- Detector

 θ_{M}

first arm)

Sample

(single crystal)

Resolution/collimators



Third arm

Reactor

Collimator

Counter

BF3

Sample rotationSample tilt

Co-aligned CaFe₂As₂ crystals

Samples need to be BIG

- ~ gram or cc
 - Counting times are long (mins/pt)







Monochromators

• Selects the incident wavevector





- Reflectivity
- focusing
- high-order contamination
 eg. λ/2 PG(004)

Mono	d(hkl)	uses
PG(002)	3.353	General
Be(002)	1.790	High k _i
Si(111)	3.135	Ν ο λ/2



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Beam monitors

High efficiency

 Low efficiency detectors for measuring beam flux

• $n + {}^{3}\text{He} \rightarrow {}^{3}\text{H} + p + 0.764 \text{ MeV}$

e⁻ drift to high voltage anode





Gas Detectors

Ionization of gas



Other triple-axis stuff



Soller Collimators

- Define beam divergence
- Q,ω resolution function

Filters

- Xtal Sapphire: fast neutron background
- Poly Be: low-energy (5 meV) band pass

0.8

0.7

transmission 9.0

0.3 0.2 0.1

0.5

1 1.5 2 2.5 3 3.5 4 4.5 5

wavelength(A)

PG: higher order contamination

Masks

- **Beam definition**
- **Background reduction**



Magic numbers

PG filter

- Best filter for rejection of $\lambda/2$ contamination
- E_f = 13.7, 14.7, 30.5, 41 meV





Sample environment



- Temperature, field, pressure
- Heavy duty for large
 sample environment
 - CCR
 - He cryostats
 - SC magnets
 - ...





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Acquiring data

- Energy gain, energy loss
 - **Detailed balance**
- Constant-Q scans
 - Most common
- Constant-ω scans
 - Used for steep dispersions





Lynn, et al., Phys. Rev. B 8, 3493 (1973).

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β-Ce 72 K

150 K

Constant-ω scans







Steep AF spin waves in CaFe₂As₂ McQueeney et al, PRL 101, 227205 (2008)

Slice of spin wave cone in Fe₃O₄ McQueeney et al, PRB 73, 174409 (2006)

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Constant-Q scans





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 $\lambda/2$ contamination

 E_i or E_f = 13.7, 14.7, 30.5, 41 meV

Configurations

• E_f-fixed mode

- Mono moves during ω -scan
- Beam monitor accounts for variations in incident flux

• E_i-fixed mode

- Analyzer moves during ω -scan
- Useful for expts requiring low background
- Analysis more complicated

Magic numbers



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Energy (meV)

40

50

60

20



 θ_{S}

No filter

April 2003

Monitor Rate (Counts/sec)

15000

10000

5000

10

incoherent – Bragg – Bragg

- Sample 2θ in Bragg condition for k_f-k_f
- Even for inelastic config, weak incoherent from mono

600

Bragg – incoherent – Bragg

• Eg. $k_i - 2k_f$

Spurions

- ħω = 41.1 meV
- E_f = 13.7 meV
 E_i = 54.8 meV
 4E_f = 54.8 meV
- Incoherent elastic scattering visible from analyzer λ/2



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Resolution

Resolution ellipsoid

- Beam divergences
- Collimations/distances
- Crystal mosaics/sizes/angles







Resolution effects







Resolution focusing



- Optimizing peak intensity
- Match slope of resolution to dispersion



Time-of-flight methods





Spallation neutron source



Pharos – Lujan Center

 Effectively utilizes time structure of pulsed neutron groups

$$t = \frac{d}{v} = \left(\frac{m}{h}d\right)\lambda$$





TOF vs. 3-axis



- epithermal (up to 2 eV)
- Total spectra (esp. powder samples)
- Absolute normalization
- Low-dimensional systems
- Hardware inflexible
- Software intensive



- High flux of thermal neutrons
- Focused studies in Q,ω (soft modes, gaps, etc.)
- Three-dimensional systems
- Hardware intensive
- Software inflexible

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Fermi Choppers

- Body radius ~ 5 cm
- Curved absorbing slats
 - B or Gd coated
 - ~mm slit size
- f = 600 Hz (max)
- Acts like shutter, $\Delta t \sim \mu s$





Figure 1. ISIS MAPS chopper and slit package assembly - exploded view





Position sensitive detectors

- ³He tubes (usu. 1 meter)
- Charge division
- Position resolution ~ cm
- Time resolution ~ 10 ns







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T-zero chopper

- Background suppression
- Blocks fast neutron flash







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Transport beam over long distances

- Background reduction
- Total external reflection
 - Ni coated glass

Guides

• Ni/Ti multilayers (supermirror)









Length = resolution

- Instruments $\sim 20 40$ m long
- E-resolution ~ 2-4% E_i

Size matters

More detectors

- SEQUOIA 1600 tubes, 144000 pixels
- Solid angle coverage 1.6 steradians
- Huge data sets
- 0.1 1 GB



SEQUOIA detector

vacuum vessel



Data visualization



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- Large, complex data from spallation sources
- Measure S(Q,ω) 4D function



Ye et al., Phys. Rev. B, 75 144408 (2007).



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Computation









Absolute normalization

Absolute normalization

- Using incoherent scattering from vanadium
- $\sigma/4\pi = 404 \text{ mbarns/Sr}$





Inelastic x-ray scattering



picomotor

piezo





 ϕ -scan of monochromator 1 meV $\Rightarrow \mu$ rad

T-scan of monochromator $1 \text{ meV}_{NXS} \underset{\text{School}}{\Rightarrow} 0.02 \text{ K}$



ω

Kinematics

- Essentially elastic scattering
- No kinematic limits

$$Q \approx 2k_i \sin \theta$$
$$h\omega = hc(k_i - k_f)$$





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SAMPLE SIZE

IXS vs. INS



IXS

- Simple scattering geometry (k_i≈k_f)
- Resolution function simpler (most angles fixed, E-scans only)
- No spurions (high-order refs. keV, no incoherent scat.)
- Can only do lattice excitations May 31, 2009 NXS School



10 20 30 40 50 60 70 80 90 100

Energy Transfer (meV)

-10 0





General neutron scattering

G. Squires, "Intro to theory of thermal neutron scattering", Dover, 1978.

S. Lovesey, "Theory of neutron scattering from condensed matter", Oxford, 1984.

R. Pynn, http://www.mrl.ucsb.edu/~pynn/.

Polarized neutron scattering

Moon, Koehler, Riste, Phys. Rev 181, 920 (1969).

Triple-axis techniques

Shirane, Shapiro, Tranquada, "Neutron scattering with a triple-axis spectrometer", Cambridge, 2002.

Time-of-flight techniques

B. Fultz, http://www.cacr.caltech.edu/projects/danse/ARCS_Book_16x.pdf