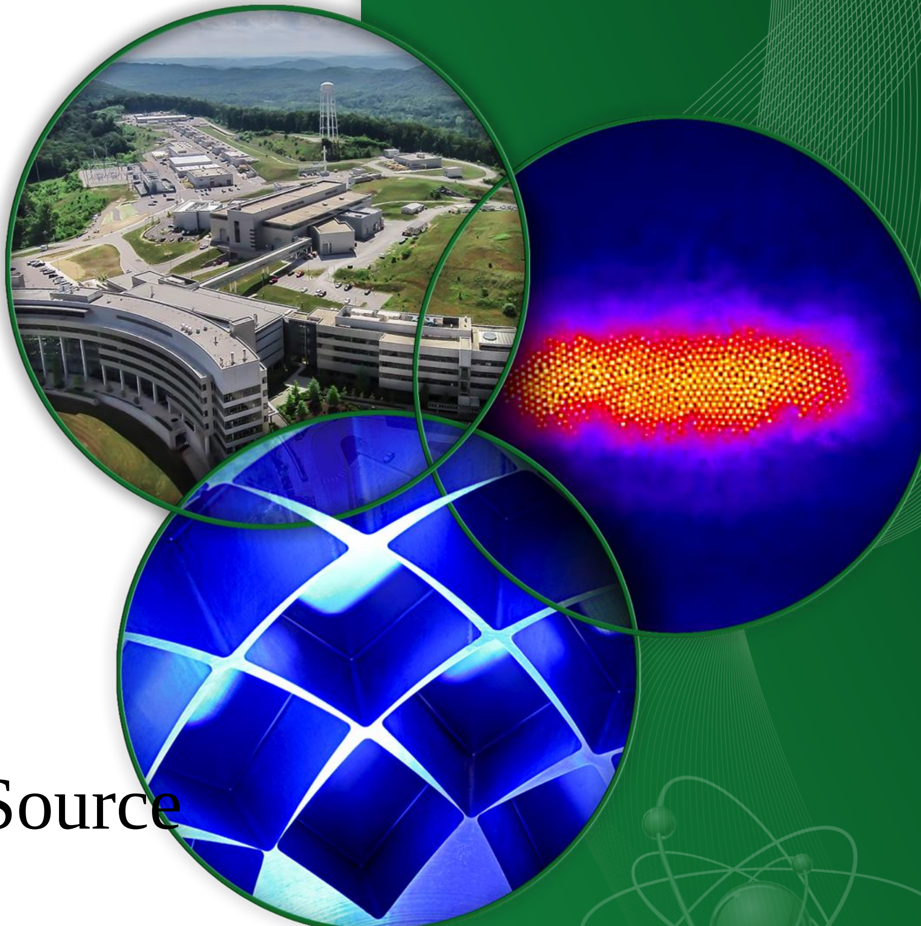
Practical Applications of Reliability Theory



George Dodson

Spallation Neutron Source

Topics

- Reliability Terms and Definitions
- Reliability Modeling as a tool for evaluating system performance
  - In the design phase what are the tradeoffs of cost vs. reliability performance?
  - In the operational phase, does the performance meet expectations?
- Analysis of the failure rate of systems or components
  - How do systems fail?
  - Is the failure rate "reasonable" ?
- Analytical calculation for the number of Spares
  - What kinds of spares are there?
  - What is a "reasonable" number of spares?

Reliability Terms

- Mean Time To Failure (MTTF) for non-repairable systems
- Mean Time Between Failures for repairable systems (MTBF)
- Reliability Probability (survival) $R(t)$
- Failure Probability (cumulative density function) $F(t)=1-R(t)$
- Failure Probability Density $f(t)$
- Failure Rate (hazard rate) $\lambda(t)$
- Mean residual life (MRL)

Important Relationships

$$R(t)+F(t)=1$$

$$f(t)=\lambda(t)\exp\left(-\int_0^t \lambda(u)du\right)=dF(t)/dt \qquad F(t)=\int_0^t f(u)du,$$

$$R(t)=1-F(t)=\exp\left(-\int_0^t \lambda(u)du\right) \qquad \lambda(t)=f(t)/R(t)$$

Where $\lambda(t)$ is the failure rate function

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MTBF

The MTBF is widely used as the measurement of equipment's reliability and performance. This value is often calculated by dividing the total operating time of the units by the total number of failures encountered. This metric is valid only when the data is exponentially distributed. This is a poor assumption which implies that the failure rate is constant if it is used as the sole measure of equipment's reliability.

Modeling

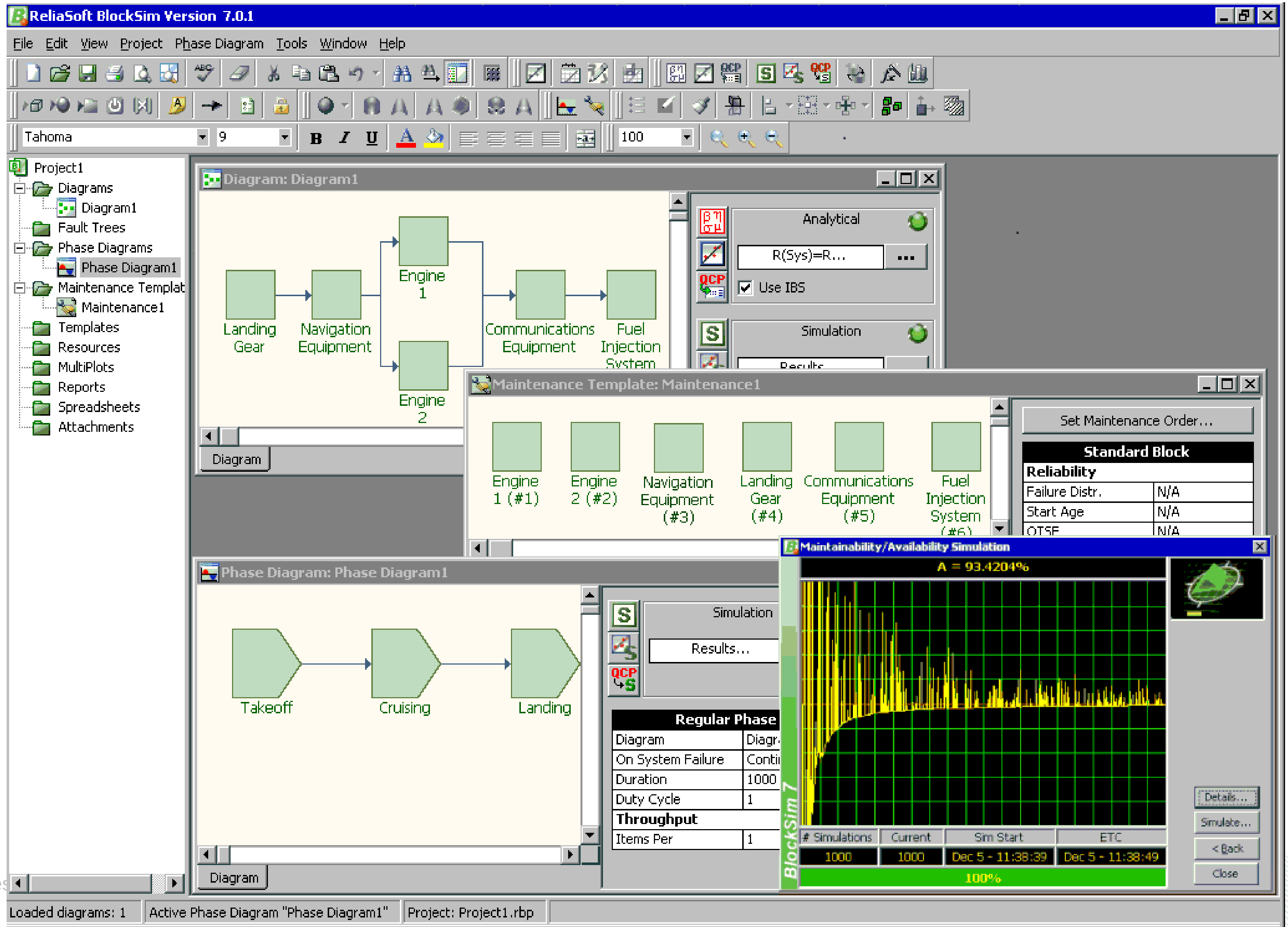
- There are essentially 2 types of models
  - Static
    - $\lambda(t)$ is constant
    - Easy, if only life were this simple
  - Dynamic
    - $\lambda(t)$ has a complex functional form
- To build a model:
  - Create a logical structure of components
  - Specify the reliability of each component
    - Drill down the structure as deep as you need to and/or have data

SNS Static Model ($\lambda(t)$ is constant) Uses Markov Chains

System	Subsyst	Assembl	Equipment/Failure Mode	Failure Rate (1/h x 10^{-6})	Failure Rate Source	MTBF (h)	Percent of Anticipated Failures	Effective MTBF for Unanticipated Failures (h)	Effective Failure Rate (1/h x 10^{-6})	No. of Equipments	No. of Spares Included	Type of Redundancy (1=hot 2=cold)	Repair (0=off-line, 1=on-line, 2=supermod	Effective Total Failure Rate (1/h x 10^{-6})
	DTL RF Transmitter													
			Transmitter	180.00	Maxwell	5556	0	5555.6	180.00	1.0	1	0	0	180.00
			HVPS	35.00	Anderson	28571	0	28571.4	35.00	1.0	1	0	0	35.00
			Klystron	20.00	Tallerico (PAC2001)	50000	0	50000.0	20.00	2.0	1	0	0	40.00
			Load	13.33	Tallerico (PAC2001)	75019	0	75018.8	13.33	2.0	1	0	0	26.66
			Circulator	20.00	Tallerico (PAC2001)	50000	0	50000.0	20.00	2.0	1	0	0	40.00
			LLRF	10.00	Tallerico (PAC 2001)	100000	0	100000.0	10.00	2.0	1	0	0	20.00
			DTL/CCL RF Window	10.00	Tallerico (PAC2001)	100000	0	100000.0	10.00	2.0	1	0	0	20.00
	ONE DTL RF STATION TOTAL							2763.3	361.88					361.66
	RFQ and DTL RF							**919.8**	**1087.24**	**3.0**	**0**	**0**	**0**	**1084.98**

Estimated Average Annual Frequency of Repairs (=Spares)	Repair Personnel Required	Mean Time To Repair MTTR (h)	MTTR Source	Other Delays $MTTR_d$ (h)	Switch-over Time MTTS (h)	Mean Down Time MDT (h)	Maximum Acceptable Total MDT/MTBF eff (x 10^{-6})	Estimated Average Total Annual Repair Time (h)	Steady State Availability	Mission Time (h)	Reliability for mission time
1.40	2	2.0	'allerico (PAC200	0		2.0	360.00	5.58	0.999640	160	0.971611
0.27	2	4.0	'allerico (PAC200	0		4.0	140.00	2.17	0.999860	160	0.994416
0.31	3	3.5	'allerico (PAC200	0		3.5	140.00	3.26	0.999860	160	0.993620
0.21	3	3.0	'allerico (PAC200	0		3.0	79.98	1.86	0.999920	160	0.995743
0.31	3	3.0	'allerico (PAC200	0		3.0	120.00	2.79	0.999880	160	0.993620
0.16	2	2.0	'allerico (PAC200	2		4.0	80.00	1.24	0.999920	160	0.996805
0.16	2	25.0	'allerico (PAC200	2		27.0	540.00	8.37	0.999460	160	0.996805
2.80	3	3.0		0.00		4.0	1459.98	25.27	0.998541		0.943777
8.41						**4.0**	**4379.94**	**75.81**	**0.995630**	**160**	**0.840545**

Dynamic Model



Uses of the Model

- Design Phase
  - Model is a simple "what if" tool for evaluating performance to compare the projected system reliability with the customer's expectations.
- Operational Phase
  - Validate model parameters with measured performance. Are you getting what you expected?
  - If not, questions to ask include, was the system:
    - Designed wrong
    - Built wrong
    - Installed wrong
    - Operated wrong
    - Maintained wrong
    - In a "sick" location

Time Distributions (Models) of the Failure Rate Function

- Exponential Distribution

$$f(t) = \lambda e^{-\lambda t}$$

➢Very commonly used, even in cases to which it does not apply (simple);
➢Applications: Electronics, mechanical components etc.

- Normal Distribution

$$f(t) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(t-\mu)^2}{2\sigma^2}}$$

➢Very straightforward and widely used;
➢Applications: Electronics, mechanical components etc.

- **Lognormal Distribution**

$$f(t) = \frac{1}{\sqrt{2\pi\sigma t}} e^{-\frac{(\ln t-\mu)^2}{2\sigma^2}}$$

➢Very powerful and can be applied to describe various failure processes;
➢Applications: Electronics, material, structure etc.

- **Weibull Distribution**

$$f(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1} e^{-\left(\frac{t}{\eta}\right)^{\beta}}$$

➢Very powerful and can be applied to describe various failure processes;
➢Applications: Electronics, mechanical components, material, structure etc.

Exponential Model

- Definition: Constant Failure Rate

$$\lambda(t) = f(t)/R(t) = \lambda$$

$$f(t) = \lambda \exp(-\lambda t) \qquad \lambda > 0, \;\; t \geq 0$$

$$R(t) = \exp(-\lambda t) = 1 - F(t)$$

$$R(x \mid t) = P_r(T > t + x \mid T > t) = \frac{\mathrm{e}^{-\lambda(t+x)}}{e^{-\lambda t}} = e^{-\lambda x} = R(x)$$

Exponential Model Cont.

- Statistical Properties

$$MTTF = \frac{1}{\lambda} \longrightarrow R(MTTF) = e^{-\lambda \times MTTF}$$

$$= e^{-1} = 0.367879$$

$$Var(T) = \frac{1}{\lambda^2}$$

$$\text{Median life} = (\ln 2)\frac{1}{\lambda} = 0.693147 \times MTTF$$

Weibull Model

- Definition

$$f(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]\qquad \beta>0,\ \ \eta>0,\ \ t\geq 0$$

$$R(t)=\exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]=1-F(t)$$

$$\lambda(t)=f(t)/R(t)=\frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}$$

- β is the Shape Parameter and
- η is the Characteristic Lifetime (1/e) survival

Weibull Model Continued:

- Statistical Properties

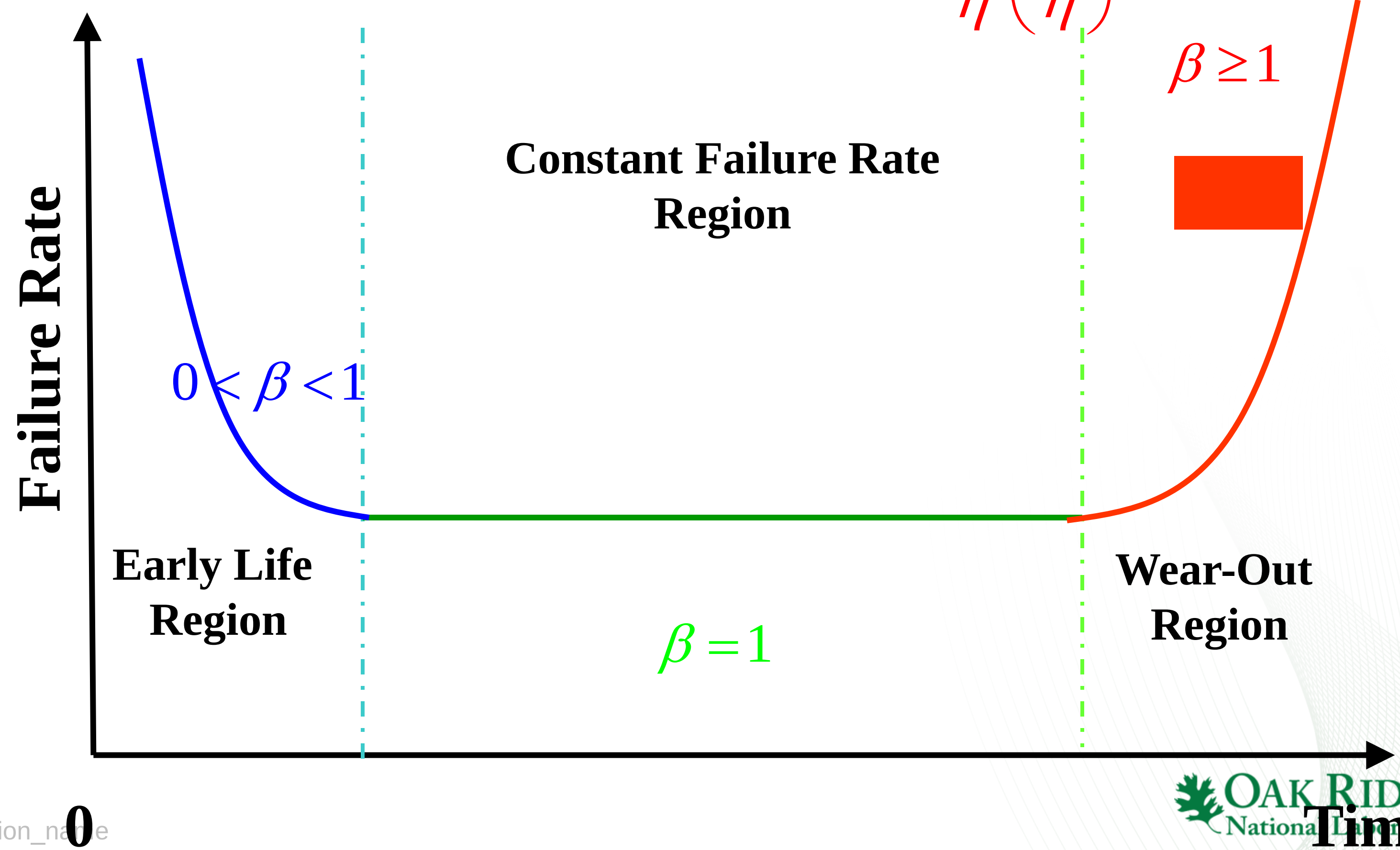
$$MTTF = \eta \int_0^{\infty} t^{1/\beta} \; e^{-t} \; dt = \eta \; \Gamma \; (1 + \frac{1}{\beta})$$

$$Var = \eta^{2} \left[\Gamma \; (1 + \frac{2}{\beta}) - \left(\Gamma(1 + \frac{1}{\beta}) \right)^2 \right]$$

$$\text{Median life} = \eta \; ((\ln 2)^{1/\beta})$$

Versatility of Weibull Model

Failure Rate: $\lambda(t) = f(t)/R(t) = \dfrac{\beta}{\eta}\left(\dfrac{t}{\eta}\right)^{\beta-1}$



Graphical Model Validation

- **Use a Q-Q Plot for checking normality**
  - **Plot Probability Quantiles (ln(ln(1/median rank)/ vs. Model Distribution Quantiles**
  - **The result should be linear in ln(time) in the middle of the plot**
- **Estimate** $Q(t_i) = 1/\hat{F}(t_i)$ **at for** t_i **using Bernard's Formula**

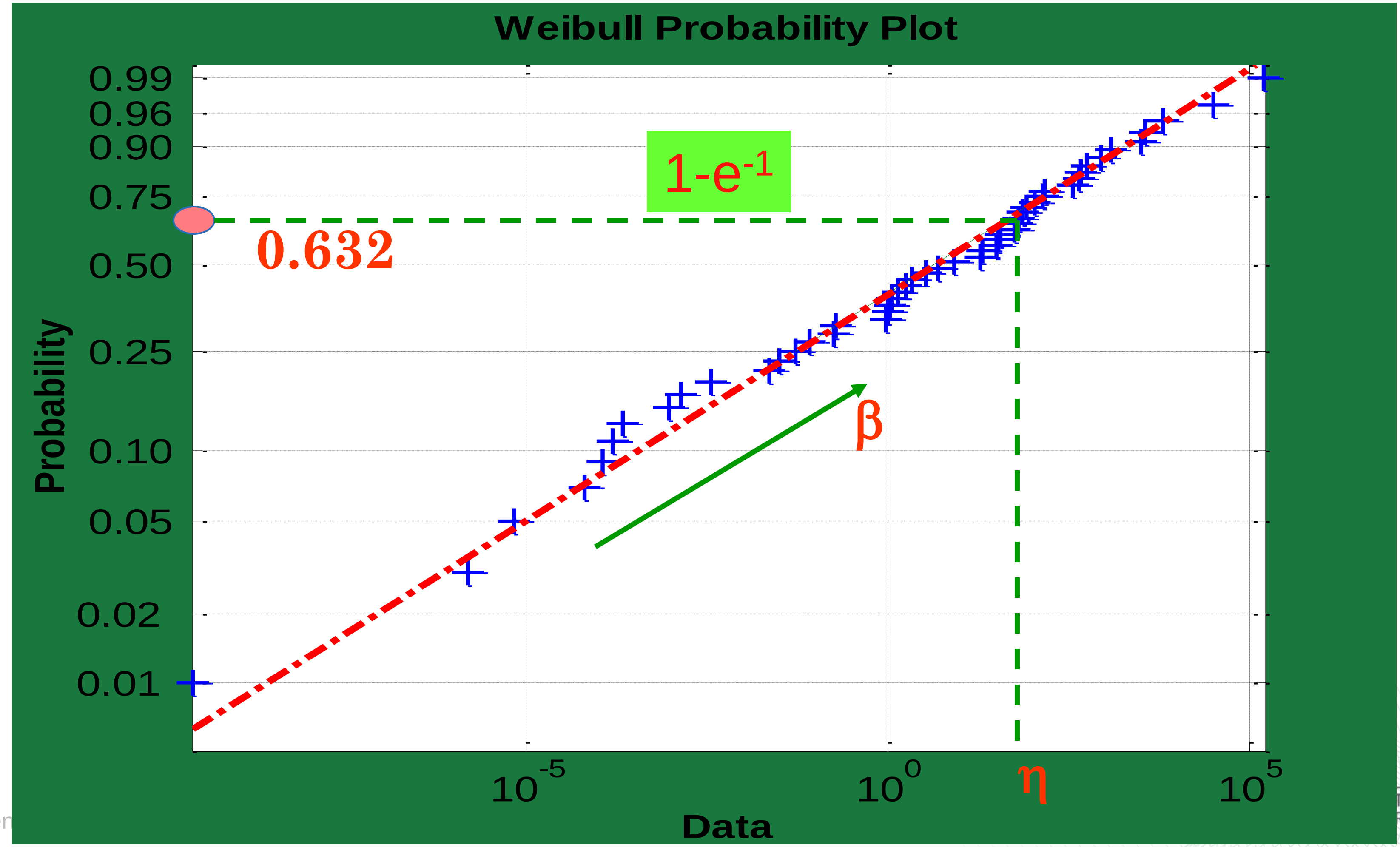
$\hat{F}(t_i)$ $(t_1, t_2, \ldots, t_i, \ldots t_n)$

For *n* observed failure time data

$$\hat{F}(t_i) = \frac{i - 0.3}{n + 0.4}$$

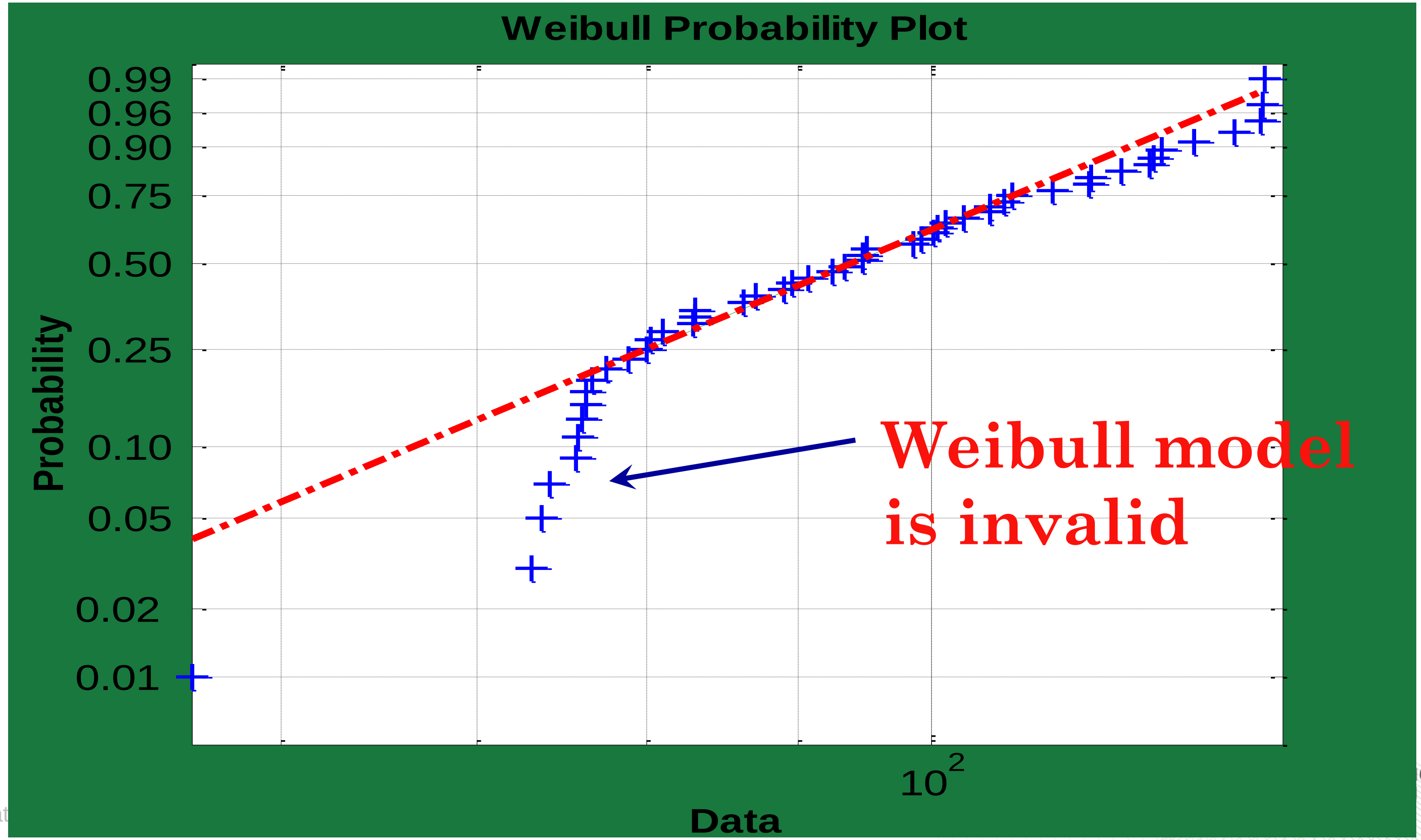
Example: Q-Q of Weibull Distribution and Weibull Fit (works well)

- T~Weibull(1, 4000) Generate 50 data points



Example: Q-Q Weibull Distribution and T-Normal Fit (obviously wrong functional form)

- T~Normal(100, 400) Generate 50 data points



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Analysis of the Failure Rate of Systems or Components

With a relatively modest failure data set you can:

- Determine what your failure rate is at any given time
- Watch this rate change with time, through Infant Failures and into Random Failures
- Predict the onset of Terminal Failures
- Alerts you to watch more closely for the predictive symptoms of failure
- Determine the cost-effectiveness of proactive replacement before failure occurs
- Closely watch your Spares (number of spares, time to repair or acquire replacements, cost)

Weibull in Excel

Using Microsoft EXCEL for Weibull Analysis

by William W. Dorner

Many people use Microsoft Excel on a daily basis. Yet few people realiz the extent of Excel's analytical capabilities. Fewer still put these capabilitie work for process improvement, product improvement and profit.Most Exce users are aware of the common formulas and charts. But with some creativ users can produce tools like control charts, Pareto charts and box-and-whis plots (see "Using Excel for Data Analysis," *Quality Digest,* October 1997). And with a little guidance, users can employ more advanced statistical

http://www.qualitydigest.com/jan99/html/body_weibull.html

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Weibull Using MS Excel

Figure 1: Failure Data

Design A		Design B	
Sample	Cycles	Sample	Cycles
1	726,044	11	529,082
2	615,432	12	729,957
3	508,077	13	650,570
4	807,863	14	445,834
5	755,223	15	343,280
6	848,953	16	959,903
7	384,558	17	730,049
8	666,686	18	730,640
9	515,201	19	973,224
10	483,331	20	258,006

Figure 2: Preparing Design A for Weibull Analysis

	A	B	C	D	E	F
1	Design A Cycles	Rank	Median Ranks	1/(1-Median Rank)	ln(ln(1/(1-Median Rank)))	ln(Design A Cycles)
2	384,558	1	0.067307692	1.072164948	-2.663843085	12.8598499
3	483,331	2	0.163461538	1.195402299	-1.72326315	13.088457
4	508,077	3	0.259615385	1.350649351	-1.202023115	13.13838829
5	515,201	4	0.355769231	1.552238806	-0.821666515	13.15231239
6	615,432	5	0.451923077	1.824561404	-0.508595394	13.33007974
7	666,686	6	0.548076923	2.212765957	-0.230365445	13.41007445
8	726,044	7	0.644230769	2.810810811	0.032924962	13.4953659
9	755,223	8	0.740384615	3.851851852	0.299032932	13.53476835
10	807,863	9	0.836538462	6.117647059	0.593977217	13.60214777
11	848,953	10	0.932692308	14.85714286	0.992688929	13.6517591

1) Order by number of cycles (in accelerators, hours to failure)
2) Calculate the “Median Rank”=((B2-0.3)/(10+0.4))
3) Be sure that the Analysis ToolPak Add-In is loaded into Excel. While on the page you just created, from the menu bar, select Tools and Data Analysis. Scroll down and highlight "Regression" and click OK. A data-entry window will pop up.
2.Under "Input Y Range," type: $E$1:$E$11.
3.For "Input X Range," type: $F$1:$F$11.
4.Click to add a checkmark in the box for "Labels."
5.For "Output Options," select "New Worksheet Ply."
6.Click to add a checkmark in the box for "Line Fit Plots."
7.Click OK. Excel will perform the regression and place the output on a new worksheet.

Figure 3: Results of Linear Regression for Design A

	A	B	C	D	E	F	G
1	SUMMARY OUTPUT						
2							
3	*Regression Statistics*						
4	Multiple R	0.98538223					
5	R Square	0.97097815					
6	Adjusted R Square	0.96735041					
7	Standard Error	0.20147761					
8	Observations	10					
9							
10	**ANOVA**						
11		*df*	*SS*	*MS*	*F*	*Significance F*	
12	Regression	1	10.86495309	10.86495309	267.6543319	1.96274E-07	
13	Residual	8	0.324745817	0.040593227			
14	Total	9	11.18969891				
15							
16		*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
17	Intercept	-57.1930531	3.464488033	-16.5083708	1.82953E-07	-65.18218204	-49.2039242
18	ln(Design A Cycles)	4.2524822	0.259929377	16.36014462	1.96274E-07	3.653083597	4.851880811
19	**Beta (or Shape Parameter) = 4.25**						
20	**Alpha (or Characteristic Life) = 693,380**						
21							

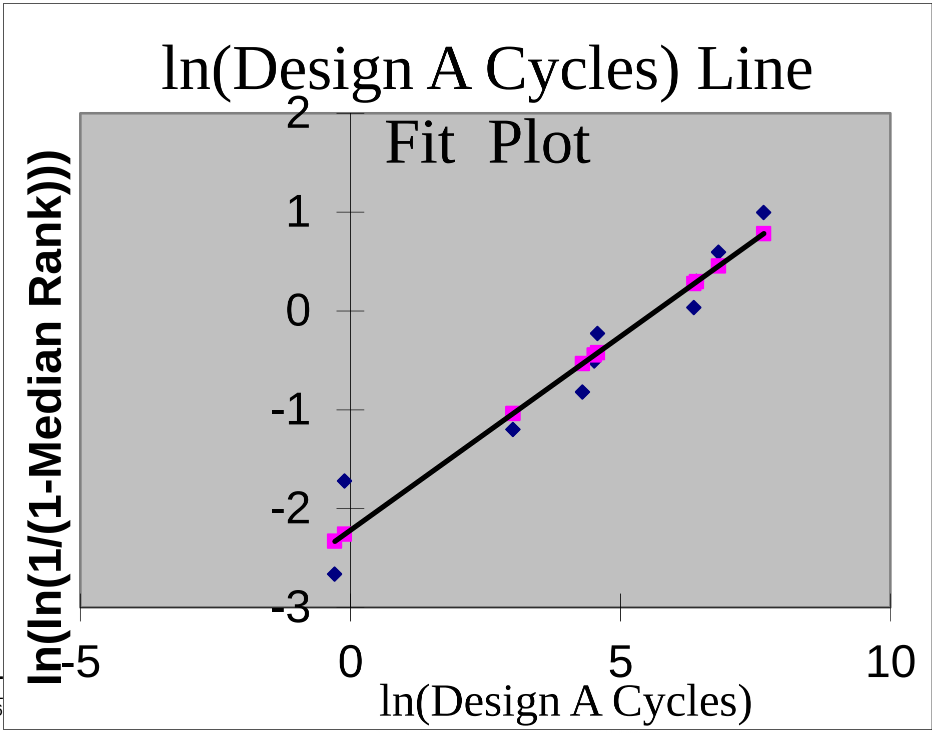
Note the:

Shape Parameter $\beta > 1$ Terminal Mortality

Characteristic Lifetime, where 1/e = 62.3% have failed.

SNS RF High Voltage Converter Modulator 2008

CCL1 hours to failure	Rank	Median Ranks	1/(1-Median Rank)	ln(ln(1/(1-Median Rank)))	ln(Design A Cycles)
0.75	1	0.067307692	1.072164948	-2.66384309	-0.28768207
0.9	2	0.163461538	1.195402299	-1.72326315	-0.10536052
20.3	3	0.259615385	1.350649351	-1.20202312	3.01062089
73.4	4	0.355769231	1.552238806	-0.82166652	4.29592394
91.8	5	0.451923077	1.824561404	-0.50859539	4.5196123
97.2	6	0.548076923	2.212765957	-0.23036544	4.57677071
578.9	7	0.644230769	2.810810811	0.032924962	6.36112975
609.2	8	0.740384615	3.851851852	0.299032932	6.41214662
912.2	9	0.836538462	6.117647059	0.593977217	6.81585926
2115	10	0.932692308	14.85714286	0.992688929	7.65681009



SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.969525286
R Square	0.93997928
Adjusted R Square	0.93247669
Standard Error	0.289744238
Observations	10

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	10.51808512	10.51808512	125.2873043	3.63718E-06
Residual	8	0.671613788	0.083951723		
Total	9	11.18969891			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-2.217586473	0.176953141	-12.53205486	1.53932E-06	-2.625641148	-1.809531798	-2.625641148	-1.809531798
ln(Design A Cycles)	0.391732899	0.034997459	11.19318115	3.63718E-06	0.311028614	0.472437184	0.311028614	0.472437184
Beta (or Shape Parameter) =	0.391732899							
Alpha (or Characteristic Life) =	287.4260525							

RESIDUAL OUTPUT

Observation	*Predicted ln(ln(1/(1-Median Rank)))*	*Residuals*
1	-2.330281006	-0.33356208
2	-2.258859654	0.535596503
3	-1.038227226	-0.16379589
4	-0.534731736	-0.286934779
5	-0.447105645	-0.061489749
6	-0.424714814	0.194349369
7	0.274277326	-0.241352364
8	0.294262312	0.00477062
9	0.452409836	0.141567381
10	0.781837942	0.210850988

Beta =0.39 (Infant Failures)
Alpha = 287
Adjusted R square =0.93

The λ for an Exponential model = 475!!

VME Crate Power Supplies (2009)

VME hours to failure	Rank	Median Ranks	1/(1-Median Rank)	ln(ln(1/(1-Median Rank)))	ln(Design A Cycles)
7536	1	0.009162304	1.009247028	-4.688058902	8.927446816
8544	2	0.022251309	1.022757697	-3.794124242	9.052984561
29136	3	0.035340314	1.036635007	-3.324794914	10.2797298
30240	4	0.048429319	1.050894085	-3.002931896	10.31692083
36024	5	0.061518325	1.065550907	-2.756842175	10.49194066
41496	6	0.07460733	1.080622348	-2.556998447	10.63335232

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.931357
R Square	0.867426
Adjusted R Square	0.834283
Standard Error	0.319986
Observations	6

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.679769	2.679769	26.17191	0.006906
Residual	4	0.409564	0.102391		
Total	5	3.089333			

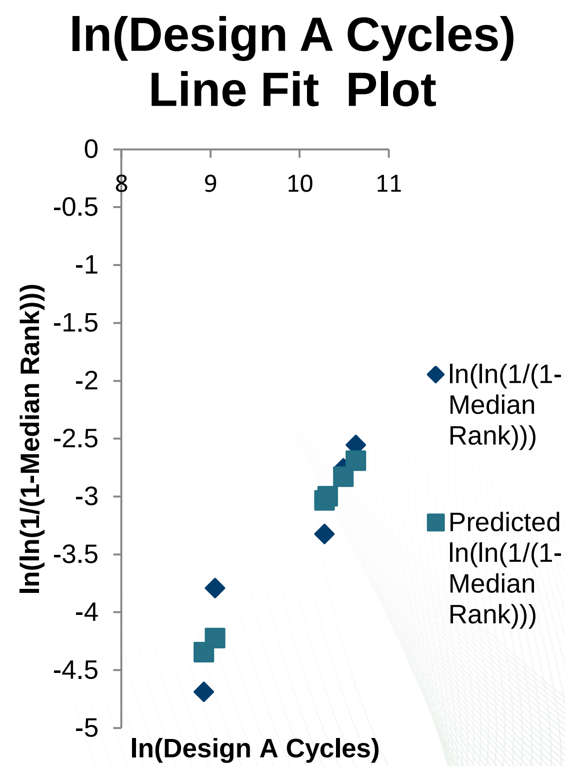
	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-12.9954	1.889141	-6.87899	0.00234	-18.2405	-7.75029	-18.2405	-7.75029
ln(Design A Cycles)	0.968949	0.189401	5.115849	0.006906	0.443086	1.494812	0.443086	1.494812
Beta (shape Parameter)	0.968949							
Alpha	667861.2	77.65828						
	Hrs	Years						

Beta=0.97

Alpha=78 Yrs.

RESIDUAL OUTPUT

Observation	*Predicted ln(ln(1/(1-Median Rank)))*	*Residuals*
1	-4.34514	-0.34292
2	-4.2235	0.42938
3	-3.03485	-0.28994
4	-2.99881	-0.00412
5	-2.82923	0.072387
6	-2.69221	0.13521



Spares

Classes of Spares

In all evaluations of Mean Time to Repair, there are assumptions on the availability of spares for systems, structures and components. In most cases, the assumption is that there is a spare of some sort available to install. There are a number of classes of spares. They include;

- A “true spare” consisting of a “like for like or equivalent” “on the shelf, tested and ready to go “, “plug compatible” replacement unit.
- A “like for like or equivalent” that is installed in some other system that is not required for operation of the accelerator systems e.g. a Test Stand.
- A system structure or component that must be modified to be used as a spare.

Only a “true spare” will not contribute to down time. In both other classes, demounting and modification of the replacement will necessarily contribute to downtime.

Spares

Beyond the “larger of 10% or 2” rule of thumb, the evaluation of the baseline number of spares should include a calculational basis which considers:

1. Number of installed units
2. Mean Time Between Failures (estimated at first, then validated against experience)
3. Mean Time to Repair or Replace in the calculation.

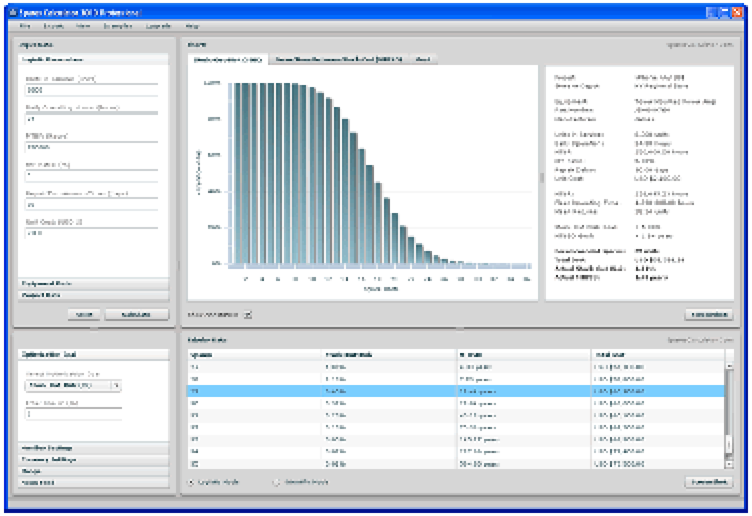
The result will be a Mean Time to Out of Stock as a function of the number of spare units.

- Spares Calculator code is available – validated against MIL Spec - U.S. Navy, Reliability Engineering Handbook, NAVAIR 01-1A-32

Spares Calculator 2010 Professional

Powerful and Easy to Learn Spare Parts Forecasting Software

Spares Calculator 2010 Professional is a powerful and accurate tool that lets you work out the risk of equipment stock-out so that you can plan your contracts and logistics operations accordingly. What's more, the package is easy to use so there's no need for specialist knowledge, complex manuals or time consuming training. Whether you supply, procure or manage spare equipment, Spares Calculator makes an essential purchase.



Key Facts

1. Easy to use spare parts forecasting software.

Use Spares Calculator Professional to determine the optimum level of spare parts inventory. Trade-off spare parts, costs, Stock-Out-Risk (SOR) and Mean-Time-Between-Stock-Out (MTBSO).

2. Justify investments, eliminate outages and eradicate redundant stock.

Justify investments by convincing customers, managers and shareholders to invest in the correct level of spare parts. Eliminate outages by finding the right level of inventory in seconds. Eradicate redundant stock by identifying slow moving stock and plan alternative strategies.

3. Based on de facto industry standard models.

Based on de facto industry standard models and proven in literally thousands of real-life and simulated trials, Spares Calculator Professional offers a scientific way to forecast spare parts.

4. Streamline budgets and costs.

Buy | Upgrade

Trial Download

Watch Demo Video

Webinar Registration

Comparison Matrix

Ask a Question

User Comment

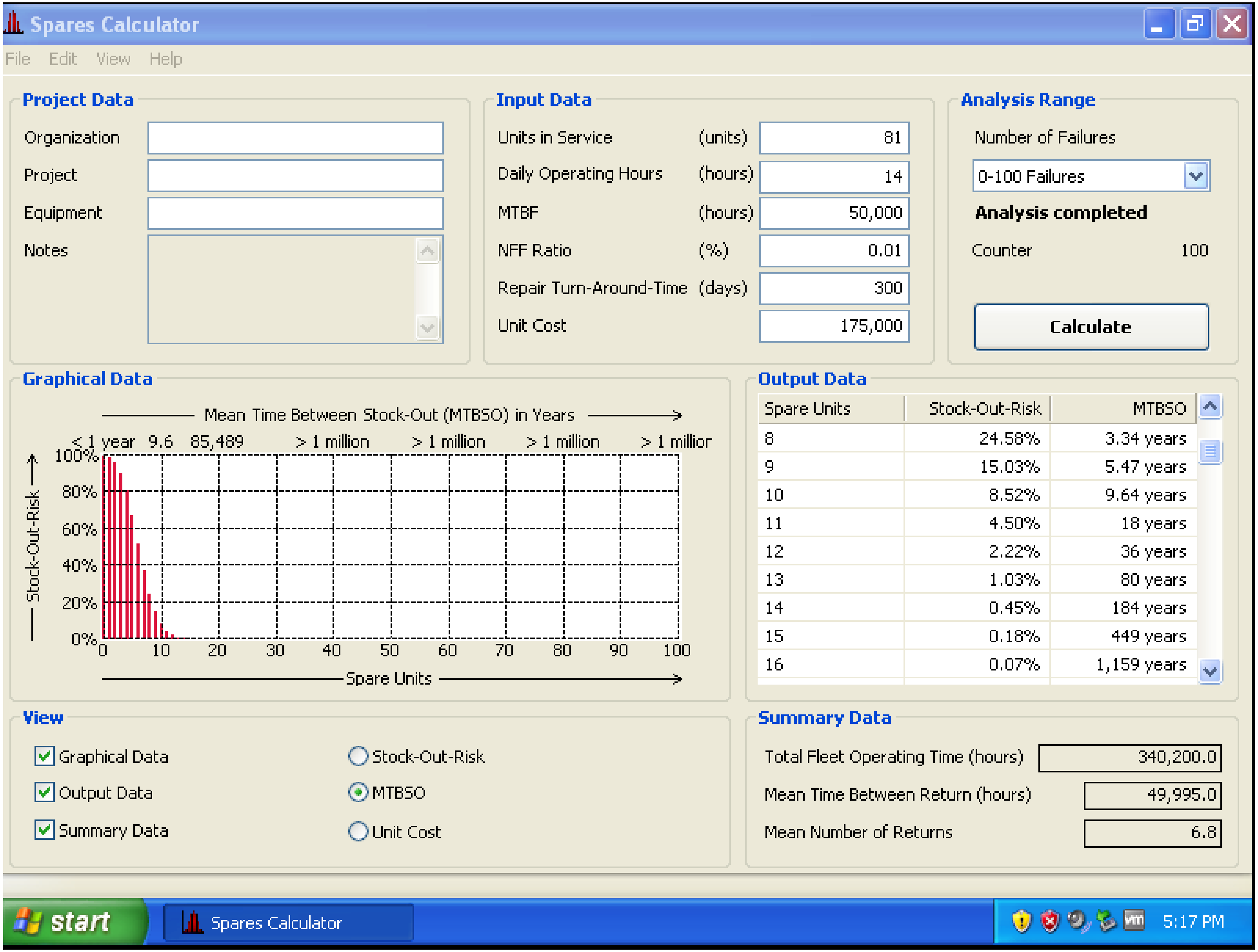
"Spares Calculator has given us a scientific way to work out how many spare parts we need. We tested Spares Calculator on a random sample of 20 line items used in our 12 operations spread across Asia. The results were amazingly accurate. We now have an enterprise licence and use Spares Calculator in all of our procurement and operational planning projects."

Customers Include

ROGERS WIRELESS

Alcatel·Lucent

GENERAL DYNAMICS


Spares Calculator
File Edit View Help
Project Data
Organization
Project
Equipment
Notes
Input Data
Units in Service (units) 81
Daily Operating Hours (hours) 14
MTBF (hours) 50,000
NFF Ratio (%) 0.01
Repair Turn-Around-Time (days) 300
Unit Cost 175,000
Analysis Range
Number of Failures
0-100 Failures
Analysis completed
Counter 100
Calculate
Graphical Data
Mean Time Between Stock-Out (MTBSO) in Years
< 1 year 9.6 85,489 > 1 million > 1 million > 1 million > 1 millior
100%
80%
60%
40%
20%
0%
Stock-Out-Risk
0 10 20 30 40 50 60 70 80 90 100
Spare Units
Output Data
Spare Units Stock-Out-Risk MTBSO
8 24.58% 3.34 years
9 15.03% 5.47 years
10 8.52% 9.64 years
11 4.50% 18 years
12 2.22% 36 years
13 1.03% 80 years
14 0.45% 184 years
15 0.18% 449 years
16 0.07% 1,159 years
View
Graphical Data
Output Data
Summary Data
Stock-Out-Risk
MTBSO
Unit Cost
Summary Data
Total Fleet Operating Time (hours) 340,200.0
Mean Time Between Return (hours) 49,995.0
Mean Number of Returns 6.8
start
Spares Calculator
5:17 PM

Spares – How Many

- Use the MTBSO to evaluate what Comfort Level you can afford to have.
- Caveat –
  - This calculation assumes a random distribution and is not accurate for NEW systems where a large number of identical are all installed at the same time.

Summary:

For a given set of performance data and an appropriate model, analysis of the data can accurately yield MTBF, MTTR for components and systems . The analysis can also yield information on where components and systems are in the lifetime curve so that you can make decisions about when to replace components and how many you should have in inventory (particularly important in long-lead-time components).

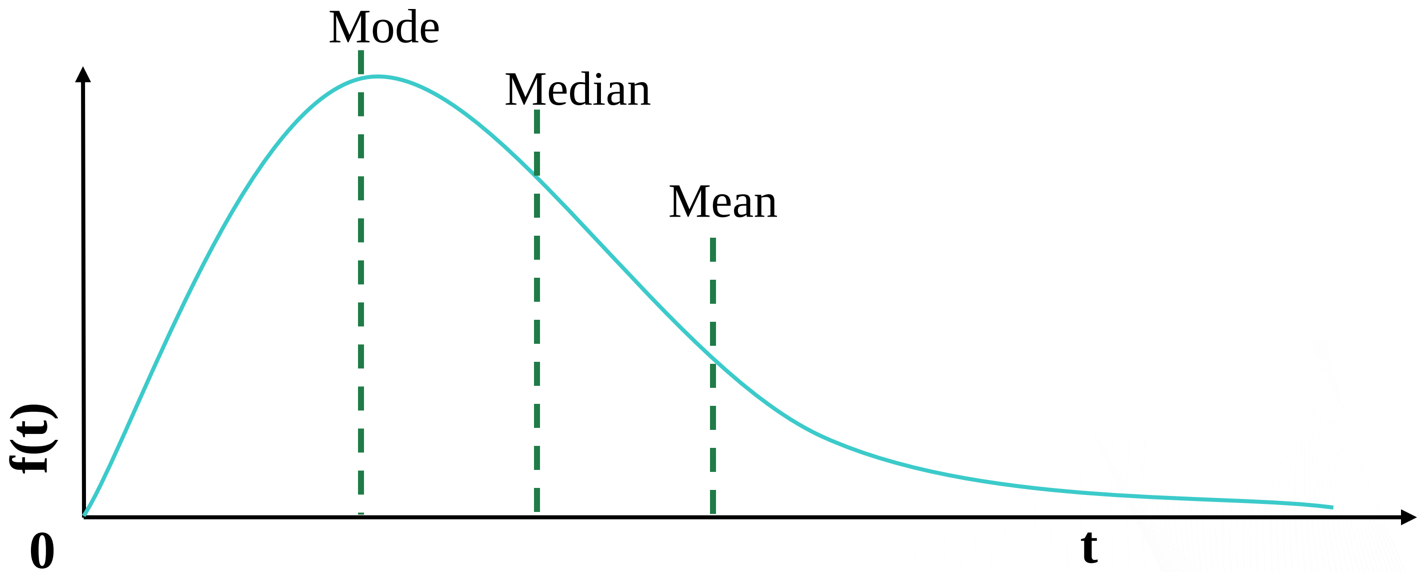
These data can be used to validate your RAMI Model of your accelerator systems.

Issues in Modeling

- "… no model is absolutely correct. In particular, however, some models are more useful than others." –
- The model should be sufficiently simple to be handled by available mathematical and statistical methods, and be sufficiently realistic such that the deducted results are of practical use.

Backup Slides

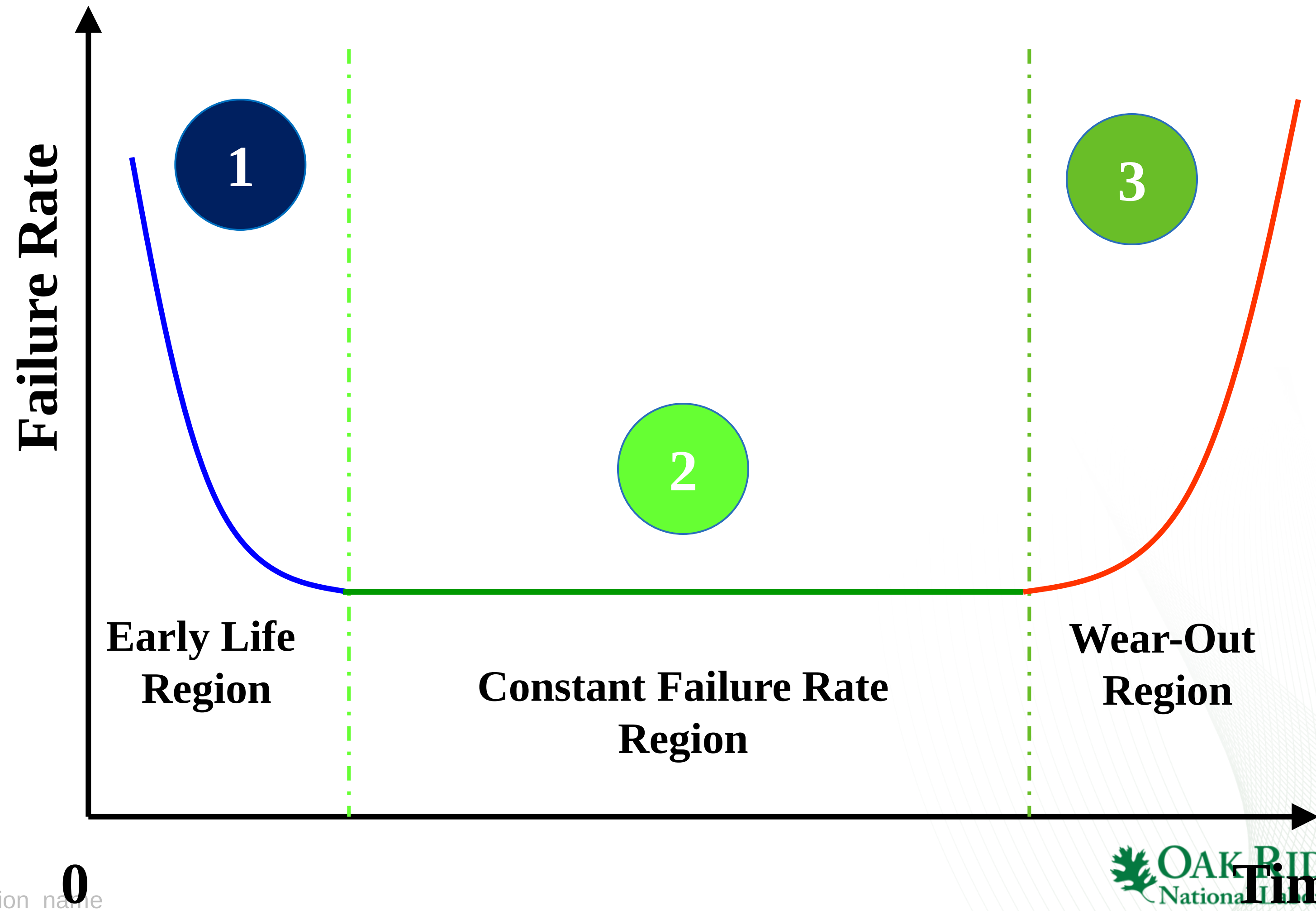
Most of these distribution functions are not Symmetric, so: Median , Mode and Mean are not the same



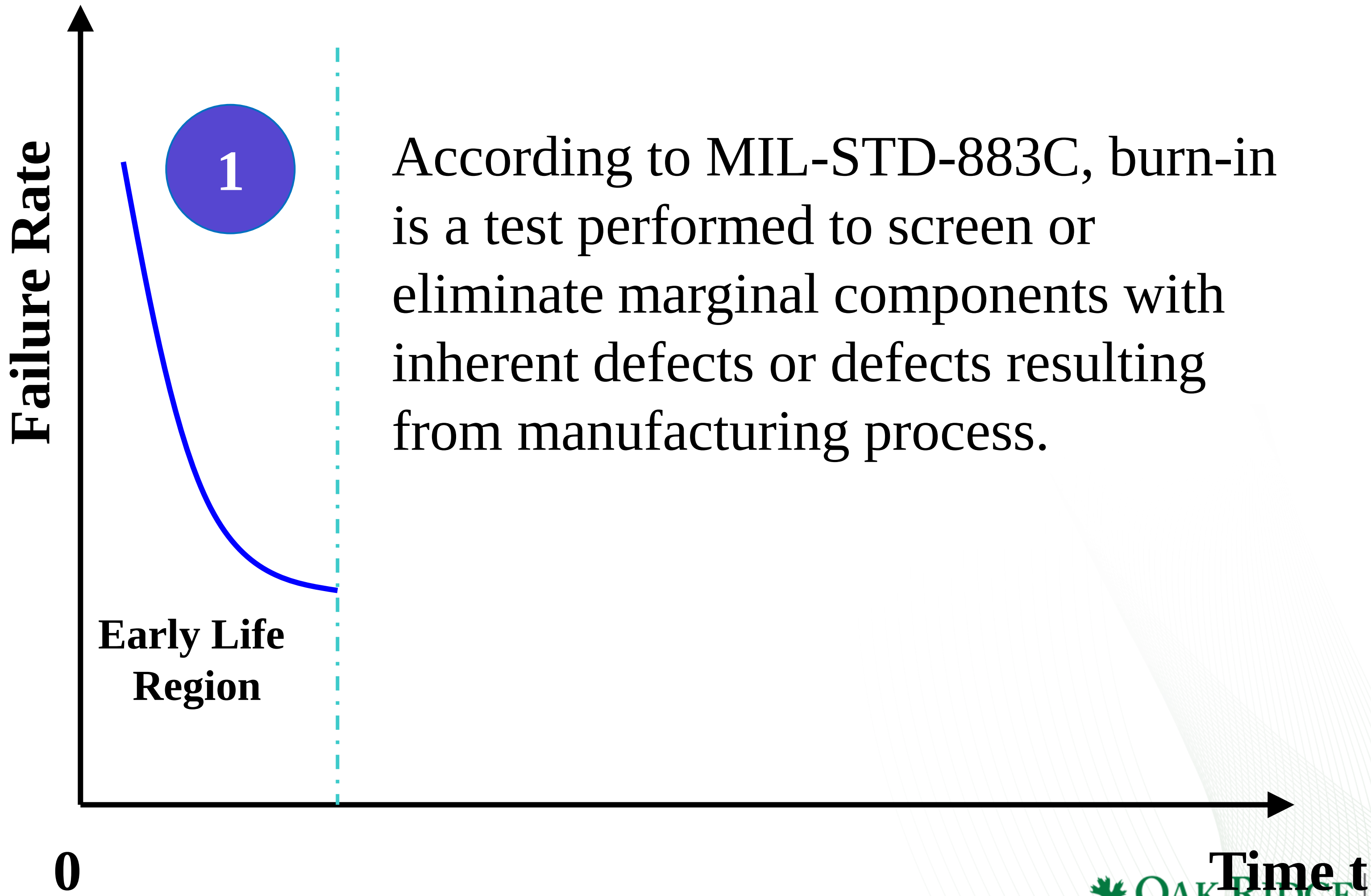
$$\text{Median - } \; t_m : \; R(t_m) = 50\%$$

$$\text{Mode - } \quad t_{\text{mode}} : \max f(t)$$

Example of a Non-Constant Failure Rate Curve: The “Bathtub” Curve



Infant Mortality or Burn-In:



According to MIL-STD-883C, burn-in is a test performed to screen or eliminate marginal components with inherent defects or defects resulting from manufacturing process.

Use of Burn-In

- One of environment stress screening (ESS) techniques

$$MRL(t) = \int_0^\infty R(x \mid t)dx = \frac{1}{R(t)} \int_0^\infty R(x+t)dx$$

  - Example: for

$$R(t) = \frac{a^2}{(a+t)^2} \quad t \geq 0$$

(A) Without burn-in

$$MTTF = \int_0^\infty R(t)dt$$

$$= \frac{-a^2}{a+t}\bigg|_0^\infty = a$$

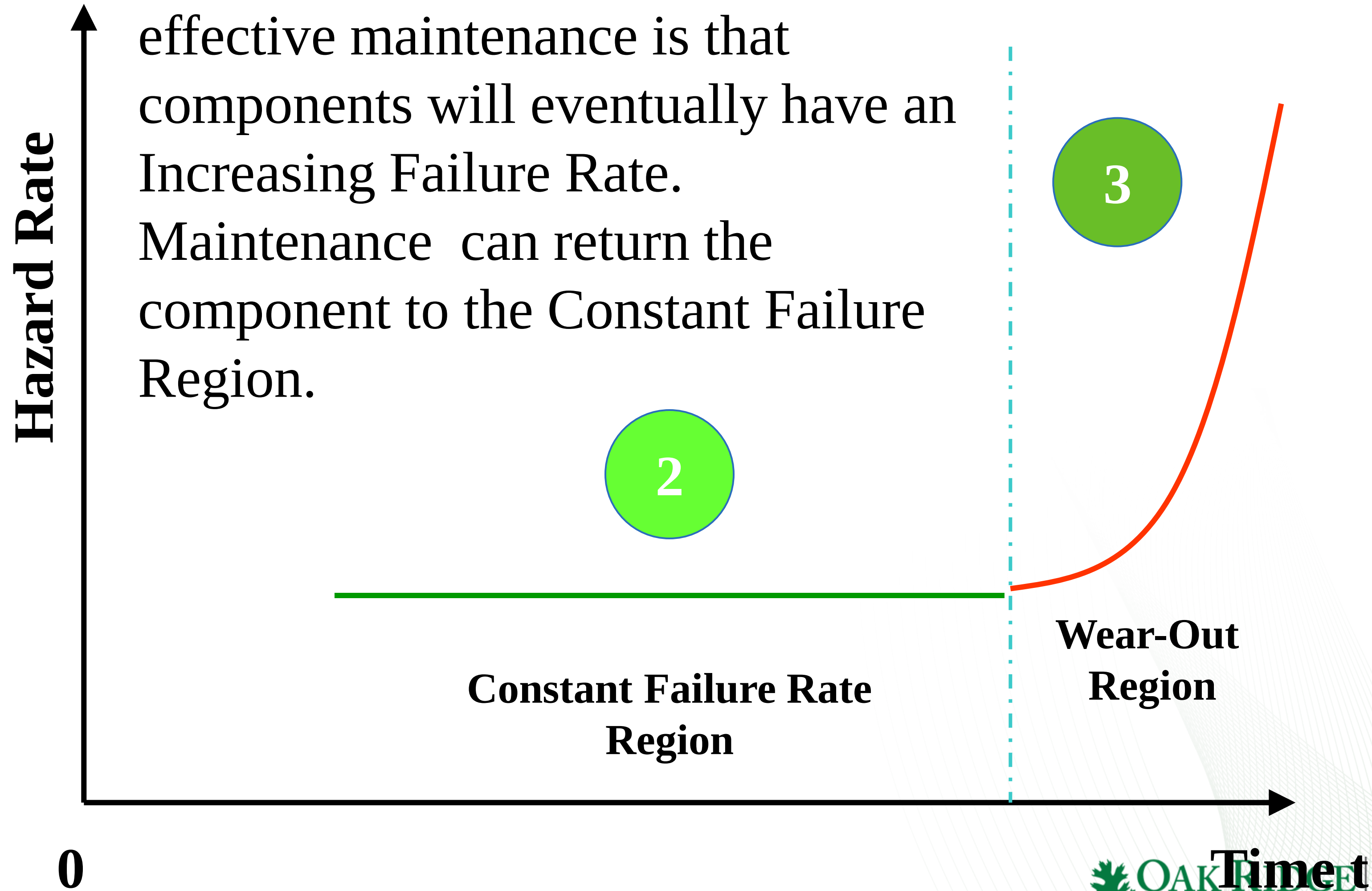
(B) After T_0

$$MRL(T_0) = \frac{1}{R(T_0)} \int_0^\infty R(x+T_0)dx$$

$$= a + T_0 > a$$

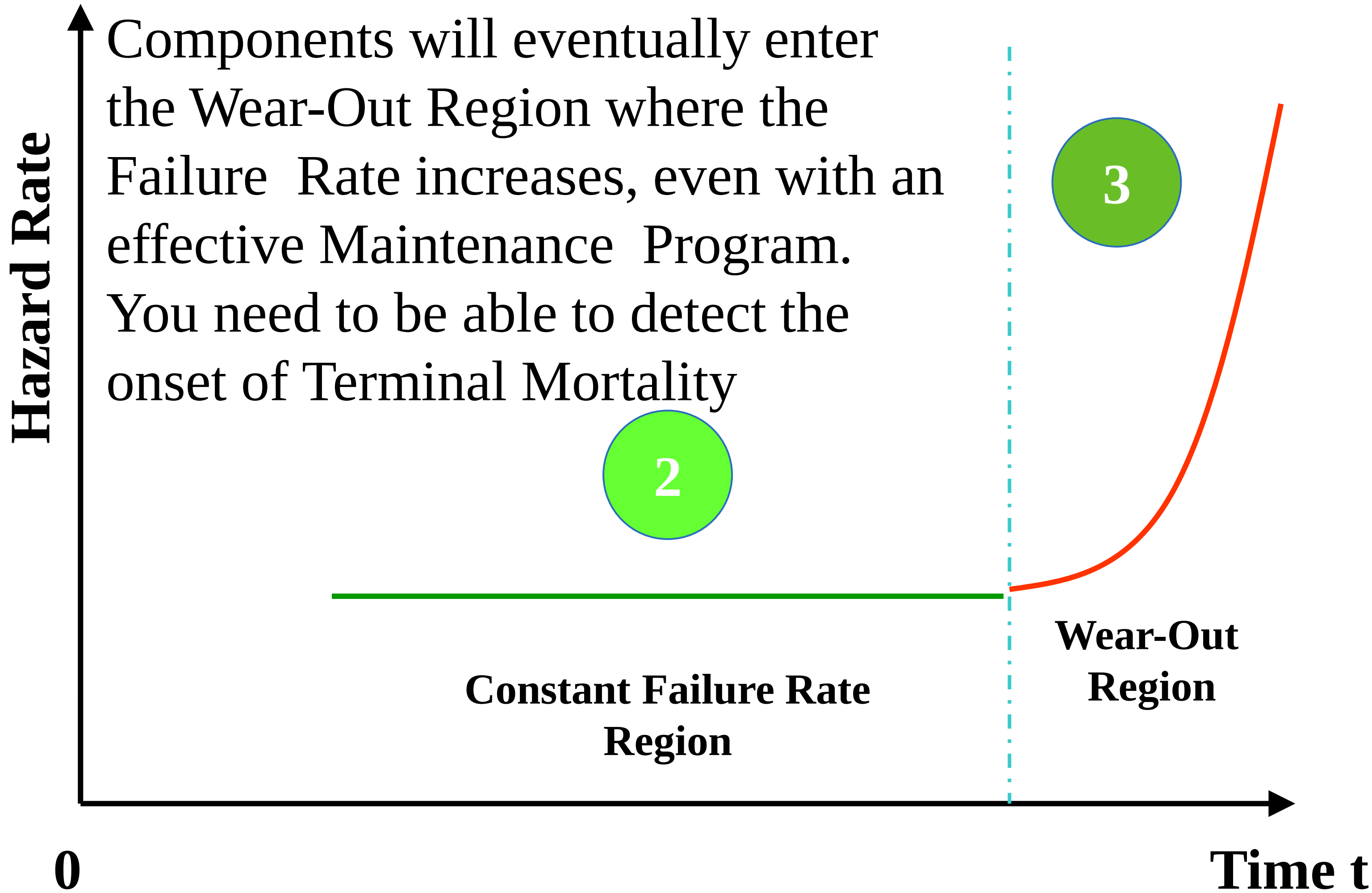
Maintenance:

- An important assumption for effective maintenance is that components will eventually have an Increasing Failure Rate. Maintenance can return the component to the Constant Failure Region.



Terminal Mortality (Wear-Out)

Components will eventually enter the Wear-Out Region where the Failure Rate increases, even with an effective Maintenance Program. You need to be able to detect the onset of Terminal Mortality



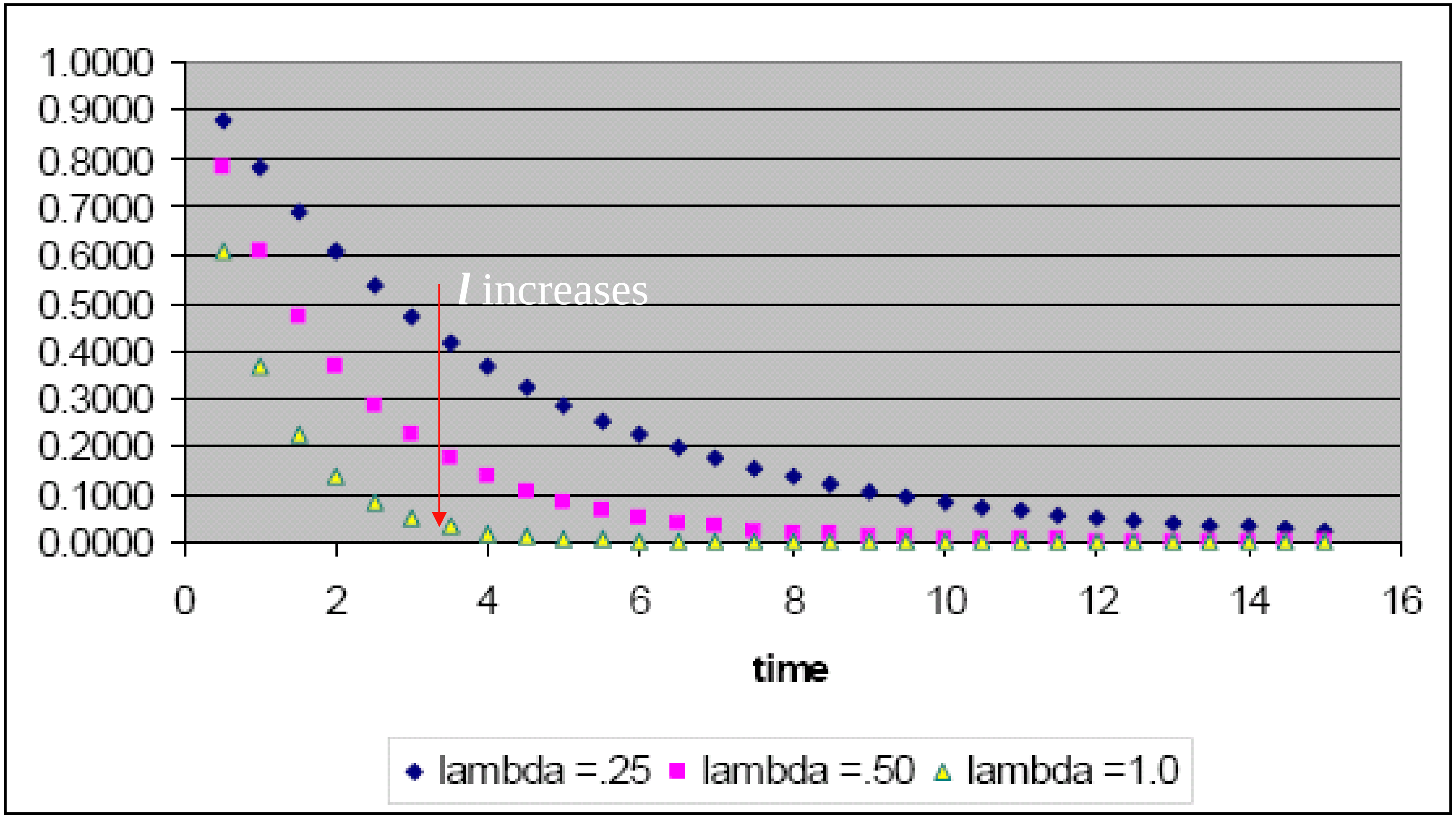
Exponential Distribution (Model)

Constant Failure Rate

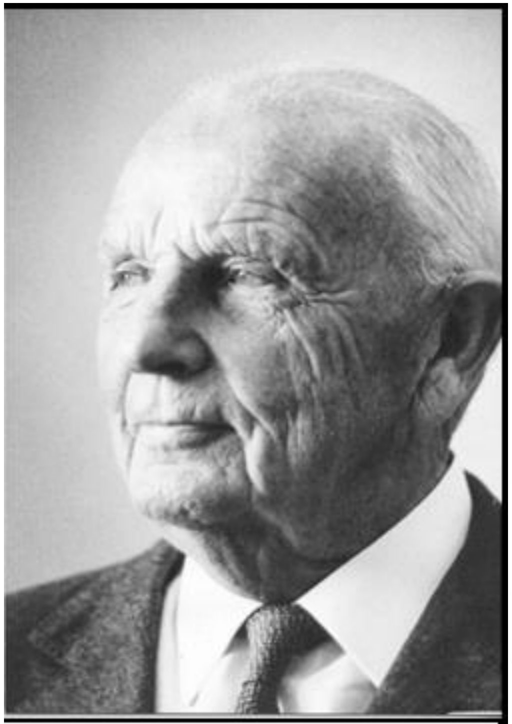
~~Single/Multiple Failure Modes~~

Example

- The higher the failure rate is, the faster the reliability drops with time



Weibull Distribution (Model) and Model Validation



Wallodi Weibull 1887-1979

Photo by Sam C. Saunders

Background of Weibull

- **Waloddi Weibull,** a Swedish inventor and engineer invented the Weibull distribution in 1937. The U.S. Air Force recognized the merit of Weibull's methods and funded his research to 1975.

- **Leonard Johnson** at General Motors improved Weibull's methods. He suggested the use of median rank values for plotting.

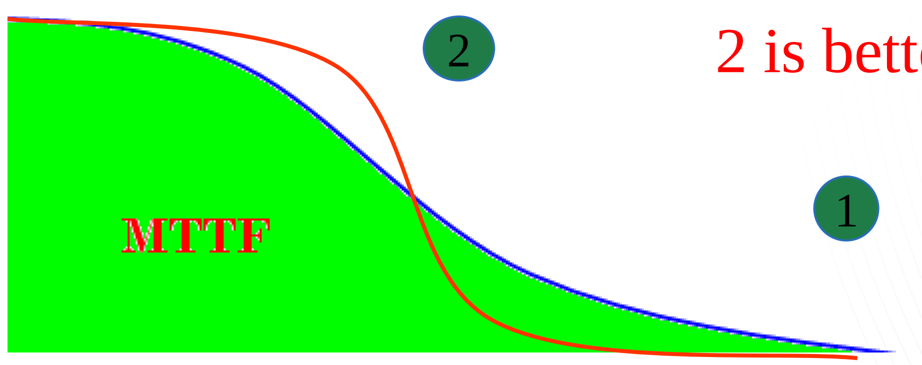
- The engineers at Pratt & Whitney found that the Weibull method worked well with extremely small samples, even for 2 or 3 failures.

- Failure Probability Density is related to the Failure Probability by:

$$f(x) = \int_0^x f(s)ds \qquad f(x) = \frac{d(F(x))}{dx}$$

- Reliability Function is related to the Failure Probability Density by:

$$R(t) = 1 - F(t) = \int_t^\infty f(u)\, du$$



2 is better than 1?

Failure Rate Function

- Increasing failure rate (IFR) v.s. decreasing failure rate (DFR)

$\lambda(t)$ □ or $\lambda(t)$ □ respectively

- Examples

$$\lambda(t) = c \quad \text{where c is a constant}$$

$$\lambda(t) = at \quad \square \quad \text{where } a > 0$$

$$\lambda(t) = \frac{1}{t+1} \quad \square \quad \text{for t} > 0$$

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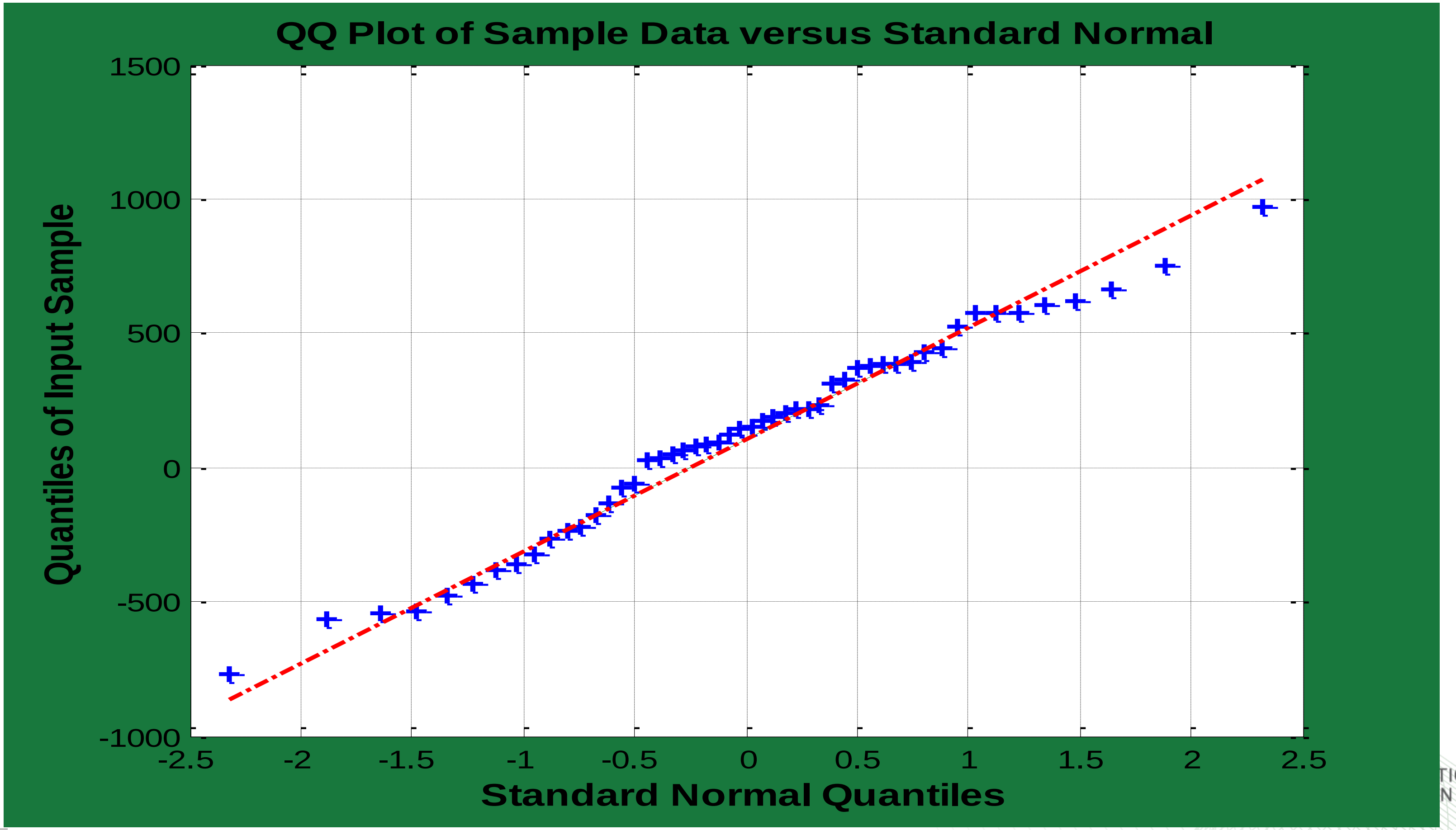
Q-Q Plot for the Normal Distribution

- T~Normal(100, 400) Generate 50 data points



Formal Statistical Test Procedures

- **Test for assumption in a more statistical way**
  - χ^2 **Goodness-of-Fit test**
  - **Bartlett's test for Exponential**
  - **Mann's test for Weibull**
  - **Komogorov-Smirnov (KS) test**

Graphical Model Validation

- Weibull Plot

$$F(t) = 1 - R(t) = 1 - \exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]$$

$$\Rightarrow \quad \ln\ln\frac{1}{1-F(t)} = \beta\ln t - \beta\ln\eta$$ is linear function of *ln*(time).

- **Estimate $\hat{F}(t_i)$ at t_i using Bernard's Formula**

For n observed failure time data $(t_1, t_2, \ldots, t_i, \ldots t_n)$

$$\hat{F}(t_i) = \frac{i-0.3}{n+0.4}$$

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