MAKING GOOD DECISIONS WITH YOUR RELIABILITY DATA: A FORMALIZED APPROACH

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Abstract

All organizations want to find ways to improve their system's reliability. Collecting and interpreting data can often be a complex and confusing process. This is especially true when both qualitative and quantitative data are used in the analysis method. Simple cost-benefit analysis techniques often result in management ignoring other important factors such as "ease of installation" or "availability of spares". The Analytic Hierarchy Process (AHP) provides a methodology to evaluate the benefits of the alternatives, initially independent of cost, utilizing a pairwise comparison technique. This technique allows the analyst to evaluate a problem's alternatives using specific criteria while also gauging (and maximizing) the decision maker's consistency. Once the benefit values are understood, costs can be integrated into the decision process as a cost-benefit ratio. A simple example will be used to explain the use of the methodology in the reliability domain.

Introduction

Selecting the best maintenance policy for an organization's equipment can be time consuming, costly, and complex. Getting consensus among stakeholders including system owners, project managers, and lab leadership can be a daunting task. Multi-Criteria Decision Analysis (MCDA) provides tools to help with problems with these complexities. MCDA problems involve a set of alternatives that are evaluated using multiple identical criteria [1]. When discussing alternatives, we are talking about the candidates for solution. In the literature terms "choice," "policy," and "candidate" are also used instead of alternative. Criteria serve as the basis for alternative evaluation. Criteria for a decision encompass all the measuring units/scales, objectives, and targets [2].

The Analytic Hierarchy Process (AHP) is an MCDA tool that is used to rank alternatives based on criteria while also incorporating the decision maker's preferences and risk tolerance. The technique offers several advantages to the decision making process including an intuitive methodology, it validates user input, and has been tested successfully in practice across 1000's of organizations throughout the world over the last 35 years [3].

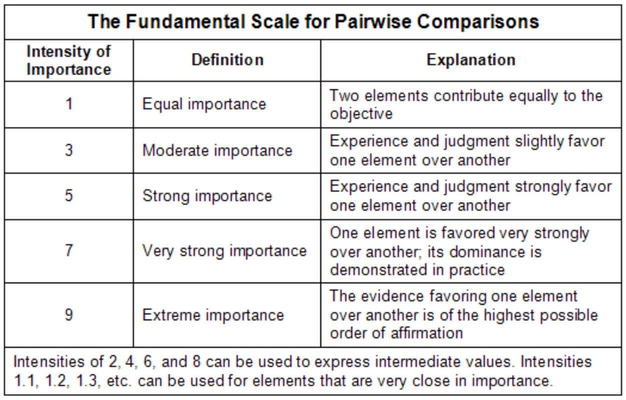
This paper will discuss the application of AHP for maintenance policy selection for accelerator systems. While there are many maintenance policies to pick from, this paper addresses three policy alternatives, corrective, preventive, and predictive maintenance with respect to six criteria: safety, machine importance, maintenance cost, failure frequency, downtime length, and component access. The numerical example provides a template for the technique that can be refined, or redefined, based on the decision maker needs. The following section provides an overview of the AHP.

Overview of the Analytic Hierarchy Process

The AHP provides the decision analyst a methodology to assess both quantitative and qualitative data using subjective assessments. Quantitative and qualitative data are normalized and synthesized in a pairwise assessment matrix. This matrix includes the relative importance of criteria used to analyze the data associated with the decision's goal. Pairwise comparison provides a redundancy mechanism which forces the decision maker to make consistent rankings. The process can be an issue for unskilled decision makers. To evaluate this potential problem, the method uses a consistency ratio to assess the consistency of the decision maker's comparisons. Allowing for some level of inconsistency is a fundamental aspect of the methodology which makes it an appealing tool for many real world problems. People often struggle to "estimate precisely measurement values even from a known scale and worse when they deal with intangibles (a is preferred to b twice and b to c three times, but a is preferred to c only five times) and ordinally intransitive (a is preferred to b and b to c but c is preferred to a)" [4, p. 86]. Broadly speaking, the AHP four step process [5] can be described as follows:

1. Set up the decision hierarchy by breaking down the problem into a hierarchy of interrelated decision elements,
2. Create input data based on pairwise comparisons of the elements
3. Determine the relative weights for the elements using the eigenvector method
4. Aggregate the weights of the decision elements to arrive at a set of ratings for the decision alternatives

The AHP's pairwise comparison is a fundamental part of the methodology. The process offers redundancy in the analysis process and provides a mechanism to evaluate the decision maker's constancy. In assessing weights, the decision maker is asked a series of questions, each of which asks how important one particular criterion is relative to another for the decision being addressed. Values are assigned to the qualitative assessment using the fundamental scale of pairwise comparisons (Table 1). When quantitative values are known, matrix entries can be derived by normalizing all quantitative values for the criteria by their sum. Once the process is complete, a consistency index (CI) can be computed and compared to a random index (RI) to determine if the data entered is consistent enough to yield meaningful results.



The Fundamental Scale for Pairwise Comparisons		
Intensity of Importance	**Definition**	**Explanation**
1	Equal importance	Two elements contribute equally to the objective
3	Moderate importance	Experience and judgment slightly favor one element over another
5	Strong importance	Experience and judgment strongly favor one element over another
7	Very strong importance	One element is favored very strongly over another; its dominance is demonstrated in practice
9	Extreme importance	The evidence favoring one element over another is of the highest possible order of affirmation
Intensities of 2, 4, 6, and 8 can be used to express intermediate values. Intensities 1.1, 1.2, 1.3, etc. can be used for elements that are very close in importance.		

Table 1

The data entry and analysis processes are straightforward in the AHP which make it popular with decision makers. However, some issues, such as rank reversal phenomenon have been pointed out as weaknesses of the method. Rank reversal phenomenon occurs when an attribute which has identical properties of another attribute is added to the model and changes the ordering of the results. To address this, Belton and Gear suggested a revised-AHP technique where each column of the AHP decision matrix is divided by the maximum entry of that column [6]. This overcame the deficiency and Saaty accepted the variant in 1994 which is now known as Ideal Mode AHP [7].

A Numerical Application

Create the Problem Hierarchy

The first step of the process is to define the decision criteria in the form of a hierarchy of objectives. This involves identifying a goal, criteria (and possibly sub-criteria), and alternatives. Figure 1 shows the hierarchy for the maintenance policy selection example.

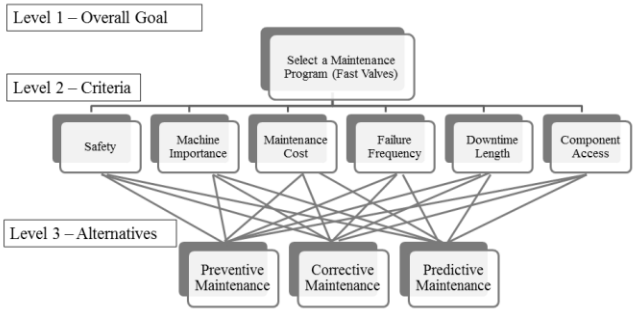


Figure 1 Policy Selection Alternative Elements

Alternative and criteria definitions were derived from Bevilacqua and Braglia [8] and adapted for this example. The alternatives were limited to corrective maintenance, preventative maintenance, and predictive maintenance to clarify the example application. Other maintenance options, such as opportunistic maintenance, would certainly be analyzed in a practical application. The alternatives are briefly explained as follows:

Corrective maintenance

- Actions are only performed when a machine breaks down.
- No interventions are made until a failure has occurred.

Preventive maintenance

- Maintenance is based on component reliability characteristics.
- Data makes it possible to analyze the behavior of the element in question and allows the maintenance engineer to define a periodic maintenance program for the machine.

Predictive maintenance

- Data are analyzed to identify temporal trends.
- Components are replaced or refurbished when the predictive parameter values reach or exceed the threshold values.

Assignment of Weights

After structuring the main criteria and measures for all alternatives, the AHP pairwise comparison method was applied to measures. The pairwise comparison matrix is used to organize the decision makers inputs based on goal-oriented preferences. Table 2 summarizes the preference assignments for the identified criteria. In this example, the preference assignments are used to assign the relative weights of the criteria. The priority vector is calculated by summing the rows of each column. The normalized priority vector is calculated by dividing the original priority vector by its sum.

	Safety	Machine Importance	Maint Cost	Failure Freq	Downtime Length	Component Access
Safety	1.00	2.00	6.00	3.00	3.00	7.00
Machine Importance	0.50	1.00	3.00	2.00	1.00	4.00
Maint Cost	0.17	0.33	1.00	1.00	0.20	1.00
Failure Freq	0.33	0.50	1.00	1.00	0.20	1.00
Downtime Length	0.33	1.00	5.00	5.00	1.00	3.00
Component Access	0.14	0.25	1.00	1.00	0.33	1.00

Table 2 - Pairwise Ranking of Measures

The matrix is normalized by dividing the values in each column by the sum of the column (A_{norm}). An approximate for w_{max} is calculated for each row by calculating the average of the rows of the normalized matrix.

A_{norm}	Safety	Machine Importance	Maint Cost	Failure Freq	Downtime Length	Component Access	w_i
Safety	0.404	0.393	0.353	0.231	0.523	0.412	0.386
Machine Importance	0.202	0.197	0.176	0.154	0.174	0.235	0.190
Maint Cost	0.067	0.066	0.059	0.077	0.035	0.059	0.060
Failure Freq	0.135	0.098	0.059	0.077	0.035	0.059	0.077
Downtime Length	0.135	0.197	0.294	0.385	0.174	0.176	0.227
Component Access	0.058	0.049	0.059	0.077	0.058	0.059	0.060

Table 3 – A_{norm} and W_{max} approximation

The original pairwise comparison matrix is multiplied by the weight matrix ($A\cdot w_{max}$) to derive the values used in the constancy index calculation.

A*wT	
Safety	2.459
Machine Importance	1.185
Maint Cost	0.370
Failure Freq	0.466
Downtime Length	1.412
Component Access	0.376

Table 4

These values are then used to compute the eigenvalue approximation (λ_{max}) using equations 1 and 2.

$$\frac{1}{n}\sum_{i=1}^{n}\frac{(A\cdot \underline{w})_i}{\underline{w}_i}=\left(\frac{1}{6}\right)\left\{\frac{2.459}{0.386}+\frac{1.185}{0.190}+\frac{0.370}{0.060}+\frac{0.466}{0.077}+\frac{1.412}{0.227}+\frac{0.376}{0.060}\right\}=6.215$$

Equation 1

$$CI=\frac{(\lambda_{\max})-n}{n-1}=\frac{6.215-6}{5}=0.043$$

Equation 2

Once the CI is calculated, it is compared to a random index (RI) to determine if the pairwise comparison inputs are consistent enough to yield analytically meaningful results. The RI table contains the values if the pairwise comparison matrix values were picked at random for n=1,2,…n. The RI values are shown in Table 5.

n	2	3	4	5	6	7	8	9	10
RI	0	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.51

Table 5 – Random Index Values

For a perfectly consistent decision maker, λ_{max} = n and CI = 0. In practice, results where CI = 0 are typically not attained during the pairwise comparison process. The CI / RI ratio results can be interpreted as follows:

- CI/RI < 0.10, consistency satisfactory
- CI/RI > 0.10, serious inconsistency exists (results may not be meaningful)

In the example, CI/RI = 0.043 / 1.24 = 0.035 < 0.10, so no serious inconsistency exists. Once the decision analyst has determined that the consistency is sufficient, the next step is to calculate the final weights from the matrix.

Eigenvector Calculations

The weights for the model are determined through the calculation of a priority vector. Some texts may refer to the priority vector as the principle eigenvector or Perron vector. The priority vector calculation can be done by successively squaring the pairwise comparison matrix until the normalized priority vector variation between iterations is sufficiently small.

$$A_0{}^2 = A_1,\ A_1{}^2 = A_2,\ A_{n-1}{}^2 = A_n$$

Equation 3

After squaring *A*, the eigenvector is calculated by summing the row values, then normalizing the resulting vector using equations 4 and 5.

$$W^i = \sum_{j=1}^{n} A_{ij}$$

Equation 4

$$W^i{}_{norm} = \frac{W^i}{\sum_{i=1}^{n} W_i}$$

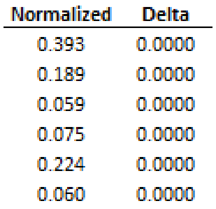
Equation 5

Saaty details a more rigorous approach to the calculation while asserting that the eigenvector solution for the priority vector calculation provided the best solution [4]. His work showed that the following two conditions must be met when computing the eigenvector:

1. *A priority vector must reproduce itself on a ratio scale meaning that it should both remain invariant under multiplication by a positive constant c, and*

2. *A priority vector should be invariant under hierarchic composition for its own judgment matrix so that one does not keep getting new priority vectors from that matrix.* [4, p. 86]

In this example, the procedure is repeated until the vector remains invariant to four decimal places. At this point the process stops and the priority vectors are recorded. Assessment of the criteria pairwise comparison matrix is shown in Table 6.



Normalized	Delta
0.393	0.0000
0.189	0.0000
0.059	0.0000
0.075	0.0000
0.224	0.0000
0.060	0.0000

Table 6

Final Prioritization and Identification of Preferred Alternative

Generically, for a problem with M alternatives and N criteria, the AHP requires the construction of N judgment matrices of order MxM (alternative preferences relative to criteria) and one judgment matrix of order NxN (for criteria weights) [7]. The previous section described the NxN criteria judgement matrix for criteria weights. The next step of the process involves the six MxM matrices for the alternatives. The process is repeated for each criterion, one at a time, to detail the decision maker's preferences with respect to the alternatives. For the example problem, the pairwise comparison is done with respect to preventive maintenance, corrective maintenance, and predictive maintenance for each of the six criteria. A single instance of the method is show for the criterion Safety in Table 4. The final priority vector, for the alternatives with respect to safety, is outlined at the bottom of table.

	Safety			Priorities	Normalized	Delta
	Prev	Corr	Pred			
Prev	1.0000	4.0000	0.3333	5.3333	0.2851	
Corr	0.2500	1.0000	0.1250	1.3750	0.0735	
Pred	3.0000	8.0000	1.0000	12.0000	0.6414	
	4.2500	13.0000	1.4583	18.7083		
Round 1 Eigenvector						
	3.0000	10.6667	1.1667	14.8333	0.2556	-0.0295
	0.8750	3.0000	0.3333	4.2083	0.0725	-0.0010
	8.0000	28.0000	3.0000	39.0000	0.6719	0.0305
				58.0417		
Round 2 Eigenvector						
	27.6667	96.6667	10.5556	134.8889	0.2560	0.0004
	7.9167	27.6667	3.0208	38.6042	0.0733	0.0007
	72.5000	253.3333	27.6667	353.5000	0.6708	-0.0011
				526.9931		
Round 3 Eigenvector						
	2296.0000	8022.9630	876.0880	11195.0509	0.2560	0.0000
	657.0660	2296.0000	250.7176	3203.7836	0.0732	0.0000
	6017.2222	21026.1111	2296.0000	29339.3333	0.6708	0.0000
				43738.1678		

Table 7

The overall goal for this example problem is to determine the maintenance solution with the high priority for the system "Fast Valves". This is determined using Equation 6:

$$A^{i}{}_{AHP} = \sum_{j-1}^{n} a_{ij} w_j \qquad for\ i = 1,2,3 \ldots M$$

Equation 6

The equation is applied to the criteria and alternative weights and the final priority vector is determined. Results for the maintenance policy selection problem are detailed in Table 8. In this example, predictive maintenance is the preferred choice with a weight of 0.495 followed by preventative maintenance at 0.304 and corrective maintenance at 0.201.

	Safety	Machine Importance	Maint Cost	Failure Freq	Downtime Length	Componer
Preventive Maintenance	0.256	0.648	0.178	0.143	0.212	0.192
Corrective Maintenance	0.073	0.122	0.751	0.714	0.062	0.634
Predictive Maintenance	0.671	0.230	0.070	0.143	0.726	0.174

Criteria Weight Vector	
Safety	0.3930
Machine Importance	0.1893
Maint Cost	0.0591
Failure Freq	0.0747
Downtime Length	0.2242
Component Access	0.0598

Preventive Maintenance	0.304
Corrective Maintenance	0.201
Predictive Maintenance	0.495

Table 8 - Hierarchy of Alternatives

Conclusion

This paper provides a systematic methodology that can be applied to decision making that allows analysts to systematically guide the process and identify rational choices when faced with complex problem situations. The application of the AHP was used for evaluation of criteria and alternatives germane to maintenance policy selection decision making. The process included the synthesis of expert's perspectives and the use of both qualitative and quantitative inputs. In this analysis, assumptions and data were gathered based on data obtained through literature and applied to a fictional case. The AHP provides redundancy for preference assignment of criteria and alternatives and a mechanism to validate consistency. As with all complex system problems, selecting the best policy for system maintenance should consider systems concepts such as darkness (never knowing everything about complex systems) and emergence (properties of the whole are often not be predictable by analyzing the individual parts). A real world application requires customization of the decision analysis application based on not only preferences, but probabilities, available quantitative data, and technical specifications. As new data is introduced, models can be updated to understand the impact on the preferred choice. To get meaningful results, the process should be viewed through a systems lens considering context, environment, darkness, emergence, politics, and the risk attitudes of the decision makers involved.

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